

Discussion Paper Series

No. **189**
CSIS Discussion Paper

October, 2026

Transport improvement and urban decentralization: The roles of commuting costs, trade costs, and residential land

Takaaki Takahashi

(Center for Spatial Information Science, The University of Tokyo)

Transport improvement and urban decentralization: The roles of commuting costs, trade costs, and residential land

Takaaki Takahashi

Center for Spatial Information Science,
The University of Tokyo.
5-1-5, Kashiwa-no-ha, Kashiwa, Chiba 277-8568, Japan

October 2026

Abstract

This study examines how transport improvement affects the spatial distributions of population and commercial activities within a city. We develop a simple two-district monocentric city model with endogenous locations of workers and firms producing differentiated services. We show that a decrease in commuting costs decentralizes both population and service production, whereas a decrease in trade costs leaves the population distribution unchanged but may either centralize or decentralize service production. When transport improvement reduces both costs, population necessarily decentralizes, while service production may become more concentrated in the city center, depending on the relative attractiveness of the city center and the relative magnitudes of the reductions in the two costs.

Keywords: urban decentralization; transport costs; residential land; service varieties; urban amenities

JEL Classification Numbers: R1; R4; R5

E-mail address: takaaki-t@csis.u-tokyo.ac.jp

I appreciate the comments from Se-il Mun, which play an important role in shaping the paper into the current form. I also thank several participants in various workshops and conferences for helpful comments. This research is partly supported by Grants in Aid for Research of the Ministry of Education, Science and Culture in Japan (18H00841).

1 Introduction

For the last several decades, many cities, in particular those in the United States and Japan, have been experiencing suburbanization of population, accompanied by decentralization of commercial activities. This process has various negative impacts on the economy of a city. For one thing, considerable amounts of existing building stock and infrastructure are left unused, resulting in inefficient resource allocation. For another, this process generates various types of negative externalities, such as an increase in environmental burdens mainly due to heavier automobile use and congestion in the suburbs, an increase in crime, and the breakdown of social networks and communities in a city center. What is more, the decentralization of commercial activities deprives the city center of agglomeration economies. Examining the mechanism behind that process and seeking remedies for such problems is an urgent task facing cities.¹

A fundamental question is what factors promote or counteract the decentralization process. One of the important forces to consider in answering this question is agglomeration economies. In particular, it is widely recognized that the concentration of differentiated products, especially the services known as *urban amenities*, such as those provided by retail shops, bars, cafes, restaurants, theaters, and concert halls, is an important source of agglomeration economies (see Glaeser et al., 2001).² Paying special attention to agglomeration economies, several studies examine the spatial distributions of both consumers (residences) and firms (workplaces) within a city. Notable examples are Picard and Tabuchi (2013), Malykhin and Ushchev (2018), Mossay et al. (2020), and Aizawa and Kono (2025). They examine the locations of residences and workplaces without presuming that firms are concentrated in the city center. They demonstrate that a city can exhibit diverse spatial patterns of residential and business land use when consumers' and firms' location decisions are interdependent through linkages, such as demand and vertical linkages, and externalities, such as technological spillovers. The monocentric pattern, in which all firms are concentrated in a certain contiguous area around the city center, is one of their equilibrium patterns.³ In this sense, their frameworks are more general than the classical monocentric framework developed by Alonso (1964), Mills (1967), and Muth (1969) (henceforth, the AMM models). At the same time, the above-mentioned studies incorporating agglomeration economies abstract from commuting costs borne by consumers and their decisions regarding the amount of land to

¹One popular approach is to make a city more compact. For studies on compact cities, see Burton (2002), Tsai (2005), Gaigné et al. (2012), OECD (2012), Lee et al. (2015), Ahlfeldt and Pietrostefani (2017), and Koziatek and Dragičević (2019), for example.

²Albouy and Stuart (2020) also find that the production of nontradable products, represented by urban amenities, plays a key role in household location decisions.

³Some researchers, such as Fujita (1988), use a narrower definition for the term "monocentric" pattern: According to this definition, it is a pattern in which no consumer is located in the central area where firms are located, that is, a pattern with no mixed use in that area.

consume for housing. However, those two elements are cornerstones of the AMM models in that the distribution of consumers is determined so as to balance the negative impact of an increase in commuting costs against the positive impact of an increase in land consumption as a consumer moves farther away from the city center. This study attempts to answer the above question while explicitly incorporating these two elements in addition to agglomeration economies.

For that purpose, we construct a simple two-region model of a city, interpreting the two regions as two districts within the city: the “city center” and the “suburb.” City residents work in the city center, and thus suburban residents pay commuting costs. The reason for returning to a monocentric city approach is to obtain an intuitive understanding of the roles of commuting costs and consumers’ demand for land in the determination of urban spatial structure. Such an understanding would be difficult to obtain in a more general framework. For instance, suppose that a nontrivial number of firms and consumers choose their locations. Then, without introducing fairly strong assumptions, it would be intractable to formulate commuting patterns and the associated commuting costs. Because we intend neither to replicate reality nor to conduct counterfactual analysis, a model that is as simple as possible is desirable for uncovering the workings of several key factors. Returning to our model, city residents consume land for housing, a homogeneous good, and nontradable differentiated services, while paying two kinds of transport costs: commuting costs, if any, and “trade costs” to acquire differentiated services produced in the respective districts. Firms in the differentiated service sector freely choose their locations, thereby determining the number of service varieties produced in each district.

One important motivation behind this study concerns policies aimed at deterring the decentralization of commercial activities. One typical arena for such policies is public transport. Since Calthorpe (1993) proposed the idea of transit-oriented development (TOD) in the field of urban planning, a number of cities around the world have introduced brand-new light rail transit (LRT) and subway lines and modernized existing lines.⁴ One important aim of these attempts is to fully exploit agglomeration economies by concentrating economic activities in the city center (Venables, 2007).⁵ More specifically, improvements in public transport are expected to encourage suburban consumers to visit the city center more often and to bring residents back there, which will have a further positive impact on businesses in the center. One example is found in Toyama City, Japan.⁶ Since the late 2000s, the city has introduced a set of policies to revitalize the city center by shifting

⁴Another term related to these policies is smart growth. The American Planning Association (2012) states that “integrating land use and transportation planning to accommodate more than just the automobile and to provide increased transportation choices, including mass transit, bicycling, and walking is a hallmark of Smart Growth.”

⁵The most important reason is said to be the alleviation of environmental burdens. By encouraging residents to switch their mode of transport from automobiles to public transport, we can reduce emissions of CO₂ and other air pollutants. For instance, Baum-Snow and Kahn (2000, 2005) show that average transit usage in a census tract decreases with the distance between the tract and a transit station.

⁶For more details on the policies implemented in Toyama city, see Iwata and Kondo (2021).

residences from the suburbs and promoting the construction of new commercial buildings in a target zone. Along with financial support for housing construction in that zone, the city has drastically improved its public transport system by constructing a new tram line and renovating an old transit line, among other measures. Those policies have actually had some effect in deterring the decentralization of commercial activities and population.⁷

Nonetheless, most existing economic studies have not found evidence supporting such policies.

On the one hand, as far as the distribution of population is concerned, many empirical studies reveal that investment in public transport tends instead to cause decentralization.⁸ For one thing, Ahlfeldt and Wendland (2011) find that the land value curve, which graphs land values as a function of distance from the city center, became flatter in Berlin from 1890 to 1936. They show that the main reason for this change was the reduction in commuting time brought about by the expansion of the railway network. Moreover, Garcia-López (2012), for railroad improvement in the Barcelona Metropolitan Region; Mayer and Trevien (2017), for the expansion of the Regional Express Rail in the greater Paris area; and Gonzalez-Navarro and Turner (2018), for subway construction around the world, all show that improvements in public transport systems lead to the decentralization of population. Finally, Heblich et al. (2020) examine changes in the spatial distributions of population and employment in London from the nineteenth century to the early twentieth century. Using reduced-form event-study analyses and rigorous structural estimation, they find that the invention and development of steam railways led to a large-scale spatial separation of workplaces and residences, which resulted in the decentralization of population. Furthermore, these empirical findings are consistent with the conventional AMM models, although these models do not allow commercial activities to locate outside the city center (for instance, see Wheaton, 1974).⁹ According to the AMM models, a fall in commuting costs brought about by transport investment raises consumers' disposable income. However, because their utility level must remain unchanged in a small open city, land rents need to rise. This geographically expands the city, which extends over the area where urban land rent exceeds a given level of agricultural rent. The same conclusion can be derived for a closed city.¹⁰

⁷For theoretical and empirical evaluations of such policies, see, for instance, Aizawa and Kono (2025) and Fujishima et al. (2025).

⁸The literature is unanimous that improvements in road transport lead to the decentralization of both residence and employment. For instance, Baum-Snow finds that the construction of new freeways causes the decentralization of population and, to a lesser degree, employment within a city (Baum-Snow, 2007b; 2020). Garcia-López obtains similar results for Barcelona Metropolitan Region (Garcia-López, 2012) and for European cities (Garcia-López, 2019). See Redding and Turner (2015) for a review of studies on investment in not only public transport but also road transport.

⁹For the reviews of the later development of the AMM models, see Anas et al. (1998), Brueckner (2001), Glaeser and Kahn (2004), and Duranton and Puga (2015), among others.

¹⁰Suppose that land rents decline in a closed city. Then, each resident, with higher disposable income, would consume a larger amount of land, and therefore, the city with a fixed population would expand. This is a contradiction, because lower urban land rents imply a smaller urban area. Hence, land rents should rise, and therefore, the city expands.

On the other hand, with respect to the distribution of commercial activities, or, more broadly, employment, few empirical studies have provided a clear-cut answer to the question of whether transport improvement deters their decentralization. One exception is the work by Heblich et al. (2020) cited earlier, which shows that employment became more concentrated in central areas of London from the nineteenth century to the early twentieth century. In addition, to the best of my knowledge, no theoretical study has examined this question. The second purpose of this study is to fill this gap. More specifically, using our model sketched earlier, we theoretically show that transport improvement may centralize, rather than decentralize, a city, depending on parameter values. The reason for this centralization is simple. The increase in consumers' disposable income due to the reduction in transport costs raises their demand for differentiated services. Because of agglomeration economies, newly entering retail firms choose to locate in the district where more service varieties are produced, which is usually the city center. In policy debates, this is probably one of the strongest arguments for the claim that transport improvement leads to a more centralized city.¹¹

The main results of this study are summarized as follows. We show that there exists a unique stable equilibrium under a certain condition. The equilibrium numbers of service varieties produced in the city center and the suburb, as well as their equilibrium populations, depend on several key parameters, such as the commuting costs paid by suburban consumers, the amount of residential land available in each district, and the strength of the preference for land. First, both the number of varieties produced in the city center and its population decrease as commuting costs and the amount of residential land available there decrease. Second, trade costs do not affect the population distribution. However, they do affect the numbers of varieties produced in the two districts. A decrease in trade costs increases the number of varieties produced in the city center if that district is endowed with a sufficiently large amount of residential land relative to the suburb, if commuting costs are sufficiently high, or if the preference for land is modest. An immediate implication is that improvement in public transport may succeed in centralizing a city in terms of the distribution of variety production, but only if the above conditions are satisfied.

As mentioned earlier, we consider a monocentric city so as to obtain an intuitive understanding of the roles of commuting costs. There are several additional justifications for doing so. First, our motivation to examine the effects of transport infrastructure investment on spatial structure provides several of these justifications. For one thing, in many cities, LRT, subway, and commuter train lines are planned to connect the city center and suburbs directly, or as feeder lines to a radial

¹¹ Another argument advanced by a number of researchers is based on the possibility of choosing between public transport and automobiles (Anas and Moses, 1979; LeRoy and Sonstelie, 1983; Sasaki, 1989, 1990; Gin and Sonstelie, 1992; DeSalvo and Huq, 1996; and Baum-Snow, 2007a, to name a few). According to these studies, residents who switch from automobiles to public transport after an improvement in public transport tend to relocate closer to the city center. This weakens the centrifugal force generated by the improvement and may overturn the decentralization result.

trunk line leading to the center. This is even more likely to be the case if we consider smaller cities that have only a few transit lines, or the lines that were constructed in the early stages of the development of larger cities. In these cases, we regard transport infrastructure as being designed on the basis of a monocentric city. Furthermore, if we take into account the endogenous determination of the locations of production activities, it becomes necessary to discuss the effects of transport infrastructure investment on the transport costs involved in production, which are likely to differ from those involved in consumption.¹² Analyzing a city with two different types of transport costs would require not only considerable elaboration but also some arbitrary assumptions about the specification of those costs. Second, although the tendency toward decentralization has been accelerating in many cities worldwide, a substantial number of cities are still monocentric. Third, this fact often leads policymakers to discuss transport infrastructure investment on the premise of a monocentric city. Fourth, the monocentric city is supported as one of the equilibrium outcomes in the works mentioned earlier. For instance, Mossay et al. (2020) show that a monocentric city emerges in equilibrium when commuting costs are sufficiently low and/or demand and vertical linkages are sufficiently strong. Thus, we can interpret our framework as a special case of a more general model.

