共同利用・共同研究拠点 Joint Usage/Research Center
東京大学 The University of Tokyo
空間情報科学研究センター
Center for Spatial Information Science

Discussion Paper Series

No. 177
CSIS Discussion Paper

October, 2022

Do transport infrastructure investments decentralize a city?

A theoretical revisit to the Alonso-Mills-Muth city with scale economies

Takaaki Takahashi
(Center for Spatial Information Science, The University of Tokyo)

Do transport infrastructure investments decentralize a city? A theoretical revisit to the Alonso-Mills-Muth city with scale economies

Takaaki Takahashi

Center for Spatial Information Science,
The University of Tokyo.
5-1-5, Kashiwa-no-ha, Kashiwa, Chiba 277-8568, Japan

October 2022

Abstract

This study explores the impacts of transport infrastructure investments on the compactness of a city. We introduce two new factors into the conventional Alonso-Mills-Muth model: First, city residents consume differentiated service varieties; and second, they pay transport costs not only to go to work but also to shop those varieties. We analyze the effects of key parameters, including the consumers' valuation of the range of service varieties, on whether the investments decentralize a city or not.

Keywords: compact city; decentralization of a city; land rent; service varieties; urban amenities

JEL Classification Numbers: R1; R4; R5

E-mail address: takaaki-t@csis.u-tokyo.ac.jp

This research is partly supported by Grants in Aid for Research of the Ministry of Education, Science and Culture in Japan (18H00841).

1 Introduction

In recent years, more and more cities have been trying to improve public transportation systems that connect their central business districts (CBDs) with suburbs. Typical examples are medium-sized cities in the EU and the US, which have been introducing brand-new light rail transit (LRT) and subway lines and updating existing lines.[1] Those attempts are carried out on various grounds, among which the following two seem to be particularly important. First and foremost, the attempts are expected to lessen environmental burdens by encouraging residents to switch the mode of transportation to use from automobile to buses and railways.[2] The decreasing dependence on automobile reduces the emissions of CO_2 and other air pollutants. Second, the attempts are intended to fully exploit agglomeration economies by concentrating economic activities in the central parts of a city (Venables, 2007). In recent years, the relative significance of a CBD has been considerably weakened in many cities. Improvements in public transportation are considered to encourage residents to visit a CBD more often, which benefits the business at the CBD and revitalizes it.

However, the evaluation of those attempts crucially depends on whether and how much they decentralize a city, because the decentralization has diverse negative impacts on the welfare of its residents. First, it imposes a greater environmental burden on a society. The reason is twofold: the decentralization increases an average commuting distance, for one thing, and promotes the use of automobile, for the other (Kahn, 2000).[3] The problem becomes particularly severe when the decentralization is accompanied by a sprawl, which is characterized by a large area of land development, a high degree of fragmentation, and a high degree of undeveloped surroundings, according to Garcia-López (2019) (for the policy implications of the urban sprawl, see OECD, 2018). In addition, the decentralization reduces open space and farm land, turning them into the land for housing and commercial buildings. This results in the loss of natural amenities and biological diversity. Second, the relocation of employment from the central parts of a city to suburbs deprives the city of part of the benefit of agglomeration economies. Third, the decentralization pushes up the costs to provide public services, which does harm to the city's fiscal health. Fourth and finally, it hurts those who do not have an access to a car, represented by low income house-

[1] This epitomizes the movement toward what is called a smart growth, as the American Planning Association (2012) mentions that "integrating land use and transportation planning to accommodate more than just the automobile and to provide increased transportation choices, including mass transit, bicycling, and walking is a hallmark of Smart Growth."

[2] Baum-Snow and Kahn (2000) show that as the distance from a transit station to a census tract decreases from 3 km to 1 km, average transit usage of that tract increases by 1.42 percentage points. They study census tracts in five cities where rail transit system was upgraded between 1980 and 1990. The same authors examine a broader range of cities and obtain a less clear-cut result (Baum-Snow and Kahn, 2005).

[3] Gaigné et al. (2012) discuss that city's compactness does not necessarily result in a lower environmental burden if one takes into account its general equilibrium effects on prices, wages and land rents. Thus, according to their arguments, the decentralization may bring about more environmentally-friendly outcomes.

holds and aged population. These adverse effects make it important to answer the question of whether and how much the improvements in public transportation cause the decentralization.

Many empirical studies show that the investments in public transportation indeed cause the decentralization of a city.[4,5] For one thing, Ahlfeldt and Wendland (2011), for instance, find that a land value curve, which describes land values as a function of the distance from the CBD, became flatter in Berlin from 1890 to 1936. They show that the main reason for this change is the reduction in commuting time brought about by the rapid advancement of railway networks. Moreover, Garcia-López (2012), for the railroad improvement in Barcelona Metropolitan Region, Mayer and Trevien (2017), for the expansion of the Regional Express Rail in the greater Paris region, and Gonzalez-Navarro and Turner (2018), for the construction of subways in the world, all show that the improvements in public transportation systems lead to the decentralization of a city. Finally, Heblich et al. (2020) examine the changes in the distributions of residence and employment in London from the 19th century to the early 20th century. By reduced-form event-study analyses and rigorous structural estimation analyses, they find that residence was decentralized but employment was rather more concentrated in central areas during that period. What is more important, they show that such spatial reorganization is largely attributed to the change in transport mode from horse-drawn buses and trams to subways.

The most conventional explanation of those decentralization results is, as in Wheaton (1974), based on the model developed in the seminal works of Alonso (1964), Mills (1967), and Muth (1969) (henceforth, the AMM model).[6] If we take a small and open city as an example, it typically goes as follows. The investments in transport infrastructure affect urban spatial structure through the decrease in commuting costs. Although it raises consumers' disposable income, their utility level must remain unchanged in a small and open city. This implies that their bid rents rise. Because cities extend over the area where urban land rent exceeds a given level of agricultural rent, the city expands. The same conclusion can be derived for a closed city.[7]

Because this explanation is so simple and powerful, it has attracted the attention of a quite large number of researchers who have an interest in the impacts of a mode choice, especially, a choice between public transportation and automobile, on urban spatial structure (Anas and

[4]See Redding and Turner (2015) for the review of the studies on the investments in not only public transportation but also road transportation.

[5]The literature is unanimous that improvements in road transportation lead to the decentralization of a city. For instance, Baum-Snow finds that the construction of new freeways causes the decentralization of residence and, to a smaller degree, employment in a city (Baum-Snow, 2007b; Baum-Snow, 2020). Garcia-López obtains similar results for Barcelona Metropolitan Region (Garcia-López, 2012) and for European cities (Garcia-López, 2019).

[6]For the reviews of the later development of the AMM model, see Anas et al. (1998), Brueckner (2001), Glaeser and Kahn (2004), and Duranton and Puga (2015), among others.

[7]Suppose that land rent declines in a closed city. Then, each resident, with higher disposable income, would consume a larger amount of land, and therefore, the city, with a fixed population, would expand. This is a contradiction, because lower urban land rent implies a smaller urban area. Hence, land rent should rise, and therefore, the city expands.

Moses, 1979; LeRoy and Sonstelie, 1983; Sasaki, 1989; Sasaki, 1990; Gin and Sonstelie, 1992; DeSalvo and Huq,1996; and Baum-Snow, 2007a, to name a few). According to them, the residents who switch the mode from automobile to public transportation after the investments in public transportation may relocate to locations closer to the CBD. This weakens the centrifugal force of those investments and possibly overturns the decentralization results. Because sizable contribution has been already made on this subject, we do not take it up in this work to focus on other factors.

We are still left with two factors that are not considered in the AMM model but may play a critical role in determining whether infrastructure investments foster the decentralization or not.

First, the AMM model fails to pay adequate attention to the importance of differentiated goods and services in residents' consumption. Typical examples are a variety of services called *urban amenities*, such as the services provided at retail shops, bars, cafes, restaurants, theaters, and concert halls. Many researchers in the urban economics believe that concentration of those services is a driving force of agglomeration (see Glaeser et al., 2001, for instance). In addition, Albouy and Stuart (2020) find that the production of nontradable goods and services, represented by the urban amenities, plays a key role in household location decisions. In a theoretical viewpoint, considering differentiated goods and services is further important, because the range of them provided in a city is to be determined endogenously according to the mechanism familiar in the literature of new economic geography (NEG) (Fujita et al., 1999; Fujita and Thisse, 2002). Nonetheless, the AMM model simply bundles them into a hypothetical homogeneous good called "composite good." This tradition prevails with the exception of only a few works, which have studied this missing factor in the context of core-periphery regional patterns. First, Tabuchi (1998) and Murata and Thisse (2005) consider two cities each of which produces a differentiated product, and examine when its production agglomerates in one city. However, their interests lie in the core-periphery patterns, not in the spatial structure of respective cities nor in the impacts of transport infrastructure investment. Second, Picard and Tabuchi (2013) and Mossay et al. (2020) study a city in which differentiated goods and services are produced. Although their results give some important insights into our research question, we rather take a different approach, which is closer to the original AMM model. We will return to this point later in this section.

Second, the AMM model does not allow for the intra-city transport costs necessary to acquire goods and services (the composite good), which we refer to as "shopping costs" in order to distinguish them from commuting costs. In the real economy, however, consumers spend not a small amount of time and money on the trips for acquiring them. One reason is that a significant share of urban amenities is provided at a CBD in small and medium-sized cities, especially those with long history. However, the AMM model implicitly assumes that all of the goods and services are "ubiquitous," that is, residents can obtain them at their backyard without paying any transport costs. In other words, that model does not consider shopping costs but considers only commuting costs. Here again, Picard and Tabuchi (2013) and Mossay et al. (2020), mentioned earlier, are rare examples that consider the shopping costs in a city.[8]

The purpose of this work is to incorporate these two factors into the AMM model and to re-examine the effects of the investments in transport infrastructure on the decentralization of a city. More specifically, we consider a small open city, where residents consume land, differentiated services produced through an increasing-returns-to-scale technology, and a homogeneous good. The service varieties are immobile and produced at the CBD, as most of the urban amenities are concentrated at that location in the real economy. Thus, consumers need to visit the CBD to acquire the varieties, paying shopping costs. The shopping costs and the commuting costs both decrease by government's investments to improve transport infrastructure.

A key observation in our analysis is that the improvements in transport infrastructure affect a consumers' utility level through two channels. One is the channel through land rent and transport costs (i.e., commuting and shopping costs). Because these two kinds of prices determine living costs for city residents, their impact on the utility level is referred to as a *living-cost effect*. The other is the channel through the range of service varieties provided in a city. The effect through this channel, a *variety effect*, is a novel feature of our framework. The improvements reduce transport costs, which raises residents' disposable income. Consequently, more varieties are provided in a city, given that the geographical size of a city remains unchanged. This signifies that the infrastructure improvements yield scale economies. However, the city size does not remain unchanged in fact, and, therefore, the range of varieties may or may not expand. In either case, the point is that how urban spatial structure changes is determined by the workings of those living-cost effect and variety effect.

We examine the effects of transport infrastructure investments on the variables related to the compactness of a city, namely, the geographical size of a city, population density, total population, and the range of service varieties provided in a city. The effects depend on several key parameters. The most important of them is the consumers' valuation of a range of service varieties. It rises as the expenditure share on those varieties increases and the elasticity of substitution among them falls. It is shown that a city expands as a result of infrastructure investments if this valuation is low, that is, if the expenditure share is small and the elasticity is high (each variant is relatively substitutable with each other). Otherwise, a city shrinks. Another important parameter is the amount of the farmland just across a city boundary that can be potentially converted to residential

[8] Picard and Tabuchi (2013) assume that each worker resides at the same place as its firm. Thus, their model does not have commuting costs.

land. Further, also important is the parameter that dictates how much transport costs from the city boundary increase as a result of the expansion of a city.

What is more, we ask if the induced increase in land rent is enough to cover the costs of infrastructure investments, and if welfare improves by the investments. It follows from our simple setting that the condition for the feasibility of investments coincides with that for their desirability. In other words, whenever investments are feasible, they are desirable, and vice versa. The condition depends on various factors, but two are particularly important. First, it becomes more likely that investments are feasible and desirable when they are allocated relatively more intensively to inner locations compared to peripheral locations. Second, the consumers' valuation of the range of service varieties affects the feasibility and the desirability, though in a complicated manner. Its impact is not monotonic but changes depending on whether a city expands or shrinks by the investments.

An additional contribution of this study is found in its policy implication. Not a small number of policymakers advocate *both* the improvements in public transportation *and* a more compact city with a more flourishing city center. On the other hand, as we saw earlier, economists have been piling up the evidences that the improvements cause the decentralization of a city. Here is a wide gap between those two groups. It seems that the variety effect helps us fill this gap. That is, our discussion suggests that if the variety effect were sufficiently strong, the improvements in public transportation would result in a more compact city with more varieties provided in a city center. We can consider that those policymakers have this case in mind, although the variety effect is actually not so strong.

This work adheres to the AMM model's monocentricity assumption that all the production activities are conducted at a city center. However, since the seminal paper of Fujita and Ogawa (1982), a considerable number of attempts are made to study their locations within a city without presuming that they are concentrated at a city center (Lucas and Rossi-Hansberg, 2002; Picard and Tabuchi, 2013; and Mossay et al., 2020, among others). They demonstrate that a city can exhibit diverse patterns of residential and business land uses when the locations of workplaces and residences are interdependent on each other through linkages, such as demand and vertical linkages, and externalities, such as technological spillovers. Their frameworks are more general than the AMM model, because they contain the monocentric city as a special case.

Nonetheless, there are several reasons to follow the monocentricity assumption. First, main interest of this work is the impacts of infrastructure investments in public transportation. In most cases, the systems of LRT, subway, and commuter train, which represent public transportation, are designed to connect the CBD and suburbs. It is certainly so if we consider smaller cities that have only a few rail lines, or if we consider the rail lines constructed at the earliest periods in bigger cities. In other words, the design of transport infrastructure is based on the monocentricity. Second, if we take into account the endogenous determination of locations of production activities, it will become necessary for us to discuss the effects of transportation infrastructure investments on the transport costs involved in production, which are supposed to differ from those involved in consumption.[9] Analyzing a city with two different types of transport costs needs not only much elaboration but also some arbitrary assumptions on the formulation of those costs. By sticking to the more familiar setting of the AMM model, we can focus on the most fundamental effects of transportation infrastructure investments. Third, although the movement toward decentralization has been accelerating at many cities in the world, it seems harmless for us to consider that a great number of cities are still monocentric. Fourth, the monocentric city is supported as one of the equilibrium outcomes in the works mentioned earlier. For instance, Mossay et al. (2020) show that the monocentric city emerges at an equilibrium when commuting costs are sufficiently low and/or demand and vertical linkages are sufficiently strong. Thus, it is possible for us to interpret our framework as focusing on such a specific situation. Fifth and finally, it is often the case that the policy discussion on transport infrastructure investments are made on the premise of a monocentric city.

The rest of the paper consists of six sections. In Section 2, the model is presented. Section 3 derives an equilibrium and explores some of its properties. In Section 4, we examine how the investments in transportation infrastructure affect a city residents' utility level. In Section 5, we analyze the impacts of the investments on some of city's characteristics related to its compactness. Welfare implications are discussed in Section 6. Section 7 concludes.

2 Basic framework

According to the tradition of the AMM model, we consider a city extended in a featureless plain around a dimensionless point, referred to as a CBD, up to $b$km from it. The city is surrounded by a rural area, which is rented out at a constant agricultural rent, $\bar{r}$. Its area within x km from the CBD is given by $S(x)$, with $S(0) = 0$ and $S'(x) \geq 0$ for any $x \in [0, b]$. Thus, no restriction is imposed on a city's geographical form. One example is a disk-shaped city with $S(x) = \int_0^x 2\pi s \, \mathrm{d}s = \pi x^2$. It is the city studied in the AMM model. Another example is a linear city. The transportation system studied in this paper is represented by a mass transit line and a highway. To the extent that such a system extends along a line, the linear city is an appropriate setting. For the linear city with a unit

[9] For instance, in the works of Picard and Tabuchi (2013) and Mossay et al. (2020), the same varieties of differentiated goods and services are used as final goods by consumers and as intermediate inputs by firms. Thus, the costs to ship the varieties of final goods and services are the same as those to ship the intermediate inputs. This simplification will not be innocuous when we study the impacts of transport infrastructure investments.

width, $S(x) = \int_0^x 1 \, \mathrm{d}s = x$. Furthermore, one of the more realistic examples is probably a city with multiple fan-shaped sectors, each of which spreads along a transportation route extending from the CBD. Our general formulation with $S(x)$ admits of all of those city types.

The city is small and open: its population changes through migration until each resident ends up receiving a given level of utility prevailing in the outside economy, which we denote by $\bar{U}$. In most part of today's economy, the mobility of workers across cities is quite high because of low barriers against migration, which are attributed, among others, to small migration costs and few rules to prohibit or restrict migration. For this reason, we concentrate on a small open city. The number of workers/residents in the city, denoted by L, is thus endogenously determined.