Among other related studies, four strands of literature are worth noting. First, some researchers take alternative approaches that do not directly rely on agglomeration economies. They include Fujita and Ogawa (1982), Fujita and Thisse (1986), Fujita (1988), Lucas and Rossi-Hansberg (2002), Konishi (2005), and Dodds and Dubrovinsky (2015). Second, another strand of literature, represented by Henkel et al (2000), Arakawa (2006), and Tabuchi (2009), discusses the spatial distribution of retail firms for a *given* distribution of consumers. Third, Tabuchi (1998) and Murata and Thisse (2005) consider two cities to examine whether and when differentiated products agglomerate in one city. Their interest lies in the emergence of a core-periphery pattern, rather than in the internal spatial structure of each city. Fourth, this study emphasizes the importance of the amount of residential land available in each district in determining the spatial distributions of consumers and firms. This is consistent with a strand of studies that underscore the effects of the amount of developable land on housing markets (Saiz, 2010; Paciorek, 2013; Hilber and Vermeulen, 2016; Baum-Snow and Han, 2024).

The rest of the paper consists of eight sections. In Section 2, the model is presented. Section 3 discusses the conditions that an equilibrium must satisfy. In Section 4, we derive the equilibrium. Section 5 shows the existence, uniqueness, and stability of the equilibrium. From Sections 6 to

¹²For instance, in the works of Picard and Tabuchi (2013) and Mossay et al. (2020), the same varieties of differentiated products are consumed as final goods and used in production as intermediate inputs. Thus, the costs of shipping the varieties as final goods are the same as those of shipping them as intermediate inputs. This simplification may not be innocuous when we study the impacts of transport infrastructure investment.

Section 8, we conduct comparative statics. In Section 6, we examine how investment in transport infrastructure affects the spatial distributions of consumers and firms. Section 7 analyzes the impacts of changes in the endowment of residential land in each district. The effects of changes in preference are discussed in Section 8. Section 9 concludes.

2 Basic framework

We consider a city that consists of two districts, a “city center,” or district 1, and a “suburb,” or district 2. The two districts are geographically separate from each other. Furthermore, each district extends over a geographical area and is endowed with S_i units of residential land ($i = 1, 2$). In this paper, we concentrate on the case in which the areas of residential land and commercial land in each district are fixed. This is because changing land use takes some time, while we are interested in the relatively short run. In addition, we limit ourselves to the case in which the amount of residential land in the city center does not exceed that in the suburb, that is, $S_1 \leq S_2$.

Two kinds of goods and services are produced in the city: differentiated products and a homogeneous composite good.

The differentiated products are services, represented by urban amenities such as those offered by cafes, bars, restaurants, theaters, concert halls, and museums. Following the convention in spatial economics, we refer to each differentiated service as a “variety.” They are nontradable in the following two senses. First, they cannot be shipped across the city boundary. Second, to consume the services, consumers need to visit the district where they are supplied.

In contrast, the homogeneous composite good is tradable: it can be shipped at no cost within the city and across the city boundary. It is taken as the numéraire. Furthermore, we assume that the good is produced in the city center, in the same spirit as conventional monocentric city models. Although this monocentric assumption is sometimes criticized for not adequately describing the spatial structure of contemporary cities, we nevertheless adhere to it for the following three reasons. First, it makes the model simple enough to enable us to interpret the derived results intuitively, thereby providing new insights. Second, it helps us use a number of existing results of the AMM models as a benchmark. Third, it prevents the analysis of the effects of transport infrastructure investment from becoming too complicated. If we allowed for more than one center, it would become necessary to identify transport routes between all pairs of centers. In that case, any plan of infrastructure investment would have to specify how to allocate them among such transport routes. Without imposing additional ad hoc assumptions, therefore, the analysis would become intractable and fail to yield any general insights.

In the city, there are workers, whose total population is fixed and normalized to unity. They

live in one of the two districts. The populations of workers in the respective districts are denoted by l_1 and l_2 , respectively ($l_1 + l_2 = 1$). Since the total population is normalized to unity, they also represent the population shares of the two districts. Each worker inelastically supplies one unit of labor and engages in the production of the composite good to earn a fixed wage, w . As in the AMM models, furthermore, we assume the presence of absentee landlords, which implies that a worker's income consists solely of the wage. Moreover, because the production of the composite good takes place in the city center, all workers need to commute to that district.

Workers consume varieties of services, a piece of land for housing, and the composite good. They have identical preferences. The preferences for a worker living in district j is represented by the following utility function ($j \in \{1, 2\}$):

$$u_j = k C_j^\alpha s_j^\beta z_j^{1-\alpha-\beta}, \quad (1)$$

where s_j and z_j are the size of land and the amount of the composite good, respectively. Furthermore,

$$C_j \equiv \left[\int_{x \in \mathbb{N}_1} c_{1j}(x)^{\frac{\sigma-1}{\sigma}} dx + \int_{x \in \mathbb{N}_2} c_{2j}(x)^{\frac{\sigma-1}{\sigma}} dx \right]^{\frac{\sigma}{\sigma-1}}$$

is the aggregate consumption of differentiated products. $c_{ij}(x)$ denotes the consumption of variety x produced in district i , and $\mathbb{N}_i$ is the set of varieties produced in that district, and its range is endogenously determined. Here, $\sigma > 1$ is the elasticity of substitution, while $\alpha \in (0, 1)$, $\beta \in (0, 1)$, and $1 - \alpha - \beta \in (0, 1)$ are the shares of expenditure on differentiated products, land, and the composite good, respectively. Finally, $k \equiv \left[\alpha^\alpha \beta^\beta (1 - \alpha - \beta)^{(1-\alpha-\beta)} \right]^{-1}$ is a positive constant. Note that the utility function reduces to that of the AMM models when $\alpha = 0$.

Workers incur two kinds of transport costs. First, recall that to consume a particular variety of services, they need to visit the district where it is supplied. This may require them to pay transport costs, which we refer to as *trade costs*. Moreover, workers may pay *commuting costs* when they commute to the city center. One simplifying assumption is that no transport costs are incurred if workers travel only within the same district. More specifically, if workers consume a variety produced in their home district, they do not have to pay trade costs, and if they live in the city center, they do not have to pay commuting costs.

We assume that the trade costs take an iceberg form, as in much of the literature in international trade theory and spatial economics. If workers living in district j consume one unit of a variety produced in district i , they need to purchase τ_{ij} units of it ($\tau_{ij} \geq 1$). Here, $\tau_{ij} - 1$ units melt away in the course of transport. Recall that when they visit a firm in their home district, they pay no trade costs, which implies that $\tau_{ij} = 1$ if $i = j$. However, when they visit the other district, they

need to pay trade costs: $\tau_{ij} = \tau > 1$ if $i \neq j$.

Furthermore, the commuting costs are also assumed to take an iceberg form. Workers living in district j effectively deliver 1 unit of labor to the city center if they supply $\mu_j > 1$ units of labor: $\mu_j - 1$ units of labor are dissipated in commuting. Thus, the effective wage rate is equal to w/μ_j and the commuting costs in monetary terms are given by $w - w/\mu_j$, which is proportional to w . One justification for this proportionality is that time costs, which can be valued at the wage rate as an opportunity cost, constitute a major component of commuting costs in addition to a monetary costs. Our formulation is analogous to that of Murata and Thisse (2005) in that disposable income after paying commuting costs is proportional to the wage rate. Since workers living in the city center pay no commuting costs, we have $\mu_1 = 1$. However, those living in the suburb pay the commuting costs: $\mu_2 = \mu > 1$. In what follows, we will use, with a slight abuse of terminology, the terms “trade costs” and “commuting costs” to refer to τ and μ , respectively.

Now, we are ready to write down a worker’s budget constraint. Let $p_i(x)$ be the price of variety x produced in district i and r_j be the rent of residential land in district j . The budget constraint for a worker living in district j is given by

$$\int_{x \in \mathbb{N}_1} \tau_{1j} p_1(x) c_{1j}(x) dx + \int_{x \in \mathbb{N}_2} \tau_{2j} p_2(x) c_{2j}(x) dx + r_j s_j + z_j = \frac{w}{\mu_j} \quad (2)$$

for $j \in \{1, 2\}$.

Finally, we turn to the production side.

First, the composite good is produced from labor using a constant-returns-to-scale technology: one unit of labor produces one unit of the good. This implies that the wage rate is equal to one. Here, we are assuming that no land is required to produce the composite good. This follows the convention of a spaceless city center in the AMM models.

Second, the differentiated products are produced from the composite good using an increasing-returns-to-scale technology. Producing each unit of a variety requires a units of the composite good as a variable input. In addition, producing each variety requires f units of the good as a fixed input. Here, we abstract from the possibility that firms use land as a fixed input, which implies that the fixed cost is independent of the location of production. The profit of a firm that produces variety x in district i is given by

$$\pi_i = [p_i(x) - a] q_i(x) - f \quad (3)$$

for $i \in \{1, 2\}$, where $q_i(x)$ is the total demand for that variety.

3 Conditions for equilibrium

This section discusses five conditions that an equilibrium must satisfy.

i) Utility maximization by workers

First, workers maximize their utility, (1), subject to their budget constraint (2). A district- j worker's consumption of the x -th variety produced in district i is equal to

$$c_{ij}(x) = \frac{\alpha}{\mu_j} \frac{P_j^{\sigma-1}}{[\tau_{ij} p_i(x)]^\sigma} \quad \text{for } x \in \mathbb{N}_i, \quad (4)$$

where $P_j \equiv \left[\sum_{i=1,2} \left\{ \int_{x \in \mathbb{N}_i} \tau_{ij}^{1-\sigma} p_i(x)^{1-\sigma} dx \right\} \right]^{\frac{1}{1-\sigma}}$ is a price index for differentiated products in district j . Furthermore, the worker's demands for land and the composite good are given by

$$s_j = \frac{\beta}{r_j \mu_j} \quad \text{and} \quad z_j = \frac{1 - \alpha - \beta}{\mu_j}, \quad (5)$$

respectively.

ii) Profit maximization by firms in the differentiated product sector

The second condition is that firms in the differentiated product sector maximize their profits (3). Equation (4) implies that the total demand for a variety produced by a district- i firm is given by

$$q_i(x) = l_1 \tau_{i1} c_{i1}(x) + l_2 \tau_{i2} c_{i2}(x) \quad \text{for } x \in \mathbb{N}_i. \quad (6)$$

Together with the assumption that firms take the price index as given, as is conventional in spatial economics, (3) and (6) yield the well-known solution to the profit maximization problem:

$$p_i(x) = p \equiv \frac{a\sigma}{\sigma-1} \quad \text{for any } x \in \mathbb{N}_i. \quad (7)$$

Here, note that the prices are equal across all varieties regardless of which district they are produced in.

iii) Market clearing of residential land

The third condition is that the residential land market clears in both districts. Since the total

demand for residential land in district j is $\beta l_j / (\mu_j r_j)$, the market clears if the land rent is equal to

$$r_j = \frac{\beta l_j}{\mu_j S_j} \quad (8)$$

for $j \in \{1, 2\}$.

iv) Firms' free entry and exit

The fourth condition is that firms in the differentiated product sector break even under free entry and exit. To obtain this condition, we need to derive the demand faced by each firm, $q_i(x)$. Since all varieties are sold at the same price, a district- j worker demands the same amount of every variety produced in a given district: $c_{ij}(x) = c_{ij}$ for any $x \in \mathbb{N}_i$. Let $\phi \equiv \tau^{1-\sigma} < 1$ be an inverse measure of trade costs, or "freeness of trade," which is frequently used in the literature on spatial economics. The price index in district j is reduced to $P_j = p N_j^{\frac{1}{1-\sigma}}$, where $N_j \equiv n_j + \phi n_{-j}$ represents the *effective number of varieties* accessible to district- j consumers, taking trade costs into account, or, more simply, *market access* ($j \in \{1, 2\}$, $-j \in \{1, 2\}$, and $-j \neq j$). Thus, (4) is reduced to

$$c_{ij}(x) = c_{ij} = \frac{\alpha}{p \mu_j \tau_{ij}^\sigma N_j} \quad \text{for any } x \in \mathbb{N}_i. \quad (9)$$

Here, note that the consumption levels are equal across all varieties produced in the same district. Furthermore, all firms in the same district face the same demand, that is, $q_i(x) = q_i$ for any $x \in \mathbb{N}_i$. It follows from (6) that

$$q_1 = \frac{\alpha}{\mu p} \left(\frac{\mu l_1}{N_1} + \frac{\phi l_2}{N_2} \right) \quad \text{and} \quad q_2 = \frac{\alpha}{\mu p} \left(\frac{\phi \mu l_1}{N_1} + \frac{l_2}{N_2} \right). \quad (10)$$

Finally, (3) implies that the profit of a district- i firm becomes 0 when

$$q_i = \frac{f}{p - a} \quad (11)$$

for $i \in \{1, 2\}$. This is the condition for zero profit.