Workers consume land for housing, a differentiated product, a homogeneous composite good, and agricultural goods. The differentiated product is represented by the services called urban amenities, such as those offered at cafes, bars, restaurants, theaters, concert halls, and museums. Thus, we refer to it as differentiated "services," although it may include manufacturing and agricultural products. Because most of the urban amenities are immobile and produced at the CBD, we assume the same for the differentiated services. Thus, consumers need to visit the CBD to acquire them. In contrast, the composite good is mobile and shipped with no costs within the city and across the city boundary. We regard it as a numéraire. Furthermore, the composite good, as well as the differentiated services, is produced at the CBD, as we consider a monocentric city in the spirit of the AMM model. Finally, the agricultural goods are produced in the rural area outside the city through a constant-returns-to-scale technology. Their price, p_A, is determined in a competitive national market and thus, regarded as given. We explicitly consider the agriculture sector only for the sake of logical completeness: it plays no significant roles in the subsequent analyses but our assumption that the land outside the city is rented out at a certain level of agricultural land rent entails the existence of that sector.

The preference of a consumer who lives at x km from the CBD is dictated by the following utility function:

$$U(x) = kC(x)^{\alpha} N(x)^{\beta} Z(x)^{\gamma} A(x)^{\delta}, \tag{1}$$

where $C(x) \equiv \left[\int_0^n c(x;i)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i \right]^{\frac{\sigma}{\sigma-1}}$ is the aggregate of the differentiated services, and n is the mass of the differentiated services produced in the city, which is endogenously determined. $N(x)$, $Z(x)$ and $A(x)$ are the amounts of the land for housing, the composite good, and the agricultural goods, respectively. Moreover, $\sigma > 1$ is an elasticity of substitution; and $\alpha \in (0,1)$, $\beta \in (0,1)$, $\gamma \in (0,1)$, and $\delta \equiv 1 - \alpha - \beta - \gamma$ are the shares of expenditure on the services, the land, the composite good, and the agricultural goods, respectively. Finally, $k \equiv {p_A}^{\delta} / \left(\alpha^{\alpha} \beta^{\beta} \gamma^{\gamma} \delta^{\delta} \right)$ is a positive constant. Note that the utility function is reduced to that in the AMM model when α and δ are equal to 0.

Each worker supplies 1 unit of labor inelastically and works in either a differentiated service sector or a composite good sector. According to the tradition of the AMM model, we assume absentee landlords. Thus, income of a worker consists solely of wage. We denote the wage rate by w.

Workers pay two types of transport costs. First, they visit the CBD to acquire differentiated services, paying transport costs, which we refer to as "shopping costs" in order to avoid the confusion with commuting costs. We assume iceberg shopping costs as is common in the literature of the international trade theory and the NEG. To consume one unit of a service variety, workers living at residential location x need to purchase $\tau(x) \geq 1$ units at the CBD. In other words, they pay the monetary equivalent of $\tau(x) - 1$ units for shopping costs. Workers living at the CBD pay no shopping costs, and the costs increase as residences move toward suburb: $\tau(0) = 1$ and $\tau'(x) > 0$ (and therefore, $\tau(x) > 1$) for any $x \in (0, b]$.

Second, workers pay commuting costs to go to work at the CBD. They are assumed to be proportional to wage income. One reason is that a time cost, which is measured by the wage rate as an opportunity cost, is a major component of the commuting costs in addition to a monetary cost. The commuting costs are assumed to be the iceberg type, that is, a fixed proportion, which depends on the distance of commuting, of wage income is dissipated during the commute. More specifically, a worker living at x with 1 dollar of wage income receives only $1/\mu(x)$ dollars after paying the commuting costs. That is, the portion $1 - 1/\mu(x)$ is dissipated as commuting costs. The commuting costs are 0 for those who live at the CBD and increasing with the distance of commuting, that is, $\mu(0) = 1$ and $\mu'(x) > 0$ (and therefore, $\mu(x) > 1$) for any $x \in (0, b]$.

Two comments are worth adding concerning the commuting costs. First, our formulation is analogous to that by Murata and Thisse (2005) in that the disposable income after the commuting costs payment becomes proportional to the wage rate. The difference is that, as a result of commuting, part of wage income dissipates in our formulation, whereas part of labor melts away in theirs. Second, one reservation is that we disregard the possibility that workers combine commuting and shopping in one sequence of a trip. Although such multipurpose trips are often observed in the real economy, incorporating them involves much more elaboration, which is beyond the scope of this paper.

Since the disposable income of a worker living at x is equal to $w/\mu(x)$, his/her budget constraint is expressed as

$$\int_0^n \tau(x)p(i)c(x;i)\,\mathrm{d}i + r(x)N(x) + Z(x) + p_A A(x) = \frac{w}{\mu(x)}, \tag{2}$$

where $p(i)$ is the producer price of the i-th service variety and $r(x)$ is land rent at x.

Now, we turn to the production side. The composite good is produced by a constant-returns-to-scale technology: one unit of labor produces one unit of the good. Thus, the wage rate is equal to 1. Furthermore, firms in the service sector use an increasing-returns-to-scale technology. To produce $q(i)$ units of the i-th service variety requires $F + vq(i)$ units of labor, where $F > 0$ and $v > 0$ are fixed and variable labor requirements, respectively. Then the profit of the firm producing the i-th variety is given by

$$\pi(i) = p(i)q(i) - \left[F + vq(i)\right]. \tag{3}$$

3 Equilibrium

This section derives an equilibrium, which satisfies the following seven conditions.

i) Utility maximization by workers

First, workers maximize their utility, (1), subject to the budget constraint, (2). Thus, we have

$$c(x;i) = \frac{\alpha}{\mu(x)} \frac{P(x)^{\sigma-1}}{\left[\tau(x)p(i)\right]^{\sigma}}, \tag{4}$$

where $P(x) \equiv \tau(x)\left[\int_o^n p(i)^{1-\sigma}\,\mathrm{d}i\right]^{\frac{1}{1-\sigma}}$ is a price index of the services for workers at x. Furthermore, their demands for land, the composite good, and the agricultural good are given by

$$N(x) = \frac{\beta}{r(x)\mu(x)}, \quad Z(x) = \frac{\gamma}{\mu(x)}, \quad \text{and} \quad A(x) = \frac{\delta}{p_A\mu(x)}, \tag{5}$$

respectively.

ii) Profit maximization by the firms in the service sector

Second, firms in the service sector maximize their profit, (3). Let $G(x) \in [0,1]$ be the cumulative population within x km from the CBD:

$$G(x) = \int_0^x \frac{1}{N(y)}\,\mathrm{d}S(y). \tag{6}$$

Then, (4) implies that the demand for each service variety is given by

$$q(i) = \int_0^b \tau(x)c(x;i)\,\mathrm{d}G(x) = \alpha p(i)^{-\sigma} \int_0^b \left[\frac{P(x)}{\tau(x)}\right]^{\sigma-1} \frac{1}{\mu(x)}\,\mathrm{d}G(x). \tag{7}$$

Let us assume that firms take the price index as given, which is a standard procedure in the NEG. (3) and (7) give us the solution to the profit maximization problem:

$$p(i) = p \equiv \frac{\sigma}{\sigma - 1} v \qquad \text{for all } i \in [0, n]. \tag{8}$$

The price is proportional to the marginal cost, and all the varieties are sold at the same price.

iii) Determination of a city boundary

Third, the city extends in the area where the land rent is no lower than the agricultural land rent. Therefore, the land rent at a city boundary becomes equal to the agricultural land rent:

$$r(b) = \bar{r}. \tag{9}$$

iv) Location equilibrium for workers

Fourth, workers have no incentive to change their location, that is, the city is at a location equilibrium.

To derive the condition for it, we compute, from (4), (5) and (8), the indirect utility for a worker living at x:

$$U(x) = \frac{n^{\lambda}}{p^{\alpha} r(x)^{\beta} t(x)}, \quad \text{where} \quad \lambda \equiv \frac{\alpha}{\sigma - 1} \quad \text{and} \quad t(x) \equiv \tau(x)^{\alpha} \mu(x). \tag{10}$$

Here, λ measures a *consumers' valuation of the range of service varieties*. It depends on two factors. First, it increases with the expenditure share on those varieties, α. Second, it decreases with the elasticity of substitution, σ. As σ declines, each variant becomes more irreplaceable, and therefore, a wide variety range becomes more valuable. Another important variable in (10) is $t(x)$, which is a combined measure of the two types of transport costs, namely, the shopping and the commuting costs. We call it *generalized transport costs*. Because the commuting costs affect the disposable income, their level matters for the consumption of not only service varieties but also land and the composite good. In contrast, the shopping costs do not affect the disposable income, but affect only the consumer price of service varieties. This is why $\mu(x)$ has no power whereas $\tau(x)$ has power α in the definition of $t(x)$.

Equation (10) shows that the indirect utility depends on three factors. First, it increases with the range of service varieties, which is a key feature of the monopolistic competition models à la Dixit-Stiglitz. Second, the indirect utility decreases with the land rent. Third and finally, it decreases with the generalized transport costs. Note that the last two factors constitute *living* *costs* for a consumer, which are given by $r(x)^{\beta}t(x)$.

The location equilibrium is realized if $U(x) = U(b)$ for any $x \in [0, b]$. Therefore, the land rent at each location is given by

$$r(x) = \bar{r}\phi(x)^{\frac{1}{\beta}} \quad \text{for any } x \in [0, b], \quad \text{where} \quad \phi(x) \equiv \frac{t(b)}{t(x)}, \tag{11}$$

where (9) is used. Here, $\phi(x)$ indicates how small the generalized transport costs from location x are, compared with the counterparts from the city boundary, b. In other words, it is a measure of *comparative transportability* (CT) at each location. (11) says that the land rent at each location increases with the CT at that location. For the analysis in the next section, it is important to note that the land rent at a particular location rises as the generalized transport costs *at the city boundary* increases, other things being equal.

Moreover, (11) implies that

$$r(x)^{\beta}t(x) = \bar{r}^{\beta}t(b) \quad \text{for any } x \in [0, b]. \tag{12}$$

At a location equilibrium, the living costs become equal at all locations. One must be familiar with this result in the AMM model: In order to keep the utility levels at the same level, the land rent must be high where the (generalized) transport costs are low. An important implication of (12) is that the utility level at any location depends on only the range of service varieties, n, and the transport costs from a city boundary, $t(b)$ (see (10).

v) Firms' free entry and exit

Fifth, firms freely enter and exit from the service sector, which drives their profit to zero. This condition enables us to derive the range of service varieties from (3):

$$n = \frac{\alpha M}{\sigma F} \quad \text{where} \quad M \equiv \int_0^b \frac{1}{\mu(x)} \, \mathrm{d}G(x). \tag{13}$$

The range of service varieties is proportional to M, which is *aggregate disposable income*, or, the sum of the consumers' incomes net of commuting costs.

vi) Land market clearing

Sixth, the land market must clear. It clears if

$$L = G(b). \tag{14}$$

vii) Utility equivalence

Finally, the small open city assumption requires that the city residents enjoy the utility level that prevails outside the city, that is,

$$U(b) = \bar{U}. \tag{15}$$

This completes the set of necessary conditions for an equilibrium. Before its derivation, it is useful to take a closer look at the equilibrium range of varieties, which will give us a good insight into the workings of our model.

Using the first equation in (5), (6), and (11) , we can rewrite the aggregate disposable income as

$$M = \frac{R}{\beta}, \quad \text{where} \quad R \equiv \bar{r} \int_0^b \phi(x)^{\frac{1}{\beta}}\, \mathrm{d}S(x) = \int_0^b r(x)\, \mathrm{d}S(x). \tag{16}$$

Here, R represents *total land rent* in a city. Why does it positively affect the average disposable income? High land rent at respective locations makes each land lot smaller. Thus, when R is high, population distribution tends to be dense, and therefore, average commuting distance becomes short. This causes low commuting costs and high average disposable income.

Now, (16) enables us to rewrite the equilibrium variety range given by (13) as

$$n = \frac{\alpha R}{\sigma F \beta}, \tag{17}$$

which immediately leads to the following observation.

Proposition 1

The range of service varieties offered in a city is proportional to the total land rent.

The reason for this result is that the range is proportional to the average disposable income, which is, in turn, proportional to the total land rent. In addition, note that the proposition shows that the range of service varieties is wide not only when the CT ($\phi(x)$) and the residential area ($S(x)$) are large at respective locations, but also when the locations with a large residential area tend to exhibit high CT.

The geographical expansion of a city increases n through the rise in the total land rent. The total land rent rises through two channels. To see this, note that

$$\frac{\mathrm{d}R}{\mathrm{d}b} = \bar{r} S'(b) + \frac{R \hat{t}_x(b)}{\beta}, \tag{18}$$

where $\hat{t}_x(b) \equiv \dfrac{\partial t(b)/\partial x}{t(b)} > 0$ is the percentage increase in the transport costs at a city boundary when the boundary infinitesimally moves outward. (In this paper, we use the circumflex to denote the relative size of a partial derivative.) Now, the first term of (18) represents the direct effect of newly incorporated areas. The second term represents the effect that the generalized transport costs from a city boundary rise, which enhances the CT at each location.

Now, we are ready to discuss the equilibrium. Substituting (10), (11), and (17) into (15) yields

$$\frac{k' R^{\lambda}}{t(b)} = \bar{U}, \tag{19}$$

where k' is a positive constant.[10] The solution of this equation gives the equilibrium location of a city boundary. It determines all the other variables including the equilibrium population, the equilibrium range of service varieties, and the land rent at each location, which are derived from (14), (17), and (11), respectively.

4 Infrastructure investments and utility level

Because the equilibrium is prescribed by the utility equivalence of (19), we can explore the effects of transportation infrastructure investments on the equilibrium spatial structure of a city by examining their effects on the indirect utility. That is the purpose of this section.

A government, which may be national, regional, or municipal, makes investments, whose amount is denoted by I, in improving transportation infrastructure. The investments are financed by part of the increment in total land rent that will be brought about by the infrastructure improvement. For a while, we assume that the improvements yield enough amount of the increment to cover the investments. Whether this assumption is satisfied or not will be discussed in Section 6.

Investments reduce both types of transport costs, the shopping and the commuting costs. In other words, $\partial\tau(x)/\partial I < 0$ and $\partial\mu(x)/\partial I < 0$ for any x. (Although transport costs are functions of I, we omit I in their expressions for brevity.) The investments may affect the two types differently: we allow for the possibility that $\partial\tau(x)/\partial I$ diverges from $\partial\mu(x)/\partial I$. Furthermore, the generalized transport costs also decrease with the investments: $\partial t(x)/\partial I < 0$.

To study the impacts of infrastructure investments on the indirect utility, it is useful to examine the impacts on the total land rent.

$$\left.\frac{\partial R}{\partial I}\right|_{b=\text{const.}} = \frac{\bar{r}B}{\beta} \quad \text{where} \quad B \equiv \int_0^b \left[-\hat{t}_I(x) + \hat{t}_I(b)\right] \phi(x)^{\frac{1}{\beta}} \, \mathrm{d}S(x). \tag{20}$$

[10] Note that $k' \equiv \left(\frac{\sigma - 1}{v}\right)^{\alpha} \left(\frac{\alpha}{\sigma^{\sigma} F \beta}\right)^{\lambda} \bar{r}^{-\beta} > 0.$

Here, $-\hat{t}_I(x) + \hat{t}_I(b)$ represents the intensity of transport cost reduction at location x compared to that at the city boundary. B is a weighted sum of such intensity, with the weight, $\phi(x)^{1/\beta}$, decreasing with the distance from the CBD. Thus, we can interpret B as a measure of how investments are biased to the locations closer to the CBD. We call it a *spatial bias* in investment allocation.

Totally differentiating (19) yields

$$\frac{\mathrm{d}U(b)}{\mathrm{d}I} = k' \frac{R^{\lambda-1}}{t(b)} \left(\frac{\mathrm{d}b}{\mathrm{d}I} U_b + U_I \right), \tag{21}$$

where

$$U_b \equiv \frac{\lambda}{\beta} \left[\bar{r}\beta S'(b) + R\hat{t}_x(b) \right] - R\hat{t}_x(b) \quad \text{and} \tag{22}$$

$$U_I \equiv \frac{\bar{r}\lambda B}{\beta} - R\hat{t}_I(b), \tag{23}$$

are the partial derivatives of $U(b)$ with respect to b and I, respectively, both divided by $k'R^{\lambda-1}/t(b) > 0$. Furthermore, $\hat{t}_I(x) \equiv \dfrac{\partial t(x)/\partial I}{t(x)} < 0$ is the rate of the reduction in the generalized transport costs from location x.

U_b measures the impact of infrastructure investments transmitted through the change in a city size. Its sign is ambiguous. On the one hand, as a city expands, the service varieties provided there increase. This is because the total land rent increases. Indeed, the term inside the square brackets at the right hand side of (22) is equal to $\mathrm{d}R/\mathrm{d}b$ up to the multiplication by a constant (see (18)). This positively affects U_b. We call this effect an *indirect variety effect*, "indirect" because it is the effect through the change in a city size. On the other hand, the increase in land rent brought about by the rise in CT at each location gives a negative impact on the indirect utility, which is represented by the last term at the right hand side of (22). This effect is referred to as an *indirect living-cost effect*.