For what follows, it is useful to consider the ratio of demands faced by firms in the two districts, $\hat{q} \equiv q_1/q_2$. In this paper, a circumflex is always used to denote the ratio of a variable for district 1 to that for district 2. The zero profit condition, (11), requires that $\hat{q} = 1$. Using (10), this requirement can be written as

$$\hat{q} = \frac{\mu \hat{l} + \phi \hat{N}(\hat{n})}{\phi \mu \hat{l} + \hat{N}(\hat{n})} = 1, \quad (12)$$

where $\hat{l} \equiv l_1/l_2$, $\hat{n} \equiv n_1/n_2$, and

$$\hat{N}(\hat{n}) \equiv \frac{N_1}{N_2} = \frac{\phi + \hat{n}}{1 + \phi\hat{n}}. \quad (13)$$

Equation (13) shows that the ratio of market access in the city center to that in the suburb increases with $\hat{n}$: A relative increase in the number of varieties produced in the city center increases market access more in that district than in the suburb. Furthermore, it is not surprising that the district that produces more varieties has greater market access:

$$\hat{N}(\hat{n}) \begin{cases} \geq \\ < \end{cases} 1 \quad \text{if} \quad \hat{n} \begin{cases} \geq \\ < \end{cases} 1. \quad (14)$$

Several remarks on $\hat{q}$ are worth adding. First, $\partial\hat{q}/\partial\hat{l} > 0$.¹³ The more workers concentrate in the city center, the greater the demand faced by a firm in that district. Second, $\hat{q}$ approaches ϕ as $\hat{l}$ goes to zero, and $1/\phi$ as $\hat{l}$ goes to infinity. Therefore, for a given value of $\hat{n} \geq 0$, $\hat{q}$ lies in the interval $(\phi, 1/\phi)$. The interval is not empty because $\phi < 1 < 1/\phi$ by construction. Finally, $\partial\hat{q}/\partial\hat{n} < 0$.¹⁴ When relatively more varieties are produced in the city center, firms in that district compete with each other more intensely, resulting in lower demand for each firm

v) Spatial equilibrium for workers

The last condition is that workers have no incentive to change their location, that is, the city is in spatial equilibrium.

To derive this condition, we compute a district- j worker's aggregate consumption of service varieties from (7) and (9):

$$C_j = \frac{\alpha}{\mu p} N_j^{\frac{1}{\sigma-1}}$$

for $j \in \{1, 2\}$. This, along with (5) and (8), yields the indirect utility of that worker:

$$v_j = \left(\frac{N_j^{\frac{1}{\sigma-1}}}{p} \right)^\alpha \left(\frac{S_j}{\beta l_j} \right)^\beta \left(\frac{1}{\mu_j} \right)^{1-\beta} \quad (15)$$

for $j \in \{1, 2\}$. This equation shows that the indirect utility depends on three factors, represented respectively by the three terms in parentheses. The first is the effective number of varieties that a worker can access. Greater market access means a lower price index. The second factor is the population of the worker's district of residence. Since the total amount of residential land in each district is fixed, the amount of land consumed per worker is determined by the district's popu-

¹³ $\frac{\partial\hat{q}}{\partial\hat{l}} = \frac{\mu(1+\phi\hat{n})(\phi+\hat{n})(1-\phi^2)}{[\phi\mu\hat{l}(1+\phi\hat{n})+\phi+\hat{n}]^2} > 0$.

¹⁴ $\frac{\partial\hat{q}}{\partial\hat{n}} = -\frac{\mu\hat{l}(1-\phi^2)^2}{[\phi\mu\hat{l}(1+\phi\hat{n})+\phi+\hat{n}]^2} < 0$.

lation. The last factor is income. Suburban workers' income is negatively related to commuting costs, μ_2 . Note that there is no income effect on land consumption. This is because a change in income is absorbed by a change in land rent, so that land consumption per capita remains constant, other things being equal. The absence of an income effect on land consumption is reflected in the fact that the term in the third pair of parentheses is raised to the power of $1 - \beta$.

It is useful to consider the ratio of the utility levels of workers in the two districts:

$$\hat{v} \equiv \frac{v_1}{v_2} = \mu^{1-\beta} \hat{N}(\hat{n})^{\frac{\alpha}{\sigma-1}} \left(\frac{\hat{S}}{\hat{l}} \right)^{\beta}, \quad (16)$$

where $\hat{S} \equiv S_1/S_2$ is the ratio of the available residential land area in the city center to that in the suburb, which is no greater than 1 by assumption. Equation (16) shows that $\hat{v}$ decreases with $\hat{l}$ and increases with $\hat{N}(\hat{n})$, as should be clear from the earlier intuitive explanation. In addition, since $\hat{N}(\hat{n})$ is increase in $\hat{n}$, $\hat{v}$ increases with $\hat{n}$. A relative increase in the number of varieties produced in the city center raises the relative utility of workers in that district.

The condition for the spatial equilibrium is given by

$$\hat{v} = 1. \quad (17)$$

4 Equilibrium

The equilibrium is a set of four variables, n_1, n_2, l_1 and l_2 , that satisfy the five conditions discussed in Section 3. Applying the first three conditions leaves us with four equations, namely, the two equations in (11), which give the fourth condition concerning zero profit, the equation in (17), which gives the last condition concerning spatial equilibrium, and $l_2 = 1 - l_1$ by definition. We can solve them for the four variables. However, it turns out that solving for $\hat{n}$ and $\hat{l}$ first and then deriving the equilibrium values of the four variables is much easier than solving directly for the four variables. Thus, we first use (12) instead of (11) and solve the system of two equations, (12) and (17).

Solving (12) enables us to obtain, for a given value of $\hat{n}$, the level of $\hat{l}$ that drives firms' profits to zero, denoted by $\hat{l}_{ZP}(\hat{n})$:

$$\hat{l}_{ZP}(\hat{n}) = \frac{\hat{N}(\hat{n})}{\mu}. \quad (18)$$

We call the locus of $\hat{l}_{ZP}(\hat{n})$ a *zero profit curve*, or a ZP curve. The ZP curve is upward sloping. An increase in the number of varieties produced in the city center reduces the profits of firms in that district because competition among them intensifies. To maintain zero profit, they must face

greater demand, which is brought about by an increase in the city center population.

Furthermore, for a given value of $\hat{n}$, (17) gives us the value of $\hat{l}$ that equalizes the utility levels of workers in the two districts, denoted by $\hat{l}_{UE}(\hat{n})$:

$$\hat{l}_{UE}(\hat{n}) = \hat{S}\mu^{\frac{1-\beta}{\beta}} \hat{N}(\hat{n})^\gamma, \quad (19)$$

where $\gamma \equiv \alpha / [\beta(\sigma - 1)] > 0$. The locus of (19) is referred to as a *utility equivalence curve*, or a UE curve. The UE curve is upward sloping.¹⁵ If more varieties are produced in the city center, workers will find living in that district more attractive. To restore utility equivalence, a worker in the city center must consume less land, which is brought about by an increase in the population of that district.

We impose one assumption regarding parameter γ .

Assumption 1

Preference parameters satisfy $\gamma < 1$, or, equivalently, $\alpha < \beta(\sigma - 1)$.

A number of empirical studies support Assumption 1. First, Bergstrand et al. (2013) find that the elasticity of substitution, σ , equals approximately 7. Anderson and van Wincoop (2004) review the literature and conclude that the parameter lies between 5 and 10. Second, a number of studies find that the share of expenditure on housing, β , is greater than 30% in most cases, although the share depends on the income levels of households and the cities where they live (see Quigley and Raphael, 2004; Gabriel and Painter, 2020; and Gong and Leung, 2024, among others). Thus, it seems innocuous to suppose, conservatively, that $\sigma \geq 5$ and $\beta \geq 0.3$ most conservatively. Then, we have $\beta(\sigma - 1) \geq 1.2$. If this is the case, $\alpha < \beta(\sigma - 1)$ is satisfied since $\alpha < 1$ by definition.

The intersection of the ZP and UE curves gives the equilibrium value of $(\hat{n}, \hat{l})$, denoted by $(\hat{n}^*, \hat{l}^*)$. Tedious computation yields

$$\hat{n}^* = \frac{\hat{A} - \phi}{1 - \phi\hat{A}} \quad \text{and} \quad \hat{l}^* = \frac{\hat{A}}{\mu}. \quad (20)$$

Here, $\hat{A} \equiv \left(\hat{S}\mu^{\frac{1}{\beta}}\right)^{\frac{1}{1-\gamma}} > 0$ is the ratio of A_1 to A_2 , where $A_i \equiv \left(S_i\mu_i^{-\frac{1}{\beta}}\right)^{\frac{1}{1-\gamma}}$ represents the *intrinsic attractiveness* of district i as a place to live ($i \in \{1, 2\}$). The attractiveness increases with S_i and decreases with μ_i . Greater affordability of residential land implies lower land rent, while higher commuting costs are associated with lower income. Moreover, γ affects the sensitivity of

¹⁵Note that $\frac{d\hat{l}_{UE}(\hat{n})}{d\hat{n}} = \frac{\gamma(1-\phi)(1+\phi)}{(\phi+\hat{n})(1+\phi\hat{n})} \cdot \hat{l}_{UE}(\hat{n}) > 0$.

the attractiveness to those two factors. High α , low β , and low σ lead to greater sensitivity. The parameter $\hat{A}$ does not depend on the spatial distribution of variety production.

It follows from (13) and (20) that in equilibrium, the ratio of market access equals the ratio of intrinsic attractiveness:

$$\hat{N}(\hat{n}^*) = \frac{\phi + \hat{n}^*}{1 + \phi\hat{n}^*} = \hat{A}. \quad (21)$$

Therefore, the market access of workers in the city center relative to that of workers in the suburb is greater when residential land is more abundant in the city center relative to the suburb and commuting costs are higher.

A necessary and sufficient condition for $\hat{n}^* > 0$ is given by¹⁶

$$\phi < \hat{A} < \frac{1}{\phi}. \quad (22)$$

In the next section, we will show that this is actually a necessary and sufficient condition for the existence of an equilibrium. Thus, in what follows, we limit our analysis to the case in which this condition is satisfied.

Assumption 2

$\phi, \hat{S}, \mu$ and preference parameters satisfy (22).

Given $\hat{n}^*$ and $\hat{l}^*$, we can uniquely determine the equilibrium values of all four variables, n_1 , n_2 , l_1 , and l_2 , denoted by n_1^* , n_2^* , l_1^* , and l_2^* , respectively. First, we obtain l_1^* and l_2^* from $\hat{l}^*$ using $l_1^* + l_2^* = 1$. To get n_1^* and n_2^* , however, we need to solve one of the two equations in (11) using the derived values of $\hat{n}^*$, l_1^* , and l_2^* .¹⁷ Tedious computation gives the following solutions:

$$(n_1^*, n_2^*) = \left(\frac{\theta(\hat{A} - \phi)}{\mu + \hat{A}}, \frac{\theta(1 - \phi\hat{A})}{\mu + \hat{A}} \right) \quad \text{and} \quad (l_1^*, l_2^*) = \left(\frac{\hat{A}}{\mu + \hat{A}}, \frac{\mu}{\mu + \hat{A}} \right), \quad (23)$$

where $\theta \equiv \alpha / [\sigma f(1 - \phi)] > 0$. It is straightforward to verify that all four variables are positive as long as Assumption 2 is satisfied.

¹⁶If (22) is satisfied, both the numerator and the denominator of $\hat{n}^*$ in (20) become positive. Therefore, (22) is a sufficient condition. On the other hand, there are two cases in which (22) is violated. First, suppose that $\hat{A} \leq \phi$, which implies that $\hat{A} \leq 1/\phi$ since $1/\phi > \phi$. Thus, the numerator is nonpositive and the denominator is nonnegative. Second, suppose that $\hat{A} \geq 1/\phi$, which implies that $\hat{A} \geq \phi$. Thus, the denominator is nonpositive and the numerator is nonnegative. In either case, $\hat{n}^*$ is nonpositive. Hence, (22) is a necessary condition.

¹⁷For instance, the equation in (11) for $i = 1$ gives

$$n_1^* = \frac{\alpha}{\mu\sigma f} \left(\frac{\mu l_1^*}{\phi + \hat{n}^*} + \frac{\phi l_2^*}{1 + \phi\hat{n}^*} \right) \hat{n}^* \quad \text{and} \quad n_2^* = \frac{\alpha}{\mu\sigma f} \left(\frac{\mu l_1^*}{\phi + \hat{n}^*} + \frac{\phi l_2^*}{1 + \phi\hat{n}^*} \right).$$

5 Properties of the equilibrium

In this section, we examine the existence, uniqueness, and stability of the equilibrium.