Next, U_I captures the other impacts, which appear even if a city size remains unchanged. The first term at the right hand side of (23) represents a *direct variety effect*. As we have seen in (20), the total land rent increases by $\bar{r}B/\beta$. This increases service varieties provided in a city. Furthermore, the second term captures a *direct living-cost effect*, which is positive. The living costs, $\bar{r}^{\beta}t(b)$, decline as the transport costs from the city boundary decrease. Unfortunately, the sign of U_I is ambiguous. One of the sufficient conditions for positive U_I is that

$$B \geq 0. \tag{24}$$

We refer to (24) as an *investment allocation condition*. It is satisfied when the investments are conducted relatively more intensively at the locations closer to the CBD.

5 Impacts of infrastructure investments on the compactness of a city

In this section, we study the impacts of transportation infrastructure investments on the "compactness" of a city. Researchers have been discussing the appropriate indices of the compactness, for the notion is associated with such various characteristics as a high population density, a high volume of buildings, a lack of unused land, and a concentration of employments (Burton, 2002; Tsai, 2005; Lee et al., 2015; Koziatek and Dragićević, 2019, among others).[11] Because the definition of the compactness is not our major concern, we do not go into detail on it, but rather pick up some important variables related to the compactness of a city, namely, the geographical size of a city, population density, total population, and the range of service varieties provided in a city.

5.1 The impact on the geographical size of a city

One of the most important measures of city's compactness is its geographical size, given by b in our model. What is more, the geographical size plays a critical role in the discussion of the impacts of transportation infrastructure investments on the other measures. That is the reason we begin the analysis with the impact on the geographical size.

Because the utility equivalence requires that $\mathrm{d}U(b)/\mathrm{d}I = 0$ (see (15)), it immediately follows from (21) that

$$\frac{\mathrm{d}b}{\mathrm{d}I} = -\frac{U_I}{U_b} = \frac{\beta R\hat{t}_I(b) - \bar{r}\lambda B}{\bar{r}\lambda\beta S'(b) + (\lambda - \beta)R\hat{t}_x(b)}. \tag{25}$$

If this derivative is negative, the city geographically shrinks as a result of infrastructure investments. If it is positive, instead, it expands.

For a while, we focus on the case in which the investment allocation condition, (24), is satisfied. Then, U_I is positive, and therefore, the sign of $\mathrm{d}b/\mathrm{d}I$ becomes the opposite to that of U_b.

First, we examine the effect of a change in λ on the sign of U_b. We can show that there is λ^0 such that $U_b \left\{ \begin{smallmatrix} \leqq \\ > \end{smallmatrix} \right\} 0$ if $\lambda \left\{ \begin{smallmatrix} \leqq \\ > \end{smallmatrix} \right\} \lambda^0$ (see the proof of Proposition 2). It turns out that

$$\lambda^0 = \frac{\beta R\hat{t}_x(b)}{\bar{r}\beta S'(b) + R\hat{t}_x(b)} \in (0, \beta). \tag{26}$$

Then, the following result immediately follows.

[11] For the review, see OECD (2012), and Ahlfeldt and Pietrostefani (2017).

Proposition 2

Suppose that the investment allocation condition is satisfied. Then, $db/dI \left\{ \begin{array}{c} \geqq \\ < \end{array} \right\} 0$ *if* $\lambda \left\{ \begin{array}{c} \leqq \\ > \end{array} \right\} \lambda^0$ *for* $\lambda^0 \in (0, \beta)$.

Proof of Proposition 2

Observe that U_b is continuous with respect to λ, $\mathrm{d}U_b/\mathrm{d}\lambda > 0$, $\lim_{\lambda \to 0} U_b < 0$, and $\lim_{\lambda \to \beta} U_b > 0$. Therefore, there exists a unique $\lambda^0 \in (0, \beta)$ that solves $U_b = 0$, for which $U_b \left\{ \begin{array}{c} \leqq \\ > \end{array} \right\} 0$ if $\lambda \left\{ \begin{array}{c} \leqq \\ > \end{array} \right\} \lambda^0$. Since $U_I > 0$ by the investment allocation condition, the proposition immediately follows from (25). ■

According to Proposition 2, whether infrastructure investments enlarge a city or not depends on the consumers' valuation of the range of service varieties. *If consumers attach a little value to the range of varieties, the investments cause a geographical expansion of a city. Instead, if they value it greatly, a city shrinks as a result of the investments.*

Why do infrastructure investments cause the expansion of a city when λ is low? Suppose that the location of a city boundary remains unchanged. The transport costs from the boundary decline as a result of investments, which raises indirect utility through the decrease in the living costs, namely, the direct living-cost effect. In addition, the direct variety effect is also positive as long as the investment allocation condition holds. Therefore, the indirect utility rises. To keep the utility at the level before the investments, the city boundary needs to move so that the utility decreases. Now, when λ is sufficiently low, $U_b < 0$ because the indirect living-cost effect outweighs the indirect variety effect (see (22)). Consequently, the city boundary needs to move farther away from the CBD. In contrast, when λ is sufficiently high, $U_b > 0$ because the indirect living-cost effect is dominated by the indirect variety effect. Therefore, the boundary must move inward rather than outward.

Recall that λ increases with α and decreases with σ. Therefore, it is more likely that the city geographically expands when the expenditure share on service varieties is low and when each variant of services is not much unique but considerably substitutable with another variant.

Furthermore, closer inspection of λ^0 gives the effects of changes in $S'(b)$ and $\hat{t}_x(b)$. First, since $\mathrm{d}\lambda^0/\mathrm{d}S'(b) < 0$, small $S'(b)$ makes $\mathrm{d}b/\mathrm{d}I$ more likely to be positive for a given value of λ. That is, when only a small amount of land is available for residential use just across a city boundary, infrastructure investments are more likely to enlarge a city. This is explained as follows. When $S'(b)$ is sufficiently small, just a small amount of land is incorporated into a city as the city's boundary moves outward by a given distance. Thus, varieties do not increase too much, that is, the positive indirect variety effect is small. As a result, the utility level falls sharply, or rises only modestly. However, recall that the utility equivalence requires the utility level to fall. This means that the city is more likely to expand. Second, since $\mathrm{d}\lambda^0/\mathrm{d}\hat{t}_x(b) > 0$, large $\hat{t}_x(b)$ makes $\mathrm{d}b/\mathrm{d}I$ more likely to be positive. In other words, when the expansion of a city brings about a sharp increase in the transport costs from a city boundary, the city is more likely to expand as a result of infrastructure investments. We can explain this result by the reasoning analogous to that for the change in $S'(b)$. These findings are summarized as follows.

Corollary 1

Suppose that the investment allocation condition is satisfied. Then, infrastructure investments are more likely to expand a city when $S'(b)$ is small and $\hat{t}_x(b)$ is large.

Proposition 2 and Corollary 1 give the conditions for the geographical expansion of a city. The empirical literature, which finds out that a city actually expands as a result of transport infrastructure investments, suggests that those conditions are satisfied. Therefore, we can expect that λ and $S'(b)$ are sufficiently low and $\hat{t}_x(b)$ is sufficiently high. If policymakers advocate compact city policies and the policies to improve public transportation systems *at the same time*, they probably have in mind the situation in which such conditions do not hold. We can argue that this is one reason for the gap between those policymakers and the empirical economists.

To close this subsection, we ask how to reconcile our results with those in the AMM model, in which infrastructure investments always bring about a spatial expansion of a city. First, the AMM model does not take into account consumers' love for variety, and therefore, assumes that $\alpha = 0$. Second, in that model, analysis is limited to the investments that involve a proportional decrease in transport costs at every location. In other words, it assumes that $\hat{t}_I(x)$ is equal to a certain fixed value for any $x \in [0, b]$. Since it implies that $\hat{t}_I(x) = \hat{t}_I(b)$ for any $x \in [0, b]$, the investment allocation condition, (24), is always satisfied. In this way, $\alpha = 0$ and the investment allocation condition hold in the AMM model, which leads to $\mathrm{d}b/\mathrm{d}I > 0$, according to Proposition 2.

5.2 The impact on the population density

One of the other important indices of city's compactness is population density. We refer to the area within x km from the CBD as a city's inner area with distance x. The density at such an inner area is defined as $D(x) \equiv G(x)/S(x)$. Since

$$\frac{\mathrm{d}D(x)}{\mathrm{d}I} = -\frac{1}{S(x)} \int_0^x \frac{1}{N(y)^2} \frac{\mathrm{d}N(y)}{\mathrm{d}I} \, \mathrm{d}S(y), \tag{27}$$

the direction of the change in the density hinges on the sign of $\mathrm{d}N(x)/\mathrm{d}I$ at each x.

Note that

$$\frac{\mathrm{d}N(x)}{\mathrm{d}I} = -N(x)\left[\hat{\mu}_I(x) + \frac{\hat{\phi}_I(x)}{\beta}\right], \tag{28}$$

where $\hat{\mu}_I(x) \equiv \dfrac{\partial\mu(x)/\partial I}{\mu(x)} < 0$, and $\hat{\phi}_I(x)$ is given by

$$\hat{\phi}_I(x) \equiv \frac{\partial\phi(x)/\partial I}{\phi(x)} = -\hat{t}_I(x) + \hat{t}_I(b) + \frac{\mathrm{d}b}{\mathrm{d}I}\hat{t}_x(b). \tag{29}$$

(28) and (29) indicate that following three factors affect the size of $\mathrm{d}N(x)/\mathrm{d}I$, among others.

First, it decreases with $\hat{\mu}_I(x)$. When $\hat{\mu}_I(x)$, which is negative, is mathematically small ($|\hat{\mu}_I(x)|$ is large), the commuting costs from x fall sharply by the investments. Because disposable income rises to a great extent, workers increase the consumption of land by a considerable amount, and, as a result, each land lot at x expands greatly.

Second, $\mathrm{d}N(x)/\mathrm{d}I$ decreases with $-\hat{t}_I(x) + \hat{t}_I(b)$. Suppose that $-\hat{t}_I(x) + \hat{t}_I(b)$ is mathematically small, that is, the reduction rate of the transport costs from x ($|\hat{t}_I(x)|$) is relatively low compared to that from the city boundary ($|\hat{t}_I(b)|$). Then, the CT at x becomes low, which implies low land rent at that location. Workers at x consume a relatively large amount of land.

Third and finally, $\mathrm{d}N(x)/\mathrm{d}I$ decreases with $\mathrm{d}b/\mathrm{d}I$, the marginal "increase" in a city size. For the sake of explanation, we focus on the case with $\mathrm{d}b/\mathrm{d}I > 0$, that is, the case in which the infrastructure investments make a city larger. The case with $\mathrm{d}b/\mathrm{d}I < 0$ can be explained in a similar manner. Now, if $\mathrm{d}b/\mathrm{d}I > 0$ is high, the CT and the land rent rises sharply at each location. Therefore, workers reduce the consumption of land lot substantially. Thus, *a greater tendency of a city toward geographical expansion is associated with a smaller size of each land lot, other things being equal*. This may be counter-intuitive, because we often take it for granted that a city that spreads over a wide area is characterized by more sparse land use. In our model, however, the decentralization enhances the locational advantage (the CT) at each inner location, which increases the land rent at that location and drives workers to consume a smaller land lot. As well, the result indicates that *a greater tendency toward geographical compactness is associated with a larger size of each land lot, other things being equal*. Here again, what works behind this seemingly counter-intuitive finding is the induced changes in the CT and land rent.

As is clear from (27), those factors that affect $\mathrm{d}N(x)/\mathrm{d}I$ act on $\mathrm{d}D(x)/\mathrm{d}I$ in the opposite direction. Thus, $\mathrm{d}D(x)/\mathrm{d}I$ tends to be high when $\hat{\mu}_I(x)$, $-\hat{t}_I(x)+\hat{t}_I(b)$, and $\mathrm{d}b/\mathrm{d}I$ are high at each x.

Although the sign of $\mathrm{d}D(x)/\mathrm{d}I$ is ambiguous in general, we can obtain an unambiguous result for a benchmark case. It is the case in which

$$\hat{t}_I(x) = \hat{t}_I \quad \text{for any } x \geq 0, \tag{30}$$

that is, transportation infrastructure is improved at a constant rate all over a city. For this case of *uniform investments*, we have the following result.

Proposition 3

Consider uniform investments. If they reduce the size of a city or keep it constant, the population density decreases in any inner area of a city.

Proof of Proposition 3 (30) implies that $\hat{\phi}_I(x) \leq 0$, which leads to $\mathrm{d}N(x)/\mathrm{d}I > 0$ and, therefore, $\mathrm{d}D(x)/\mathrm{d}I < 0$. ■

The above proposition shows that the shrinkage of a city is accompanied by a decrease in population density, as long as uniform investments are concerned. However, the expansion of a city does not necessarily lead to the increase in population density, even if investments are uniform. It may result in the decrease in population density depending on parameter values.

The ambiguity of the direction of a change in the density can be a source of a policy debate. If the density increases, city residents will be forced to live in smaller housing. At the same time, the decrease in the use of automobile will alleviate environmental burdens. If the density decrease, to the contrary, we will see the trade-off in the opposite direction. Thus, whether the density increases or not greatly affects the evaluation of infrastructure investment policies.

5.3 The impact on the total population

To see the impact of infrastructure investments on the total population, we derive

$$\frac{\mathrm{d}L}{\mathrm{d}I} = \frac{\mathrm{d}b}{\mathrm{d}I}\left[\frac{S'(b)}{N(b)} + \frac{\hat{t}_x(b)L}{\beta}\right] + \int_0^b \frac{1}{N(x)}\left[\hat{\mu}_I(x) + \frac{1}{\beta}\{-\hat{t}_I(x)+\hat{t}_I(b)\}\right]\,\mathrm{d}S(x) \tag{31}$$

from (6), (14), (28), and (29). Since the term in the first pair of square brackets at the right hand side is positive, $\mathrm{d}L/\mathrm{d}I$ is positively related to $\mathrm{d}b/\mathrm{d}I$: city population is more likely to increase when city expands more as a result of investments.

To obtain better insight, let us focus on the uniform investments, for which the integral in (31) becomes negative. Three cases are distinguished. First, if the size of a city remains unchanged ($\mathrm{d}b/\mathrm{d}I = 0$), total population decreases by infrastructure investments. Second, if the city shrinks ($\mathrm{d}b/\mathrm{d}I < 0$), total population again decreases. Finally, if the city expands ($\mathrm{d}b/\mathrm{d}I > 0$), total population may increase or decrease. It decreases when commuting costs from respective locations decline sharply ($|\hat{\mu}_I(x)|$ is sufficiently large for respective x). In that case, workers' disposable income increases greatly, and they consume such a larger amount of land that a city can accommodate only a fewer number of them. The next proposition sums up these observations.

Proposition 4

Consider uniform investments. If they reduce the size of a city or keep it constant, total population decreases. Instead, if they enlarge a city, the total population may increase or decrease depending on parameter values.

5.4 The impact on the range of service varieties

Next, we examine how transportation infrastructure investments affect the range of service varieties. Since they are produced and sold at the CBD, the range is associated with the liveliness and the prosperity of business at a city center. Thus, we can argue that the investments revitalize a city center if they widen the range of service varieties.

It follows from (17) that

$$\frac{\mathrm{d}n}{\mathrm{d}I} = \frac{n}{\beta R}\left[\frac{\mathrm{d}b}{\mathrm{d}I}\left\{\bar{r}\beta S'(b) + R\hat{t}_x(b)\right\} + \bar{r}B\right].$$

Now, suppose that the investment allocation condition is satisfied. Three cases are distinguished. First, if the size of a city remains unchanged ($\mathrm{d}b/\mathrm{d}I = 0$), service varieties increase by infrastructure investments. Second, if the city expands ($\mathrm{d}b/\mathrm{d}I > 0$), varieties again increase. Finally, if the city shrinks ($\mathrm{d}b/\mathrm{d}I < 0$), varieties may increase or decrease. They increase when the spatial bias in investment allocation, B, is relatively large compared with the effect through a change in the location of a city boundary. The next proposition sums up these observations.

Proposition 5

Suppose that the investment allocation condition is satisfied. If infrastructure investments increase the size of a city or keep it constant, the service varieties provided at a city increase. Instead, *if they reduce the size of a city, the varieties may increase or decrease depending on parameter values.*

6 Feasibility and desirability of infrastructure investments

Our final task is to discuss welfare implications of infrastructure investments.

In our model, the change in a welfare level brought about by infrastructure investments is measured by that in total land rent. First, consumers enjoy the same level of utility before and after the investments, because we are considering a small open city. Second, producers always receive the same amount of profit, namely, zero profit, which results from the assumption of free entry and exit. Third, transport costs affect equilibrium outcomes only through the consumers' budget constraint. Therefore, what is left as a source of welfare change is solely the change in total land rent, which is taken over by absentee landlords.

We are assuming that transportation infrastructure investments are financed from the increase in total land rent. Therefore, a government can make investments if and only if that increase is at least equal to the cost of investments. Furthermore, welfare does not worsen by investments if and only if the same condition is satisfied. Therefore, *whenever investments are feasible, they are desirable, and vice versa*. The government has no conflict between the implementation of the policy and its financing. (In what follows, we will refer to only the feasibility to save the space, but the feasibility must be read as the feasibility *and* the desirability.)