5.1 Existence and uniqueness

It is straightforward to demonstrate the existence and uniqueness of the equilibrium using the same framework.

Consider the vertical distance between the UE and ZP curves:

$$D(\hat{n}) \equiv \hat{l}_{UE}(\hat{n}) - \hat{l}_{ZP}(\hat{n}) = \frac{1}{\mu} \hat{N}(\hat{n}) \tilde{D}(\hat{n}),$$

$$\text{where } \tilde{D}(\hat{n}) \equiv \frac{\hat{A}^{1-\gamma}}{\hat{N}(\hat{n})^{1-\gamma}} - 1 = \hat{S} \mu^{\frac{1}{\beta}} \left(\frac{1 + \phi \hat{n}}{\phi + \hat{n}} \right)^{1-\gamma} - 1. \quad (24)$$

It follows from (24) that an equilibrium occurs at a value of $\hat{n}$ for which $D(\hat{n}) = 0$, or, equivalently, $\tilde{D}(\hat{n}) = 0$. If $\tilde{D}(\hat{n}) > 0$ for a given $\hat{n}$, then $\hat{l}_{UE}(\hat{n}) - \hat{l}_{ZP}(\hat{n}) > 0$. In that case, the relative population of the city center that equalizes utility levels is larger than the actual relative population, given that firms are distributed according to the zero profit requirement. Thus, workers will migrate from the suburb to the city center and, as a result, a wider range of varieties will be produced. In this sense, $\tilde{D}(\hat{n})$, which we call a *divergence function*, measures the potential for further agglomeration in the city center, which we refer to more briefly as its *agglomeration potential*.

Several properties of the divergence function are important. First, it is strictly decreasing. This is because $\hat{A}$ does not depend on $\hat{n}$ and $\hat{N}(\hat{n})$ is increasing.¹⁸ This implies that the upward-sloping ZP curve is steeper than the upward-sloping UE curve at any $\hat{n}$. Second, the divergence function takes the value $(\hat{A}/\phi)^{1-\gamma} - 1$ at $\hat{n} = 0$. Third and finally, it approaches $(\hat{A}\phi)^{1-\gamma} - 1$ as $\hat{n}$ goes to positive infinity. Fig. 1 depicts a typical divergence function.¹⁹ Because the curve is downward sloping, the necessary and sufficient condition for an equilibrium with $\hat{n}^* \in (0, \infty)$ to exist is that both inequalities $(\hat{A}/\phi)^{1-\gamma} - 1 > 0$ and $(\hat{A}\phi)^{1-\gamma} - 1 < 0$ are satisfied. This is exactly the condition given in (22). Furthermore, since $\tilde{D}(\hat{n})$ is strictly decreasing, $\hat{n}^*$ is unique if it exists.

————— Insert Fig. 1 around here. —————

So far, we have established that $\hat{n}^*$ exists and is unique, provided that Assumption 2 is satis-

¹⁸Indeed, we have $\frac{d\tilde{D}(\hat{n})}{d\hat{n}} = -(1-\gamma)(1+\phi)(1-\phi)\hat{A}^{1-\gamma}(\phi\hat{n}+1)^{-\gamma}(\hat{n}+\phi)^{\gamma-2} < 0$ as long as $\gamma < 1$.

¹⁹It is readily verified that the divergence function is convex.

fied. Recall that all four equilibrium variables are uniquely determined if $\hat{n}^*$ is given. Therefore, the existence of a unique $\hat{n}^*$ implies the existence of a unique equilibrium. Thus, we have established the following proposition.

Proposition 1

There exists an equilibrium if and only if Assumption 2 is satisfied. Furthermore, the equilibrium is unique if it exists.

5.2 Stability

Recall that one of the equilibrium conditions is utility equivalence between workers in the two districts. This implicitly assumes that workers migrate between the districts according to the utility difference between the two districts. From this perspective, our equilibrium can be interpreted as a steady state of a dynamical system of migration. Under this interpretation, the steady state must be stable for the comparative statics analysis that we conduct in the next section. Thus, in this subsection, we examine the stability of the equilibrium.

Suppose that workers migrate gradually because migration involves some frictional costs. As is common in the literature on spatial economics, we assume simple replicator dynamics:

$$\frac{dl_1}{dt} = \varepsilon(v_1 - v_2),$$

where dl_1 / dt is the time derivative of l_1 and ε is an adjustment speed. We say that an equilibrium is stable if $v_1 - v_2$ decreases with l_1 in a neighborhood of $v_1 - v_2 = 0$. Note that $v_1 - v_2$ decreases with l_1 if and only if $\hat{v}$ decreases with $\hat{l}$.²⁰ Therefore, an equilibrium is stable if

$$\frac{d\hat{v}}{d\hat{l}} < 0 \tag{25}$$

at the equilibrium.

We consider two scenarios regarding how workers anticipate the effects of their migration on the ranges of varieties produced in the two districts.

The first scenario assumes “myopic” workers, who do not take into account the changes in the ranges of varieties brought about by their migration. This may be because they lack sufficient knowledge or information to fully anticipate such changes, or because firm entry and exit take

²⁰This is because $\frac{d\hat{v}}{d\hat{l}} = \frac{l_2^2}{v_2} \left(\frac{dv_1}{dl_1} - \frac{dv_2}{dl_1} \right)$ provided that $v_1 = v_2$.

longer than the migration process, so that the ranges of varieties produced adjusted only slowly. Because myopic workers assume that $\hat{n}$ is constant, (25) implies that the equilibrium is stable if $\partial\hat{v}/\partial\hat{l}|_{\hat{n} \text{ given}} < 0$ at the equilibrium. It immediately follows from (17) that

$$\left. \frac{\partial\hat{v}}{\partial\hat{l}} \right|_{\hat{n} \text{ given}} = -\frac{\beta}{\hat{l}} < 0,$$

where the derivative is evaluated at $\hat{v} = 1$. Therefore, the equilibrium is always stable in this scenario.

The second scenario considers “far-sighted” workers, who take into account the resulting changes in variety ranges brought about by their migration. One plausible hypothesis is that they expect the ranges to change to satisfy the zero profit requirement. In other words, they expect $\hat{n}$ to change along the ZP curve according to (18). Some computation yields

$$\frac{d\hat{v}}{d\hat{l}} = \left. \frac{\partial\hat{v}}{\partial\hat{l}} \right|_{\hat{n} \text{ given}} + \frac{d\hat{n}}{d\hat{l}} \frac{d\hat{v}}{d\hat{n}} = \frac{1}{\hat{l}} \left(\frac{\alpha}{\sigma-1} - \beta \right),$$

where all the derivatives are evaluated at $\hat{v} = 1$. This derivative becomes negative as long as Assumption 1 is satisfied.

We have established the following result.

Proposition 2

- i) When workers are myopic, the equilibrium is always stable.*
- ii) When workers are far-sighted, the equilibrium is stable if Assumption 1 is satisfied.*

To sum up, under Assumption 1, the equilibrium is stable whether workers are myopic or far-sighted.

6 Effects of decreases in transport costs

Having verified that the equilibrium exists, is unique, and is stable, we can now conduct comparative statics analysis. To begin, we answer whether the transport infrastructure investments succeed in revitalizing the city center, which is one of the main interests of this paper. For that purpose, we examine how transport costs affect the equilibrium distributions of firms and workers between the two districts.

6.1 Effects of a decrease in commuting costs

To begin, suppose that commuting costs decrease as a result of improvements in transport infrastructure. The following results immediately follow from (23).

Proposition 3

- i) $\partial n_1^* / \partial \mu > 0$ and $\partial n_2^* / \partial \mu < 0$.
- ii) $\partial l_1^* / \partial \mu > 0$ and $\partial l_2^* / \partial \mu < 0$.

In other words, as commuting costs decrease, the numbers of both firms and workers decrease in the city center and increase in the suburb. This result is consistent with the results obtained in the AMM models.

To understand the reason, it is useful to note that both $\hat{n}^*$ and $\hat{l}^*$ fall as commuting costs decrease. The changes in these relative variables can be explained intuitively by the shifts in the ZP and UE curves. First, the ZP curve shifts upward since $\partial \hat{l}_{ZP}(\hat{n}) / \partial \mu < 0$. This is because the profits of suburban firms increase as suburban workers' disposable income rises. To equalize firms' profits in the two districts, therefore, the number of suburban workers relative to that of city center workers needs to decrease. Second, the UE curve shifts downward since $\partial \hat{l}_{UE}(\hat{n}) / \partial \mu > 0$. The decrease in commuting costs affects the utility level of suburban consumers through two channels. First, it enhances their disposable income, which has a favorable effect on their utility. Second, this increase in disposable income raises land rent in the suburb, which has an adverse effect on their utility. However, it turns out that the former effect dominates the latter, and, therefore, the relative utility of suburban consumers rises. To restore utility equivalence, the suburban population needs to increase, which results in a downward shift of the UE curve. In Fig. 2, the ZP curve shifts from the thick solid ZP° curve to the thin solid ZP' curve, while the UE curve shifts from the thick dashed UE° curve to the thin dashed UE' curve. The intersection of the ZP and UE curves moves from point E° to E' . That is, the equilibrium values of $\hat{n}$ and $\hat{l}$ change from $\hat{n}^{*\circ}$ to $\hat{n}^{*l}$ and from $\hat{l}^{*\circ}$ to $\hat{l}^{*l}$, respectively. Since the two curves have positive slopes and the ZP curve is steeper than the UE curve at the equilibrium, E' is located to the southwest of E° . Therefore, both $\hat{n}^*$ and $\hat{l}^*$ decrease.

————— Insert Fig. 2 around here. —————

6.2 Effects of a decrease in trade costs

The next task is to examine the effects of a decrease in trade costs, that is, an increase in ϕ .

First, changes in n_1^* and n_2^* are given by

$$\frac{\partial n_1^*}{\partial \phi} = \frac{\theta(\hat{A} - 1)}{(1 - \phi)(\mu + \hat{A})} \quad \text{and} \quad \frac{\partial n_2^*}{\partial \phi} = -\frac{\partial n_1^*}{\partial \phi}, \quad (26)$$

respectively. (26) implies that the signs of the derivatives depend on the value of $\hat{A}$. Note that $\hat{A} \begin{cases} \geq \\ < \end{cases} 1$ if and only if $\hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}$. Second, (23) shows that neither l_1^* nor l_2^* depends on ϕ . These observations immediately lead to the following results.

Proposition 4

- i) $\frac{\partial n_1^*}{\partial \phi} \begin{cases} \geq \\ < \end{cases} 0$ and $\frac{\partial n_2^*}{\partial \phi} \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}$.
- ii) $\frac{\partial l_1^*}{\partial \phi} = \frac{\partial l_2^*}{\partial \phi} = 0$.

Thus, as trade costs decrease, the number of varieties produced in the city center increases and that in the suburb decreases if any of the following conditions holds: the city center is endowed with a sufficiently large amount of residential land relative to the suburb, the commuting costs are sufficiently high, or the preference for land is modest. Conversely, if none of these conditions holds, fewer varieties will be produced in the city center and more in the suburb. In addition, note that the equilibrium ratio of the numbers of varieties, $\hat{n}^*$, moves in the same direction as n_1^* because n_1^* and n_2^* move in opposite directions. Moreover, the population distribution over the two districts does not change.

Note that it follows from (21) that

$$\hat{n}^* \begin{cases} \geq \\ < \end{cases} 1 \quad \text{if} \quad \hat{A} \begin{cases} \geq \\ < \end{cases} 1 \quad \text{or, equivalently, if} \quad \hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}. \quad (27)$$

Therefore, as ϕ rises, n_1^* increases and n_2^* decreases when $\hat{n}^* > 1$, while n_1^* decreases and n_2^* increases when $\hat{n}^* < 1$. In other words, as trade costs decrease, more varieties will be produced in a district that initially produces more varieties than the other district. Conversely, fewer varieties will be produced in a district that initially produces fewer varieties than the other district. In this sense, the decrease in trade costs has a snowball effect that enlarges the spatial divergence in variety production. This is perhaps not surprising considering that the literature on spatial economics emphasizes that a decrease in trade costs reinforces agglomeration forces.

To gain an intuitive understanding of the results concerning the changes in $\hat{n}^*$ and $\hat{l}^*$, it is

helpful to examine how the ZP and the UE curves shift as ϕ increases. Note that

$$\frac{\partial \ln \hat{l}_{ZP}}{\partial \ln \phi} = \frac{\phi(1 + \hat{n})(1 - \hat{n})}{\hat{N}(\hat{n})(1 + \phi\hat{n})^2} \quad \text{and} \quad \frac{\partial \ln \hat{l}_{UE}}{\partial \ln \phi} = \frac{\phi\gamma(1 + \hat{n})(1 - \hat{n})}{\hat{N}(\hat{n})(1 + \phi\hat{n})^2}.$$

Therefore, both curves shift downward for $\hat{n}$ greater than 1 and upward for $\hat{n}$ smaller than 1. In other words, they rotate clockwise about the point at $\hat{n} = 1$. This is explained as follows. As trade costs decrease, workers find that imported varieties become relatively less expensive. Therefore, the effective number of varieties increases more in the district where more imported varieties are consumed. Suppose that $\hat{n}$ is greater than 1. Then, it is the suburb that imports more varieties, and, therefore, that district sees a larger increase in the effective number of varieties. On the one hand, because the competition among varieties intensifies more in the suburb, the demand for each variety decreases relatively more in the suburb, other things being equal. However, firm's profits must be equal across the two districts, which requires their demand levels to be equal. Therefore, the demand for a suburban variety must increase relative to that for a city center variety, which is accomplished by the increase in the relative population of the suburb. As a result, the ZP curve shifts downward. On the other hand, the larger increase in the effective number of varieties in the suburb makes the suburban workers' relative utility rise (see (15)). To restore the utility equivalence, the relative population of the suburb must increase so that residents there consume less land. This results in a downward shift of the UE curve.

Now, we are ready to explain the changes in $\hat{n}^*$ and $\hat{l}^*$. For the sake of exposition, let us concentrate on the case with $\hat{S} > \mu^{-\frac{1}{\beta}}$, which corresponds to the case with $\hat{n}^* > 1$. Fig. 2 illustrates the shifts of the ZP and UE curves. The ZP curve shifts from the thick solid ZP° curve to the thin solid ZP' curve, while the UE curve shifts from the thick dashed UE° curve to the thin dashed UE' curve. The figure reflects two facts. First, both curves shift downward in the neighborhood of the equilibrium because they rotate clockwise about points C^{ZP} and C^{UE} , respectively, both of which are located at $\hat{n} = 1$. Second, the ZP curve shifts downward more than the UE curve.²¹ The equilibrium moves from point E° to E' , both of which lie on the same horizontal line. Thus, the equilibrium value of $\hat{n}$ changes from $\hat{n}^{*\circ}$ to $\hat{n}^{*'}$, whereas that of $\hat{l}$ remains unchanged. We can similarly explain the other case with $\hat{S} < \mu^{-\frac{1}{\beta}}$.

————— Insert Fig. 3 around here. —————

²¹This can be understood from the fact that $\hat{N}(\hat{n})$ in the equation of $\hat{l}_{ZP}$, (18), is raised to the power of 1, whereas that in the equation of $\hat{l}_{UE}$, (19), is raised to the power of $\gamma < 1$.

One policy implication is that the attempts to reduce trade costs may not succeed in expanding the number of service firms in a city center. Whether such attempts succeed depends on the parameter values. As we have seen in Proposition 4, the number of service firms in a city center is more likely to expand when that district is endowed with a large amount of residential land relative to the suburb, commuting costs are high, and preference for land is modest. This prediction is a matter for empirical examination, which is beyond the scope of this paper.

6.3 Effects of decreases in both commuting and trade costs

Finally, we examine the general case in which *both* commuting and trade costs decrease as a result of transport infrastructure investments. Here, we face the difficult issue of how to formulate the effects of investments on each of the two kinds of transport costs. For one thing, the magnitude of the effect on commuting costs may differ from that of the effect on trade costs. What is more, that difference probably depends on a number of factors, such as the transport mode targeted by the investments, the amount of investment, the quality of the transport infrastructure before the investments, the number of users, and the availability of other transport systems. To avoid making arbitrary assumptions about such particularities, we instead put these details into a black box and assume only that a given amount of investments, I , reduces μ and τ by $\eta > 0$ units and 1 unit, respectively. Here, η represents the relative strength of the effect of investments on commuting costs compared to that on trade costs. An $\eta > 1$ means that investments reduce commuting costs more than trade costs, while $\eta < 1$ means the reverse. Then, we can write the total effects of investments on n_i^* and l_i^* as

$$\begin{cases} \frac{dn_i^*}{dI} = -\eta \frac{dn_i^*}{d\mu} - \frac{dn_i^*}{d\tau} = -\eta \frac{dn_i^*}{d\mu} + (\sigma - 1)\phi^{\frac{\sigma}{\sigma-1}} \frac{dn_i^*}{d\phi} & \text{and} \\ \frac{dl_i^*}{dI} = -\eta \frac{dl_i^*}{d\mu} - \frac{dl_i^*}{d\tau} = -\eta \frac{dl_i^*}{d\mu} + (\sigma - 1)\phi^{\frac{\sigma}{\sigma-1}} \frac{dl_i^*}{d\phi} \end{cases} \quad (28)$$

for $i \in \{1, 2\}$, respectively.

Combining (28) with our earlier findings immediately leads to several observations. First, recall that both $dl_1^*/d\phi$ and $dl_2^*/d\phi$ are zero and that $dl_1^*/d\mu$ is positive while $dl_2^*/d\mu$ is negative. Therefore, dl_1^*/dI is negative and dl_2^*/dI is positive: Because a reduction in trade costs has no effect on the population distribution, while a reduction in commuting costs promotes suburbanization, any infrastructure investment leads to suburbanization. Second, recall that if $\hat{S} \leq \mu^{-\frac{1}{\beta}}$, $dn_1^*/d\phi$ is negative or 0 (see Proposition 4). In that case, dn_1^*/dI is negative since $dn_1^*/d\mu$ is always positive. By similar reasoning, we can conclude that dn_2^*/dI is positive if $\hat{S} \leq \mu^{-\frac{1}{\beta}}$. Third, suppose that $\hat{S} > \mu^{-\frac{1}{\beta}}$. Then, $dn_1^*/d\phi$ becomes positive. Therefore, there is a critical value, η_1° ,

such that dn_1^*/dI is positive when η is below the critical value and negative when it is above the critical value. Similarly, there is a critical value, η_2° , such that dn_2^*/dI is negative when η is below the critical value and positive when it is above the critical value. Fourth and finally, we can state the contrapositive of the second observation: If dn_1^*/dI is nonnegative or if dn_2^*/dI is nonpositive, then $\hat{A}$ must be greater than 1. Those findings are summarized as follows.

Proposition 5

- i) $dl_1^*/dI < 0$ and $dl_2^*/dI > 0$.
- ii) If $\hat{S} \leq \mu^{-\frac{1}{\beta}}$, then $dn_1^*/dI < 0$ and $dn_2^*/dI > 0$.
- iii) Suppose that $\hat{S} > \mu^{-\frac{1}{\beta}}$. Then, there exists η_1° such that $\frac{dn_1^*}{dI} \begin{cases} \geq \\ < \end{cases} 0$ if $\eta \begin{cases} \leq \\ > \end{cases} \eta_1^\circ$. Similarly, there exists η_2° such that $\frac{dn_2^*}{dI} \begin{cases} \leq \\ > \end{cases} 0$ if $\eta \begin{cases} \leq \\ > \end{cases} \eta_2^\circ$.
- iv) If $dn_1^*/dI \geq 0$ or if $dn_2^*/dI \leq 0$, then $\hat{S} > \mu^{-\frac{1}{\beta}}$.

We can combine ii) and iii) of the proposition and restate them as follows. Fewer varieties are produced in the city center and more varieties are produced in the suburb if the relative attractiveness of the city center is too low or investments reduce commuting costs to a sufficiently greater extent than trade costs. Conversely, more varieties are produced in the city center and fewer varieties are produced in the suburb if the relative attractiveness of the city center is sufficiently high and investments reduce commuting costs to a sufficiently smaller extent than trade costs.

7 Effects of a change in the endowment of residential land

Next, we examine the effects of a change in the relative endowment of residential land, $\hat{S}$. (23) immediately leads to the following proposition.

Proposition 6

- i) $\partial n_1^*/\partial \hat{S} > 0$ and $\partial n_2^*/\partial \hat{S} < 0$.
- ii) $\partial l_1^*/\partial \hat{S} > 0$ and $\partial l_2^*/\partial \hat{S} < 0$.

Thus, when the city center is endowed with a larger amount of residential land relative to the suburb, both firms and workers concentrate more in the city center. Because n_1^* and n_2^* move in opposite directions, their ratio, $\hat{n}^*$, moves in the same direction as n_1^* . A similar remark holds for $\hat{l}^*$.

Again, we can explain the changes in the relative variables, $\hat{n}^*$ and $\hat{l}^*$, in terms of shifts in the ZP and UE curves. On the one hand, since $\hat{l}_{ZP}(\hat{n})$ does not depend on $\hat{S}$, the ZP curve does not shift. On the other hand, $\hat{l}_{UE}(\hat{n})$ rises for any given value of $\hat{n}$. This is explained as follows. The increase in $\hat{S}$ softens the residential land market in the city center, which puts downward pressure on land rent there. This would raise the utility level of city center workers, other things being equal. To offset this effect, the population of that district needs to increase, which brings about an upward shift of the UE curve. In addition, recall that both curves are upward-sloping, with the ZP curve being steeper than the UE curve at their intersection. Therefore, the intersection moves to the northwest.

8 Effects of changes in preferences

Finally, we examine the effects of changes in the parameters related to preferences.

8.1 Change in the elasticity of substitution

To begin, we analyze a change in σ . Some studies, including De Loecker and Unger (2020), find that firms' markup or profitability has increased in recent decades.²² In our CES utility setting, such a development corresponds to a fall in σ . As σ falls, workers become less willing to substitute one variety for another, that is, their "love of varieties" strengthens. This strengthens economies of scale and firms' monopoly power. Several factors may account for the fall in σ . For one thing, it may be attributed to an increase in workers' income. As their income rises, they tend to have a stronger preference for specific varieties. For another, the fall in σ may be driven by a decrease in the costs of obtaining information about individual varieties, caused by the advances in information technology and the emergence of large online commercial platforms.

First, we examine the effects on the numbers of varieties produced in the two districts. Paying attention to the fact that those effects operate through θ and γ (see (23)), we obtain

$$\frac{\partial n_1^*}{\partial \sigma} = \frac{\theta}{\mu + \hat{A}} \Omega_1(\hat{A}) \quad \text{and} \quad \frac{\partial n_2^*}{\partial \sigma} = \frac{\theta}{\mu + \hat{A}} \Omega_2(\hat{A}), \quad (29)$$

where

$$\begin{cases} \Omega_1(\hat{A}) & \equiv -\frac{\hat{A} - \phi}{\sigma} - \frac{\gamma(\phi + \mu)}{(\sigma - 1)(1 - \gamma)} \cdot \frac{\hat{A} \ln \hat{A}}{\mu + \hat{A}} \quad \text{and} \\ \Omega_2(\hat{A}) & \equiv -\frac{1 - \phi \hat{A}}{\sigma} + \frac{\gamma(1 + \phi \mu)}{(\sigma - 1)(1 - \gamma)} \cdot \frac{\hat{A} \ln \hat{A}}{\mu + \hat{A}}. \end{cases}$$

²²According to them, average markups increased from 21% above cost in 1980 to 61% above cost in 2016, and profitability, which takes into account fixed costs, also increased over that period.

Now, because $\hat{A}$ depends on μ and $\hat{S}$, we can express the effects in terms of one of these two parameters. Here, we choose $\hat{S}$. It is convenient to introduce a function $s(x) \equiv \mu^{-\frac{1}{\beta}} x^{1-\gamma}$, which gives $\hat{S}$ for each value of $x = \hat{A}$, given μ . For instance, we have $s(1) = \mu^{-\frac{1}{\beta}}$ since $\hat{S}$ becomes equal to $\mu^{-\frac{1}{\beta}}$ for $\hat{A} = 1$. Note that $s(1) < 1$ since $\mu > 1$. The function $s(x)$ is strictly increasing for any $x > 0$.

Recall that our parameters are subject to two sets of constraints. First, we are assuming that the city center is endowed with a smaller amount of residential land than the suburb, that is, $\hat{S} < 1$. Second, the relative intrinsic attractiveness, $\hat{A}$, is constrained by lower and upper bounds, ϕ and $1/\phi$, respectively (see Assumption 2). This implies that $\hat{S}$ also has lower and upper bounds, given by $s(\phi)$ and $s(1/\phi)$, respectively. Taking these two constraints into account, we can obtain the following result from (29).

Proposition 7

- i) There exists $\hat{S}^\circ$ with $s(\phi) < \hat{S}^\circ < s(1) < 1$ such that $\frac{\partial n_1^*}{\partial \sigma} \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \hat{S}^\circ$.
- ii) There exists $\mu' > 1$ that separates the following two cases.
 - a) If $\mu > \mu'$, there exists $\hat{S}'$ with $s(1) < \hat{S}' < 1 < s(1/\phi)$ such that $\frac{\partial n_2^*}{\partial \sigma} \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{S} \begin{cases} \leq \\ > \end{cases} \hat{S}'$.
 - b) If $\mu \leq \mu'$, $\partial n_2^* / \partial \sigma < 0$ for any $\hat{S} < 1$.