When the cost of infrastructure investments, I, is sufficiently small, the increase in total land rent brought about by investments is approximated by $I \cdot \mathrm{d}R/\mathrm{d}I$. In that case, therefore, that increase is not smaller than the cost of investments if and only if $\mathrm{d}R/\mathrm{d}I \geq 1$. In other words, the investments are feasible if and only if

$$\frac{\mathrm{d}R}{\mathrm{d}I} = \bar{r}\left[S'(b)\frac{\mathrm{d}b}{\mathrm{d}I} + \frac{B}{\beta}\right] \geq 1. \tag{32}$$

The first term in the square brackets represents the effect of the land that is added to or removed from a city. If it is positive, it describes the effect of the land newly incorporated to a city. Instead, if it is negative, it indicates the effect of the land transformed into agricultural ground. The second term represents the effect of the rise in land rent at each location, which is caused by the decrease in transport costs due to the infrastructure investments.

(32) shows that infrastructure investments are feasible when one or some of the following conditions are satisfied. First, $S'(b)$ is large. That is, a large area is added to a city when it

expands, and a small area is removed from it when it shrinks. Second, $\mathrm{d}b/\mathrm{d}I$ is large: the city expands greatly or shrinks only slightly. Third and finally, B is large, that is, the allocation of investments is strongly biased to inner locations. This condition has an important implication. Even when investments reduce the size of a city, they may be feasible if infrastructure is improved particularly in its inner parts such as the areas surrounding the CBD. In contrast, if the main purpose of investments is the development of a new sub-center at a city's fringe, for example, they may not be feasible.

To explore the condition for the feasibility of investments further, we examine the effect of a change in the preference parameter λ on $\mathrm{d}R/\mathrm{d}I$. For that purpose, it is useful to rewrite $\mathrm{d}R/\mathrm{d}I$ as follows.

$$\frac{\mathrm{d}R}{\mathrm{d}I} = \frac{\bar{r}\lambda^0}{\beta^2(\lambda - \lambda^0)}\Big[(\lambda - \beta)B - \beta^2\Psi\Big], \quad \text{where} \quad \Psi \equiv -S'(b)\frac{\hat{t}_I(b)}{\hat{t}_x(b)} > 0, \tag{33}$$

and λ^0 has been defined in (26). Here, we avoid unnecessary complications by assuming that the allocation condition, (24), is satisfied. The following properties of $\mathrm{d}R/\mathrm{d}I$ immediately follow from (33). First, it is not continuous at $\lambda = \lambda^0$. Second, except for that point, it increases with λ.[12] Third, it goes to positive infinity as λ approaches λ^0 from below, and to negative infinity as λ approaches λ^0 from above. Fourth, it takes a positive value, R_I^0, at $\lambda = 0$, and approaches a positive constant, R_I^∞, as λ diverges to positive infinity, where

$$R_I^0 \equiv \frac{\bar{r}}{\beta}\big(B + \beta\Psi\big) > 0, \quad \text{and} \quad R_I^\infty \equiv \frac{\bar{r}}{\beta^2}\lambda^0 B > 0.$$

Here, it is straightforward to see that $R_I^0 > R_I^\infty$. Those four properties are illustrated in Fig. 1, which shows the relation between λ and $\mathrm{d}R/\mathrm{d}I$.

Insert Fig. 1 around here.

Recall that λ^0 is a critical value of λ that determines the direction of a change in the geographical size of a city (see Proposition 2). The discontinuity of $\mathrm{d}R/\mathrm{d}I$ at λ^0 stems from the divergence in that direction. Given that a city expands by investments, $\lambda^0 - \epsilon$ is a sufficiently high value of λ that total land rent increases enough to cover the investment costs. However, given that it shrinks, $\lambda^0 + \epsilon$ is not a sufficiently high value of λ, but λ needs to be much higher.

[12] It follows from $\lambda^0 < \beta$ that $\dfrac{\mathrm{d}^2R}{\mathrm{d}\lambda\,\mathrm{d}I} = \dfrac{\bar{r}\lambda^0}{\beta^2(\lambda - \lambda^0)^2}\Big[(\beta - \lambda^0)B + \beta^2\Psi\Big] > 0.$

Let λ^* be the value of λ that solves $\mathrm{d}R/\mathrm{d}I = 1$:

$$\lambda^* \equiv \beta\lambda^0 \cdot \frac{\bar{r}\big(B + \beta\Psi\big) - \beta}{\bar{r}\lambda^0 B - \beta^2}.$$

We can distinguish three cases depending on the sizes of R_I^0 and R_I^∞. First, suppose that $R_I^0 < 1$. Then, λ^* lies in the interval $(0, \lambda^0)$. Therefore, for $\lambda \in [0, \lambda^0)$, $\mathrm{d}R/\mathrm{d}I \left\{ \begin{smallmatrix} \lesseqqgtr \\ > \end{smallmatrix} \right\} 1$ if $\lambda \left\{ \begin{smallmatrix} \lesseqqgtr \\ > \end{smallmatrix} \right\} \lambda^*$. Furthermore, since $R_I^0 > R_I^\infty$, we have $R_I^\infty < 1$. Therefore, $\mathrm{d}R/\mathrm{d}I < 1$ for any $\lambda > \lambda^0$. Second, suppose that $R_I^0 \geq 1$ and $R_I^\infty \leq 1$. The former inequality implies that $\mathrm{d}R/\mathrm{d}I \geq 1$ for any $\lambda \in [0, \lambda^0)$. However, it follows from the latter inequality that $\mathrm{d}R/\mathrm{d}I < 1$ for any $\lambda > \lambda^0$. Third, suppose that $R_I^\infty > 1$, which implies that $R_I^0 \geq 1$. Again in this case, $\mathrm{d}R/\mathrm{d}I \geq 1$ for any $\lambda \in [0, \lambda^0)$. Furthermore, λ^* lies at the right of λ^0. Therefore, for any $\lambda > \lambda^0$, $\mathrm{d}R/\mathrm{d}I \left\{ \begin{smallmatrix} \lesseqqgtr \\ > \end{smallmatrix} \right\} 1$ if $\lambda \left\{ \begin{smallmatrix} \lesseqqgtr \\ > \end{smallmatrix} \right\} \lambda^*$.

Let $\underline{B} \equiv \beta\left(\dfrac{1}{\bar{r}} - \Psi\right)$ and $\bar{B} \equiv \dfrac{\beta^2}{\bar{r}\lambda^0}$, where $\underline{B} < \bar{B}$. With some manipulations of the conditions prescribing respective cases, we can summarize these three cases as follows.

Proposition 6
i) Suppose that $B < \underline{B}$. Then, $\lambda^* \in (0, \lambda^0)$, and investments are feasible if and only if $\lambda \in [\lambda^*, \lambda^0)$.
ii) Suppose that $B \in \left[\underline{B}, \bar{B}\right]$. Then, investments are feasible if and only if $\lambda \in [0, \lambda^0)$.
iii) Suppose that $B > \bar{B}$. Then, $\lambda^* > \lambda^0$, and investments are feasible if and only if $\lambda \in [0, \lambda^0) \cup (\lambda^*, \infty)$.

We make several remarks concerning Proposition 6.

First, recall that a city becomes smaller by infrastructure investments if $\lambda > \lambda^0$. Therefore, in cases i) and ii), *investments are not feasible whenever they reduce the geographical size of a city*. In contrast, the city expands if $\lambda < \lambda^0$. Therefore, in cases ii) and iii), *investments are feasible whenever they result in the expansion of a city.* Thus, we can say that not a shrinkage but an expansion of a city is more likely to be associated with feasible investments.

Furthermore, in case iii), the feasibility changes in a non-monotonic manner as λ gradually increases from $\lambda = 0$. At first, investments remain feasible as long as λ is smaller than λ^0. However, as soon as it reaches λ^0, they become infeasible. Then, the feasibility is recovered when λ exceeds λ^*.[13]

Moreover, Proposition 6 indicates that high B enhances the feasibility of investments through two channels. The first channel is related to local changes. As B rises, λ^* decreases. This widens

[13] Here, λ^* is bounded, and λ diverges to positive infinity as σ approaches 1 from above. Therefore, for sufficiently low σ, we have $\lambda > \lambda^*$. Hence, there necessarily exists the possibility of the recovery of the feasibility.

the interval $[\lambda^*, \lambda^0)$ in case i) and the interval (λ^*, ∞) in case iii), both of which are the intervals of λ that make investments feasible. The second channel is related to a global change. As B rises from a sufficiently low level, the relevant case switches from i) to ii) and then from ii) to iii). Correspondingly, the range of λ for feasible investments, which we refer to as a feasibility range, widens in a stepwise manner. The reason is simple. When B is high, investments are abundantly allocated to inner locations compared to the city boundary. Thus, land rent at those locations increases considerably, which guarantees the feasibility.