The proof of Proposition 7 is relegated to Appendices. According to this proposition, as σ falls, n_1^* is more likely to increase when the city center is endowed with a relatively large amount of residential land. In contrast, n_2^* is more likely to increase when the opposite holds.

This conclusion becomes even more pronounced when we consider the *relative* number, not the *absolute* number, of varieties. Since (20) implies that

$$\frac{\partial \hat{n}^*}{\partial \sigma} = -\frac{\gamma(1+\phi)(1-\phi)}{(\sigma-1)(1-\gamma)^2} \cdot \frac{\hat{A}}{(1-\phi\hat{A})^2} \cdot \ln(\hat{S}\mu^{\frac{1}{\beta}}),$$

$\frac{\partial \hat{n}^*}{\partial \sigma} \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}$. That is, the relative number of varieties produced in the city center increases if $\hat{S}$ is high, and μ and β are low. In other words, the relative number is more likely to increase when the city center is endowed with a relatively large amount of residential land, commuting costs are low, and/or the preference for land is modest.

Furthermore, the relation of $\hat{n}^*$ and $\hat{A}$ described by (27) indicates that the fall in σ has a snow-ball effect similar to that of a fall in trade costs: $\hat{n}^*$ increases if $\hat{n}^* > 1$, while it decreases if $\hat{n}^* < 1$. Again, this is consistent with a common result in spatial economics that a fall in σ strengthens

agglomeration forces.

Next, we examine the effect on the population distribution. Since

$$\frac{\partial l_1^*}{\partial \sigma} = -\frac{\gamma\mu}{(\sigma-1)(1-\gamma)^2} \cdot \frac{\hat{A}}{(\mu+\hat{A})^2} \cdot \ln(\hat{S}\mu^{\frac{1}{\beta}}),$$

$\frac{\partial l_1^*}{\partial \sigma} \begin{cases} \leq \\ \geq \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ \leq \end{cases} \mu^{-\frac{1}{\beta}}$. It is clear that $\hat{l}^*$ moves in the same direction as l_1^* because populations of the two districts must sum to a constant.

We can provide an intuitive explanation for the directions of changes in $\hat{n}^*$ and $\hat{l}^*$ by examining how the ZP and UE curves move. As σ changes, the UE curve shifts whereas the ZP curve does not. To examine the direction in which the UE curve shifts, we observe

$$\frac{\partial \ln \hat{l}_{UE}(\hat{n})}{\partial \ln \sigma} = -\frac{\sigma\gamma}{\sigma-1} \ln \hat{N}(\hat{n}).$$

Therefore, $\frac{\partial \hat{l}_{UE}(\hat{n})}{\partial \sigma} \begin{cases} \leq \\ \geq \end{cases} 0$ if $\hat{N} \begin{cases} \geq \\ \leq \end{cases} 1$, or, equivalently, if $\hat{n} \begin{cases} \geq \\ \leq \end{cases} 1$ (see (14)). As σ decreases, the UE curve rotates counterclockwise about the point at $\hat{n} = 1$. The reason is simple. As discussed above, a fall in σ strengthens workers' "love of varieties." Thus, the residents of the district where a greater number of varieties are produced benefit more. To offset this effect, the population of that district relative to that of the other district needs to increase, which effectively reduces the land consumption of each resident there. Now, suppose that $\hat{n}^*$ is greater than 1, which is the case when $\hat{S} > \mu^{-\frac{1}{\beta}}$. As σ falls, the new UE curve lies above the original UE curve in the neighborhood of $\hat{n}^*$. Since both the UE and the ZP curves are upward-sloping and the ZP curve is steeper than the UE curve in the neighborhood of the equilibrium, the equilibrium moves to the northeast. Both $\hat{n}^*$ and $\hat{l}^*$ increase. By the same reasoning, we can explain why both $\hat{n}^*$ and $\hat{l}^*$ decrease when $\hat{n}^* < 1$.

8.2 Change in the preference between differentiated products and the homogeneous good

Next, we suppose that workers develop a stronger preference for differentiated products relative to the homogeneous good. More specifically, their expenditure share on differentiated products increases, whereas that on the homogeneous good decreases by the same amount. In other words, α rises while β does not change.

First, we examine the effects on the numbers of varieties produced in the two districts. It is

readily verified that

$$\frac{\partial n_1^*}{\partial \alpha} = \frac{\theta}{\alpha\beta(\sigma-1)(1-\gamma)(\mu+\hat{A})^2} \Psi_1(\hat{A}) \quad \text{and} \quad \frac{\partial n_2^*}{\partial \alpha} = \frac{\theta}{\alpha\beta(\sigma-1)(1-\gamma)(\mu+\hat{A})^2} \Psi_2(\hat{A}), \quad (30)$$

where

$$\begin{cases} \Psi_1(\hat{A}) & \equiv \beta(\sigma-1)(1-\gamma)(\mu+\hat{A})(\hat{A}-\phi) + \alpha(\phi+\mu)\hat{A} \ln \hat{A} \quad \text{and} \\ \Psi_2(\hat{A}) & \equiv \beta(\sigma-1)(1-\gamma)(\mu+\hat{A})(1-\phi\hat{A}) - \alpha(1+\phi\mu)\hat{A} \ln \hat{A}. \end{cases}$$

By the same reasoning as in the analysis of a change in σ , we can obtain the following result from (30).

Proposition 8

- i) There exists $\hat{S}^+$ with $s(\phi) < \hat{S}^+ < s(1) < 1$ such that $\frac{\partial n_1^*}{\partial \alpha} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \hat{S}^+$.
- ii) There is a critical value of μ, μ'' , that separates the following two cases.
 - a) If $\mu > \mu''$, there exists $\hat{S}''$ with $s(1) < \hat{S}'' < 1 < s(1/\phi)$ such that $\frac{\partial n_2^*}{\partial \alpha} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{S} \begin{cases} \leq \\ > \end{cases} \hat{S}''$.
 - b) If $\mu \leq \mu''$, $\partial n_2^* / \partial \alpha > 0$ for any $\hat{S} < 1$.

The proof of Proposition 8 is relegated to Appendices.

Thus, the effects of a rise in α are qualitatively similar to those of a fall in σ : n_1^* is more likely to increase when the city center is endowed with a relatively large amount of residential land. In contrast, n_2^* is more likely to increase when the city center is endowed with a relatively small amount. In short, the number of varieties produced is more likely to increase in a district with a relatively greater amount of residential land.

As in the analysis of a change in σ , we can obtain a stronger result for the relative number of varieties.

$$\frac{\partial \hat{n}^*}{\partial \alpha} = \frac{(1+\phi)(1-\phi)}{\beta(\sigma-1)(1-\gamma)^2} \cdot \frac{\hat{A}}{(1-\phi\hat{A})^2} \cdot \ln(\hat{S}\mu^{\frac{1}{\beta}}).$$

Therefore, $\frac{\partial \hat{n}^*}{\partial \alpha} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}$. Again, this result is the same as that obtained for a fall in σ . That is, the relative number of varieties produced in the city center increases if $\hat{S}$ is high, and μ and β are low, or, in other words, if the city center has a relatively large amount of residential land, commuting costs are low, and the preference for land is modest. Furthermore, the relation of $\hat{n}^*$ and $\hat{A}$ described by (27) indicates that the increase in α has a snowball effect similar to that of the fall in σ : Again, this is consistent with a common result in spatial economics that an increase in α strengthens agglomeration forces.

Next, we examine the effect on the population distribution. Since

$$\frac{\partial l_1^*}{\partial \alpha} = \frac{\mu}{\beta(\sigma-1)(1-\gamma)^2} \cdot \frac{\hat{A}}{(\mu + \hat{A})^2(1-\phi\hat{A})^2} \cdot \ln(\hat{S}\mu^{\frac{1}{\beta}}),$$

$\frac{\partial l_1^*}{\partial \alpha} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{S} \begin{cases} \geq \\ < \end{cases} \mu^{-\frac{1}{\beta}}$. Furthermore, $\hat{l}^*$ moves in the same direction as l_1^* .

We again explain the directions of changes in $\hat{n}^*$ and $\hat{l}^*$ in terms of shifts in the ZP and UE curves. Although the ZP curve does not move as α changes, the UE curve shifts. The direction of the shift, which is dictated by

$$\frac{\partial \ln \hat{l}_{UE}(\hat{n}^*)}{\partial \ln \alpha} = \frac{\alpha \ln \hat{A}}{\beta(\sigma-1)(1-\gamma)} \cdot \ln(\hat{S}\mu^{\frac{1}{\beta}}),$$

depends on whether $\hat{S}\mu^{\frac{1}{\beta}}$ exceeds 1. This is explained as follows. The increase in α enhances the importance of the effective number of varieties for workers. Therefore, the workers in the district with the greater effective number of varieties benefit more from the increase. If $\hat{S} > \mu^{-\frac{1}{\beta}}$, they are city center residents, whereas if $\hat{S} < \mu^{-\frac{1}{\beta}}$, they are suburban residents. Consequently, the utility of city center residents relative to that of suburban residents increases if $\hat{S} > \mu^{-\frac{1}{\beta}}$, whereas it decreases if $\hat{S} < \mu^{-\frac{1}{\beta}}$, other things being equal. As we have seen in the analysis of a change in σ , those changes in the relative utility must be accommodated by a change in the population distribution. That is, if $\hat{S} > \mu^{-\frac{1}{\beta}}$, the population of the city center relative to that of the suburb must increase, and if $\hat{S} < \mu^{-\frac{1}{\beta}}$, it must decrease. This implies that the UE curve shifts upward if $\hat{S} > \mu^{-\frac{1}{\beta}}$, whereas it shifts downward if $\hat{S} < \mu^{-\frac{1}{\beta}}$. Now, if the UE curve shifts upward in the neighborhood of the equilibrium, the intersection of the ZP and the UE curves moves to the northeast, and thus, both $\hat{n}^*$ and $\hat{l}^*$ increase. Conversely, if the UE curve shifts downward, the opposite holds.

8.3 Change in the preference between land and the homogeneous good

The final exercise examines the effects of a change in the preference between land and the homogeneous good. Suppose that β rises while α remains unchanged. Note that

$$\begin{cases} \frac{\partial n_1^*}{\partial \beta} = -\theta\Lambda(\phi + \mu) \cdot \ln(\hat{S}\mu^{\frac{\sigma-1}{\alpha}}) & \text{and} & \frac{\partial n_2^*}{\partial \beta} = \theta\Lambda(1 + \phi\mu) \cdot \ln(\hat{S}\mu^{\frac{\sigma-1}{\alpha}}), \\ \frac{\partial l_1^*}{\partial \beta} = -\mu\Lambda \ln(\hat{S}\mu^{\frac{\sigma-1}{\alpha}}) & \text{and} & \frac{\partial l_2^*}{\partial \beta} = \mu\Lambda \ln(\hat{S}\mu^{\frac{\sigma-1}{\alpha}}), \end{cases} \quad (31)$$

where $\Lambda \equiv \frac{\alpha \hat{A}}{\beta^2(\sigma-1)(1-\gamma)^2(\mu + \hat{A})^2} > 0$. Equation (31) immediately leads to the following proposition.

Proposition 9

If $\hat{S} \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} \mu^{-\frac{\sigma-1}{\alpha}}$, then, $\frac{\partial n_1^*}{\partial \beta} \left\{ \begin{smallmatrix} \leq \\ > \end{smallmatrix} \right\} 0$, $\frac{\partial n_2^*}{\partial \beta} \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} 0$, $\frac{\partial l_1^*}{\partial \beta} \left\{ \begin{smallmatrix} \leq \\ > \end{smallmatrix} \right\} 0$, and $\frac{\partial l_2^*}{\partial \beta} \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} 0$.

Furthermore, since n_1^* and n_2^* move in opposite directions, their ratio, $\hat{n}^*$, moves in the same direction as n_1^* . Similarly, $\hat{l}^*$ moves in the same direction as l_1^* .

As in the previous analyses, we can intuitively explain the directions of changes in $\hat{n}^*$ and $\hat{l}^*$ in terms of shifts in the ZP and UE curves. Suppose that β rises. Although the ZP curve does not move, the UE curve shifts according to

$$\frac{\partial \ln \hat{l}_{UE}(\hat{n}^*)}{\partial \ln \beta} = -\frac{\alpha \beta (\sigma-1)}{[\beta(\sigma-1) - \alpha]^2} \ln(\hat{S} \mu^{\frac{\sigma-1}{\alpha}}).$$

Therefore, the UE curve shifts downward if $\hat{S} > \mu^{-\frac{\sigma-1}{\alpha}}$, and upward if $\hat{S} < \mu^{-\frac{\sigma-1}{\alpha}}$. The reason is that the rise in β affects the utility level of city center workers relative to that of suburban workers through the following two channels. First, city center workers enjoy a higher disposable income than suburban workers because they do not have to pay commuting costs. However, there is no income effect on land consumption: the amount of land consumed is the same regardless of income. Therefore, as land consumption becomes more important, city center workers' advantage from having a higher disposable income diminishes, which reduces their relative utility level. Second, if $\hat{N}$ is sufficiently high, city center workers have the additional advantage of being able to consume a greater effective number of varieties than suburban workers. As β rises, this advantage also diminishes, which reduces the relative utility level of city center workers. Conversely, if $\hat{N}$ is low, they have a disadvantage rather than an advantage. Then, the rise in β mitigates this disadvantage, thereby raising their relative utility level. Consequently, if $\hat{N}$ is sufficiently high, both effects work to reduce the relative utility level of city center workers. Then, $\hat{l}$ must decrease to restore utility equivalence, and therefore the UE curve shifts downward. However, if $\hat{N}$ is too low, the second effect dominates the first and the relative utility level rises. In this case, $\hat{l}$ must increase and the UE curve shifts upward. Finally, we examine how the equilibrium moves. Since $\hat{N}(\hat{n}^*) = (\mu^{\frac{1}{\beta}} \hat{S})^{\frac{1}{1-\gamma}}$, $\hat{N}(\hat{n}^*) \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} \mu^{-\frac{\sigma-1}{\alpha}}$ if $\hat{S} \left\{ \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} \right\} \mu^{-\frac{\sigma-1}{\alpha}}$. Hence, if $\hat{S} > \mu^{-\frac{\sigma-1}{\alpha}}$, the UE curve shifts downward in the neighborhood of the equilibrium and both $\hat{n}^*$ and $\hat{l}^*$ decrease. In contrast, if $\hat{S} < \mu^{-\frac{\sigma-1}{\alpha}}$, it shifts upward and both $\hat{n}^*$ and $\hat{l}^*$ increase.

9 Concluding remarks

This study has examined the spatial distributions of population and service production within a city using a simple two-district monocentric model that incorporates commuting costs, trade costs, residential land, and agglomeration economies arising from differentiated services. We have shown that, under certain conditions, there exists a unique and stable equilibrium and have examined how its spatial structure responds to changes in transport costs, residential land endowments, and preferences. A decrease in commuting costs decentralizes both population and service production, whereas a decrease in trade costs does not affect the population distribution but may either centralize or decentralize service production. In particular, lower trade costs promote the concentration of service production in the city center when residential land there is sufficiently abundant relative to the suburb, commuting costs are sufficiently high, or the preference for land is modest. When transport infrastructure investment reduces both commuting and trade costs, population necessarily decentralizes, while service production may become more concentrated in the city center if the relative attractiveness of the city center is sufficiently high and the reduction in trade costs is sufficiently large relative to that in commuting costs. Our analysis also shows that residential land endowments and preferences play important roles in determining the spatial distributions of both workers and service firms.

This work has many limitations. To conclude the paper, we discuss some of the most important ones and suggest directions for possible future research.

First and foremost, we have not empirically examined the validity of our results. It is true that the purpose of this work is to obtain insights into the basic mechanism through which infrastructure investments affect urban spatial structure, but we have also obtained several comparative statics results concerning the effects of changes in various parameters on the spatial structure of a city. The empirical validity of these results should be examined in future research. For that purpose, it is necessary to remove some of the simplifying assumptions in our model to make it more realistic as a foundation for empirical analysis.

Second, one of the simplifying assumptions that need to be relaxed for empirical analysis is the monocentric city assumption, under which all firms producing the homogeneous good are concentrated in the city center and all workers commute to that district. A natural next step is to remove the assumption that the locations of firms producing the homogeneous good are fixed. However, allowing firms producing the homogeneous good, in addition to firms producing differentiated products, to choose their locations complicates the analysis, because we need to consider the effects of infrastructure investments not only on the locations of consumers and firms in the differentiated product sector but also on the locations of firms in the homogeneous

good sector. What is more, transport costs for shipping inputs used in the production process play an important role in firms' location decisions. Thus, we need to consider two different kinds of transport costs: those for shipping inputs and those for shipping final goods and services. Because infrastructure investments arguably have different impacts on these two kinds of transport costs, we expect the analysis to become much more involved.

Third, both commuting costs and trade costs are assumed to take an iceberg form, which implies that no one receives transport cost payments. In reality, however, these payments can be an important financial resource for infrastructure investments. In particular, for local governments that provide public transport services, revenue from passenger fares is often one of the major funding sources. Thus, future research needs to take into account the monetary flows associated with transport costs.

Fourth and finally, our model assumes that land rent accrues to absentee landlords. Although this assumption is useful for simplifying the analysis, it may not be innocuous. One reason is that, in a small open city model, a change in welfare is reflected in a change in total land rent. Therefore, investments affect only the welfare of absentee landlords, not that of city residents. To analyze the welfare implications further, we need to use an alternative framework.²³

²³Other limitations not discussed here include the absence of mode choice and the possibility that workers combine commuting and shopping in a single trip, to name a few.

References

- Ahlfeldt, G. M, and N. Wendland. (2011) "Fifty years of urban accessibility: The impact of the urban railway network on the land gradient in Berlin 1890–1936," *Regional Science and Urban Economics*, 41, 77–88.
- Ahlfeldt, G. M, and E. Pietrostefani. (2017) "The compact city in empirical research: A quantitative literature review," *SERC discussion paper*, 215.
- Aizawa, H., and T. Kono. (2025) "How should place-based policies be designed to efficiently promote retail agglomeration?" *Journal of Regional Science*, 65, 1423–1448.
- Albouy, D., and B. A. Stuart. (2020) "Urban population and amenities: The neoclassical model of location," *International Economic Review*, 61, 127–158.
- Alonso, W. (1964) *Location and Land Use*, Harvard University Press, Cambridge, MA, US.
- American Planning Association. (2012) *APA Policy Guide on Smart Growth*, ratified in 2002 and updated in 2012. <https://www.planning.org/policy/guides/adopted/smartgrowth.htm> (accessed May 27, 2022).
- Anas, A., R. Arnott, and K. A. Small. (1998) "Urban spatial structure," *Journal of Economic Literature*, 36, 1426–1464.
- Anas, A. and L. N. Moses. (1979) "Mode choice, transport structure and urban land use," *Journal of Urban Economics*, 6, 228–246.
- Anderson, J. E. and E. van Wincoop. (2004) "Trade costs," *Journal of Economic Literature*, 42, 691–751.
- Arakawa, K. (2006) "A model of shopping centers," *Journal of Regional Science*, 46, 969–990.
- Baum-Snow, N. (2007a) "Suburbanization and transportation in the monocentric model," *Journal of Urban Economics*, 62, 405–423.
- Baum-Snow, N. (2007b) "Did highways cause suburbanization?" *The Quarterly Journal of Economics*, 122, 775–805.
- Baum-Snow, N. (2020) "Urban transport expansions and changes in the spatial structure of U.S. cities: Implications for productivity and welfare," *The Review of Economics and Statistics*, 102, 929–945.

- Baum-Snow, N., and L. Han. (2024) "The microgeography of housing supply," *Journal of Political Economy*, 132, 1897–1946.
- Baum-Snow, N., and M. E. Kahn. (2000) "The effects of new public projects to expand urban rail transit," *Journal of Public Economics*, 77, 241–263.
- Baum-Snow, N., and M. E. Kahn. (2005) "Effects of urban rail transit expansions: Evidence from sixteen cities, 1970-2000," *Brookings-Wharton Papers on Urban Affairs*, (2005) 147–206.
- Bergstrand, J. H., P. Egger and M. Larch. (2013) "Gravity Redux: Estimation of gravity-equation coefficients, elasticities of substitution, and general equilibrium comparative statics under asymmetric bilateral trade costs," *Journal of International Economics*, 89, 110–121.
- Brueckner, J. K. (2001) "Urban sprawl: Lessons from urban economics," in *Brookings-Wharton Papers on Urban Affairs* ed. by J. K. Brueckner, E. Mills, and M. Kremer, Brookings Institution Press, 65–97.
- Burton, E. (2002) "Measuring urban compactness in UK towns and cities," *Environment and Planning B*, 29, 219–250.
- Calthorpe, P. (1993) *The Next American Metropolis: Ecology, Community, and the American Dream*, Princeton Architectural Press, New York, NY, US.
- De Loecker, J., J. Eeckhout, and G. Unger. (2020) "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 135, 561–644.
- DeSalvo, J., and M. Huq. (1996) "Income, residential location, and mode choice," *Journal of Urban Economics*, 40, 84–99.
- Dodds, S., and M. Dubrovinsky. (2015) "Retail amenities and urban sprawl," *Journal of Regional Science*, 55, 280–297.
- Duranton, G., and D. Puga. (2015) "Urban land use," in *Handbook of Regional and Urban Economics* vol. 5 ed. by G. Duranton, J. V. Henderson, and W. C. Strange, Elsevier, Amsterdam, Netherlands, 467–560.
- Fujishima, S., T. Hoshino, and S. Sugawara. (2025) "Exploring practical treatment assignment rules for place-based policies: A Japanese case study," *Journal of Regional Science*, doi.org/10.1111/jors.70006.
- Fujita, M. (1988) "A monopolistic competition model of spatial agglomeration: Differentiated product approach," *Regional Science and Urban Economics*, 18, 87–124.

- Fujita, M., and H. Ogawa. (1982) "Multiple equilibria and structural transition of non-monocentric urban configurations," *Regional Science and Urban Economics*, 12, 161–196.
- Fujita, M., and J.-F. Thisse. (1986) "Spatial competition with a land market: Hotelling and von Thunen unified," *Review of Economic Studies*, 53, 819–841.
- Gaigné, C., S. Riou, and J.-F. Thisse. (2012) "Are compact cities environmentally friendly?" *Journal of Urban Economics*, 72, 123–136.
- Gabriel, S., and G. Painter. (2020) "Why affordability matters," *Regional Science and Urban Economics*, 80, 103378.
- Garcia-López, M.-À. (2012) "Urban spatial structure, suburbanization and transportation in Barcelona," *Journal of Urban Economics*, 72, 176–190.
- Garcia-López, M.-À. (2019) "All roads lead to Rome... and to sprawl? Evidence from European cities," *Regional Science and Urban Economics*, 79, 103467.
- Gin, A., and J. Sonstelie. (1992) "The streetcar and residential location in nineteenth century Philadelphia," *Journal of Urban Economics*, 32, 92–107.
- Glaeser, E. L., and M. E. Kahn. (2004) "Sprawl and urban growth," in *Handbook of Regional and Urban Economics* vol. 4 ed. by J. H. Henderson, and J.-F. Thisse, Elsevier, Amsterdam, Netherlands, 2481–2527.
- Glaeser, E. L., J. Kolko, and A. Saiz (2001) "Consumer city," *Journal of Economic Geography*, 1, 27–50.
- Gong, Y., and C. K. Y. Leung. (2024) "Does space matter? The case of the housing expenditure cap," *Regional Science and Urban Economics*, 104, 103974.
- Gonzalez-Navarro, M., and M. A. Turner. (2018) "Subways and urban growth: Evidence from earth," *Journal of Urban Economics*, 108, 85–106.
- Heblich, S., S. J. Redding, and D. M. Sturm. (2020) "The making of the modern metropolis: Evidence from London," *The Quarterly Journal of Economics*, 135, 2059–2133.
- Henkel, J., K. Stahl, and U. Walz. (2000) "Coalition building in a spatial economy," *Journal of Urban Economics*, 47, 136–163.
- Hilber, C. A. L., and W. Vermeulen. (2016) "The impact of supply constraints on house prices in England," *The Economic Journal*, 126, 358–405.

- Iwata, S., and K. Kondo. (2021) "The spillover effects of compact city policy on incumbent retailers: Evidence from Toyama City," *RIETI Discussion Paper Series*, 21-E-085.
- Konishi, H. (2005) "Concentration of competing retail stores," *Journal of Urban Economics*, 58, 488–512.
- Koziatek, O., and S. Dragičević. (2019) "A local and districtal spatial index for measuring three-dimensional urban compactness growth," *Environment and Planning B*, 46, 143–164.
- Lee, J., K. Kurisu, K. An, and K. Hanaki. (2015) "Development of the compact city index and its application to Japanese cities," *Urban Studies*, 52, 1054–1070.
- LeRoy S. F., and J. Sonstelie. (1983) "Paradise lost and regained: Transportation innovation, income, and residential location," *Journal of Urban Economics*, 13, 67–89.
- Lucas, R., and E. Rossi-Hansberg. (2002) "On the internal structure of cities," *Econometrica*, 70, 1445–1476.
- Malykhin, N., and P. Ushchev. (2018) "How market interactions shape the city structure," *Regional Science and Urban Economics*, 71, 122–136.
- Mayer, T., and C. Trevien. (2017) "The impact of urban public transportation evidence from the Paris district," *Journal of Urban Economics*, 102, 1–21.
- Mills, E. S. (1967) "An aggregative model of resource allocation in a metropolitan area," *American Economic Review*, 57, 197–210.
- Mossay, P., P. M. Picard, and T. Tabuchi. (2020) "Urban structures with forward and backward linkages," *Regional Science and Urban Economics*, 83, 103522.
- Muth, R. F. (1969) *Cities and Housing*, University of Chicago Press, Chicago, IL, US.
- Murata, Y., and J.-F. Thisse. (2005) "A simple model of economic geography à la Helpman–Tabuchi," *Journal of Urban Economics*, 58, 137–155.
- OECD. (2012) *Compact city policies: A comparative assessment*, OECD Green Growth Studies, OECD Publishing, Paris, France.
- Paciorek, A. (2013) "Supply constraints and housing market dynamics," *Journal of Urban Economics*, 77, 11–26.
- Picard, P. M., and T. Tabuchi. (2013) "On microfoundations of the city," *Journal of Economic Theory*, 148, 2561–2582.

- Quigley, J. M., and Raphael, S. (2004) "Is housing unaffordable? Why isn't it more affordable?" *Journal of Economic Perspectives*, 18, 191–214.
- Redding, S. J., and Turner, M. A. (2015) "Transportation costs and the spatial organization of economic activity," in *Handbook of Regional and Urban Economics*, vol. 5 ed. by G. Duranton, J. V. Henderson, and W. C. Strange, Elsevier, Amsterdam, Netherlands, 1339–1398.
- Saiz, A. (2010) "The geographic determinants of housing supply," *The Quarterly Journal of Economics*, 125, 1253–1296.
- Sasaki, K. (1989) "Transportation system change and urban structure in two-transport mode setting," *Journal of Urban Economics*, 25, 346–367.
- Sasaki, K. (1990) "Income class, modal choice, and urban spatial structure," *Journal of Urban Economics*, 27, 322–343.
- Tabuchi, T. (1998) "Urban agglomeration and dispersion: A synthesis of Alonso and Krugman," *Journal of Urban Economics*, 44, 333–351.
- Tabuchi, T. (2009) "Self-organizing marketplaces," *Journal of Urban Economics*, 66, 179–185.
- Tsai, Y.-H. (2005) "Quantifying urban form: compactness versus 'sprawl'," *Urban Studies*, 42, 141–161.
- Venables, A. J. (2007) "Evaluating urban transport improvements: Cost-benefit analysis in the presence of agglomeration and income taxation," *Journal of Transport Economics and Policy*, 41, 173–188.
- Wheaton, W. C. (1974) "A comparative static analysis of urban spatial structure," *Journal of Economic Theory*, 9, 223–237.

Appendices

Proof of Proposition 7

i) First, note that $\Omega_1(\hat{A}) < 0$ for any $\hat{A} \geq 1$. Second, $\Omega_1(\phi) > 0$. Third and finally, $d^2\Omega_1(\hat{A})/d\hat{A}^2 < 0$ for any $\hat{A} \in (0, 1]$, that is, $\Omega_1(\hat{A})$ is concave in that interval. These three observations imply that there exists $\hat{A}^\circ$ in $(\phi, 1)$ such that $\Omega_1(\hat{A}) \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\circ$. Therefore, $\frac{\partial n_1^*}{\partial \sigma} \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\circ$. Now, let us denote $s(\hat{A}^\circ)$ by $\hat{S}^\circ$, which lies in the interval $(s(\phi), s(1))$ since $\hat{A}^\circ \in (\phi, 1)$. Then, $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\circ$ is equivalent to $\hat{S} \begin{cases} \geq \\ < \end{cases} \hat{S}^\circ$. This establishes our result.

ii) First, note that $\Omega_2(\hat{A}) < 0$ for any $\hat{A} \leq 1$. Second, $\Omega_2(1/\phi) > 0$. Third and finally, $d\Omega_2(\hat{A})/d\hat{A} > 0$ for any $\hat{A} \geq 1$, that is, $\Omega_2(\hat{A})$ is increasing in that interval. These three observations imply that there exists $\hat{A}'$ in $(1, 1/\phi)$ such that $\Omega_2(\hat{A}) \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}'$. Therefore, $\frac{\partial n_2^*}{\partial \sigma} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}'$. Now, let us denote $s(\hat{A}')$ by $\hat{S}'$, which lies in the interval $(s(1), s(1/\phi))$ since $\hat{A}' \in (1, 1/\phi)$. Note that $\hat{S}'$ exceeds 1 if $\mu \leq \mu' \equiv (\hat{A}')^{\beta(1-\gamma)}$. In that case, any feasible $\hat{S}$ with $\hat{S} \leq 1$ is smaller than $\hat{S}'$, and therefore, $\partial n_2^*/\partial \sigma$ is negative. This corresponds to case b) in the proposition. ■

Proof of Proposition 8

i) First, note that $\Psi_1(\hat{A}) > 0$ for any $\hat{A} \geq 1$. Second, $\Psi_1(\phi) < 0$. Third and finally, $d^2\Psi_1(\hat{A})/d\hat{A}^2 > 0$ for any $\hat{A} \in (0, 1]$, that is, $\Psi_1(\hat{A})$ is convex in that interval. These three observations imply that there exists $\hat{A}^\dagger$ in $(\phi, 1)$ such that $\Psi_1(\hat{A}) \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\dagger$. Therefore, $\frac{\partial n_1^*}{\partial \alpha} \begin{cases} \geq \\ < \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\dagger$. Now, let us denote $s(\hat{A}^\dagger)$ by $\hat{S}^\dagger$, which lies in the interval $(s(\phi), s(1))$ since $\hat{A}^\dagger \in (\phi, 1)$. Then, $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}^\dagger$ is equivalent to $\hat{S} \begin{cases} \geq \\ < \end{cases} \hat{S}^\dagger$. This establishes our result.

ii) First, note that $\Psi_2(\hat{A}) > 0$ for any $\hat{A} \leq 1$. Second, $\Psi_2(1/\phi) < 0$. Third and finally, $d^2\Psi_2(\hat{A})/d\hat{A}^2 < 0$ for any $\hat{A} \in [1, 1/\phi]$, that is, $\Psi_2(\hat{A})$ is concave in that interval. These three observations imply that there exists $\hat{A}''$ in $(1, 1/\phi)$ such that $\Psi_2(\hat{A}) \begin{cases} \leq \\ > \end{cases} 0$ if $\hat{A} \begin{cases} \geq \\ < \end{cases} \hat{A}''$. Now, let us denote $s(\hat{A}'')$ by $\hat{S}''$, which lies in the interval $(s(1), s(1/\phi))$ since $\hat{A}'' \in (1, 1/\phi)$. Note that $\hat{S}''$ exceeds 1 if $\mu \leq \mu'' \equiv (\hat{A}'')^{\beta(1-\gamma)}$. In that case, any feasible $\hat{S}$ with $\hat{S} \leq 1$ is smaller than $\hat{S}''$, and therefore, $\partial n_2^*/\partial \alpha$ is positive. This corresponds to case b) in the proposition. ■

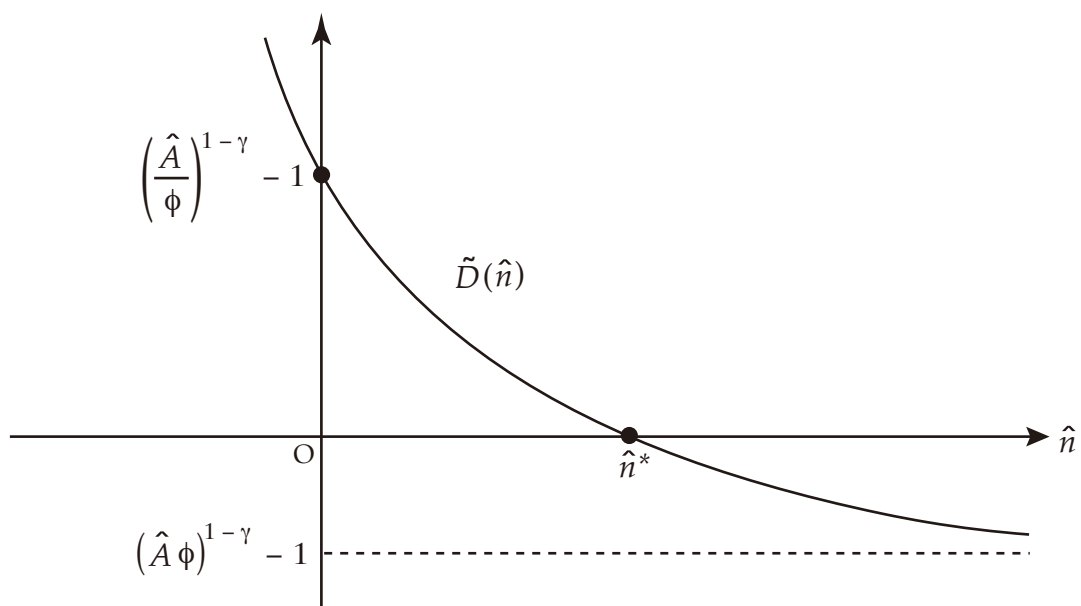


Fig. 1. Divergence function

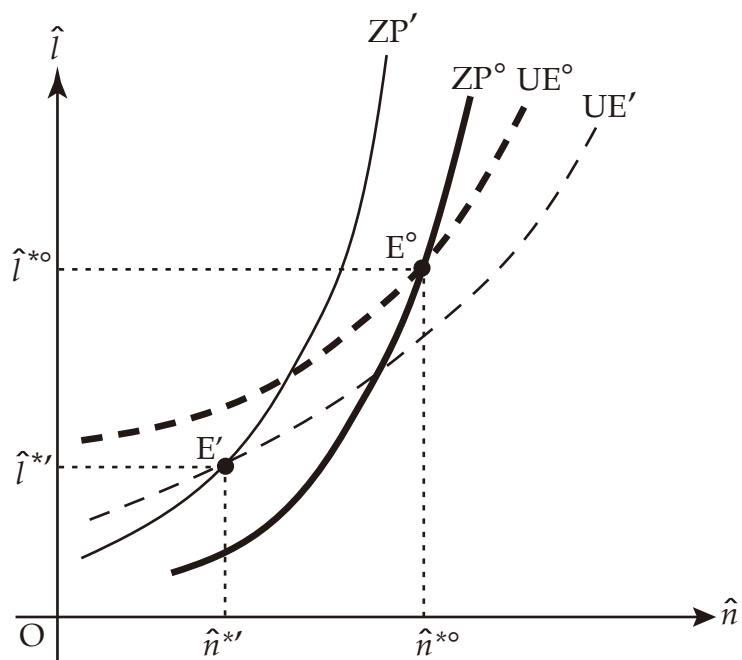


Fig. 2. Effects of a decrease in commuting costs

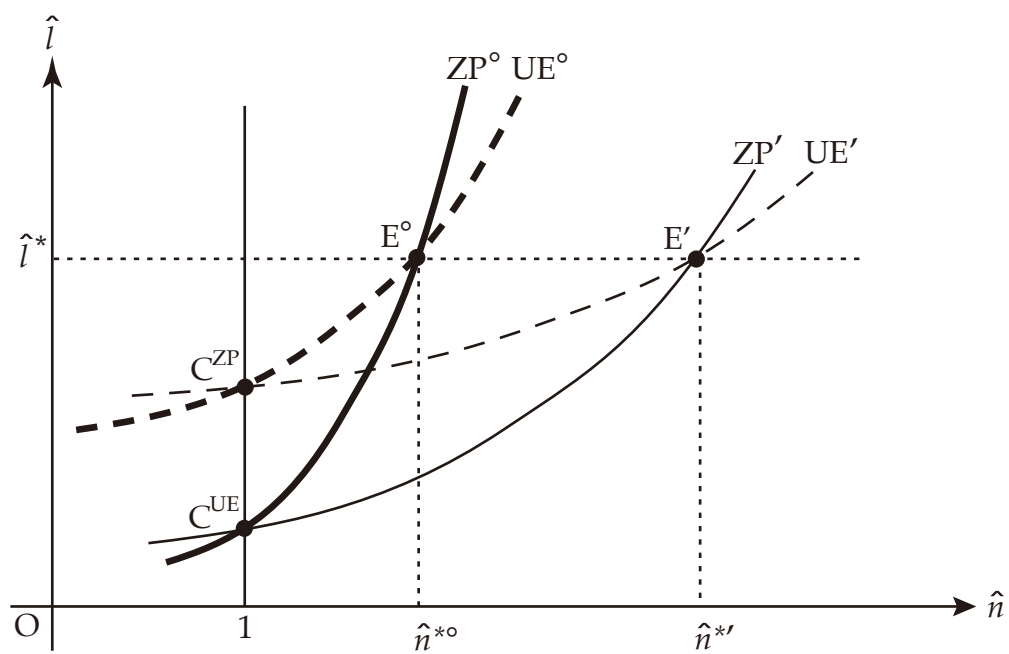


Fig. 3. Effects of a decrease in trade costs