Other parameters affect the feasibility of investments through the local and the global channels as well. As an example, take a look at a change in $S'(b)$. First, we examine the local effects. As $S'(b)$ increases, λ^0 decreases. Furthermore, it turns out that λ^* also decreases in case i) but increases in case iii).[14] Therefore, the interval $[\lambda^*, \lambda^0)$ in case i) expands or shrinks, the interval $[0, \lambda^0)$ in cases ii) and iii) shrinks, and the interval (λ^*, ∞) in case iii) shrinks. Thus, high $S'(b)$ reduces the feasibility of investments in cases ii) and iii), but the effect is ambiguous in case i). Second, as $S'(b)$ increases, Ψ increases, which makes $\underline{B}$ smaller. Thus, case i) becomes less likely to occur and case ii) becomes more likely to occur. This is the global effect.

This ambiguity in the effects of a parameter change also applies to the change in $\hat{t}_x(b)$. All the effects operate in the opposite direction to that in the case of a change in $S'(b)$. Thus, for a local effect, the rise in $\hat{t}_x(b)$ strengthens the feasibility of investments in cases ii) and iii), but the result depends in case i). For a global effect, the rise in $\hat{t}_x(b)$ increases the likelihood for case i) to occur but decreases that for case ii) to occur.

7 Concluding remarks

This study explores the impacts of transport infrastructure investments on the compactness of a city. To answer the questions as to whether and how the investments cause decentralization, we extend the AMM model by incorporating two novel factors. First, city residents consume differentiated service varieties represented by urban amenities. Second, they pay transport costs not only for commute but also to shop such varieties. With these factors considered, infrastructure investments increase the number of varieties provided in a city, given that the size of a city remains unchanged. This is because the induced fall in transport costs enhances the disposable income of city residents. Whether a city decentralizes or not is determined by the relative magnitudes of two effects, namely, the variety effects of the change in a range of service varieties and the living-cost

[14] It is readily verified that $\dfrac{\partial \lambda^*}{\partial S'(b)} = \lambda^* \left[\dfrac{\partial \lambda^0}{\partial S'(b)} \Theta_1 + \dfrac{\partial \Psi}{\partial S'(b)} \Theta_2 \right]$, where $\Theta_1 \equiv \dfrac{\beta^2}{\lambda^0(\beta^2 - \bar{r}\lambda^0 B)}$ and $\Theta_2 \equiv \dfrac{\bar{r}\beta}{\bar{r}(B + \beta\Psi) - \beta}$. Now, $\partial \lambda^0 / \partial S'(b) < 0$ and $\partial \Psi / \partial S'(b) > 0$. Furthermore, $\Theta_1 > 0$ and $\Theta_2 < 0$ in case i), whereas $\Theta_1 < 0$ and $\Theta_2 > 0$ in case iii). Therefore, $\partial \lambda^* / \partial S'(b)$ is negative in case i) and positive in case iii).

effects of the changes in land rent and transport costs.

We show that infrastructure investments reduce the size of a city if workers attach a sufficiently high value to the range of service varieties. Furthermore, we examine the impacts of investments on other variables, namely, population density, total population, and the range of service varieties provided in a city. Finally, we obtain the conditions for the feasibility and the desirability of investments. Those conditions are more likely to be satisfied when the consumers' valuation on the range of service varieties is low, and when the investments are abundantly allocated to inner locations compared to peripheral ones.

To conclude the paper, we discuss limitations of this work and the directions of future research.

First, as has been discussed in the introduction, this work focuses on a monocentric city. Equipped with some insights on the basic mechanism through which infrastructure investments affect urban spatial structure, we can advance to the next step by removing the assumption of firms' fixed locations. However, allowing for firms' location decisions complicates the analysis, because we need to consider the effects of infrastructure investments not only on consumers' locations but also on firms' locations. What is more, in the location decisions of firms, the transport costs to ship inputs in production processes play an important role. Thus, we need to consider two completely different kinds of transport costs, those to ship the inputs and those to ship final goods and services. Because the impacts of infrastructure investments are also different between those two kinds of transport costs, we expect that the analysis becomes much more involved.

Second, in this paper, we abstract away the possibility of a mode choice to focus on the two novel factors mentioned above. However, the improvement in a particular transportation mode induces some residents to switch from other modes to that mode. These changes affect the spatial structure of a city. To go into this line is left for future research.

Third, both the commuting costs and the shopping costs are assumed to be the iceberg type, which implies that no one receives transport cost payments. In the reality, however, they can be an important financial resource for infrastructure investments. In particular, for the local governments that provide public transportation services, the revenue from passenger fares is often one of the major funding sources. Thus, the future research needs to take into account the monetary flows of transportation costs.

Fourth and last, our model does not consider the possibility that workers combine commuting and shopping in one sequence of a trip. However, this is not a realistic assumption, because not a small number of shopping behaviors take place during a trip to go to work in the real economy. Although allowing for multi-purpose trips makes the analysis much more complicated, expanding the theoretical model in that direction will help us acquire a better understanding of urban spatial structure.

References

Ahlfeldt, G. M, and N. Wendland. (2011) "Fifty years of urban accessibility: The impact of the urban railway network on the land gradient in Berlin 1890–1936," *Regional Science and Urban Economics*, 41, 77–88.

Ahlfeldt, G. M, and E. Pietrostefani. (2017) "The compact city in empirical research: A quantitative literature review," *SERC discussion paper*, 215.

Albouy, D., and B. A. Stuart. (2020) "Urban population and amenities: The neoclassical model of location," *International Economic Review*, 61, 127–158.

Alonso, W. (1964) *Location and Land Use*, Harvard University Press, Cambridge, MA, US.

American Planning Association. (2012) *APA Policy Guide on Smart Growth*, ratified in 2002 and updated in 2012. https://www.planning.org/policy/guides/adopted/smartgrowth.htm (accessed May 27, 2022).

Anas, A., R. Arnott, and K. A. Small. (1998) "Urban spatial structure," *Journal of Economic Literature*, 36, 1426–1464.

Anas, A. and L. N. Moses. (1979) "Mode choice, transport structure and urban land use," *Journal of Urban Economics*, 6, 228–246.

Baum-Snow, N. (2007a) "Suburbanization and transportation in the monocentric model," *Journal of Urban Economics*, 62, 405–423.

Baum-Snow, N. (2007b) "Did highways cause suburbanization?" *The Quarterly Journal of Economics*, 122, 775–805.

Baum-Snow, N. (2020) "Urban transport expansions and changes in the spatial structure of U.S. cities: Implications for productivity and welfare," *The Review of Economics and Statistics*, 102, 929–945.

Baum-Snow, N., and M. E. Kahn. (2000) "The effects of new public projects to expand urban rail transit," *Journal of Public Economics*, 77, 241–263.

Baum-Snow, N., and M. E. Kahn. (2005) "Effects of urban rail transit expansions: Evidence from sixteen cities, 1970-2000," *Brookings-Wharton Papers on Urban Affairs*, (2005) 147–206.

Brueckner, J. K. (2001) "Urban sprawl: Lessons from urban economics," in *Brookings-Wharton Papers on Urban Affairs* ed. by J. K. Brueckner, E. Mills, and M. Kremer, Brookings Institution Press, 65–97.

Burton, E. (2002) "Measuring urban compactness in UK towns and cities," *Environment and Planning B*, 29, 219–250.

DeSalvo, J., and M.Huq. (1996) "Income, residential location, and mode choice," *Journal of Urban Economics*, 40, 84–99.

Duranton, G., and D. Puga. (2015) "Urban land use," in *Handbook of Regional and Urban Economics* vol. 5 ed. by G. Duranton, J. V. Henderson, and W. C. Strange, Elsevier, Amsterdam, Netherlands, 467–560.

Fujita, M., P. Krugman, and A. J. Venables. (1999) *The Spatial Economy: Cities, Regions, and International Trade*, MIT Press, Cambridge, MA, US.

Fujita, M., and H. Ogawa. (1982) "Multiple equilibria and structural transition of non-monocentric urban configurations," *Regional Science and Urban Economics*, 12, 161–196.

Fujita, M., and J.-F. Thisse. (2002) *The Economics of Agglomeration*, Cambridge University Press, Cambridge, MA, US.

Gaigné, C., S. Riou, and J.-F. Thisse. (2012) "Are compact cities environmentally friendly?" *Journal of Urban Economics*, 72, 123–136.

Gin, A., and J. Sonstelie. (1992) "The streetcar and residential location in nineteenth century Philadelphia," *Journal of Urban Economics*, 32, 92–107.

Garcia-López, M.-À. (2012) "Urban spatial structure, suburbanization and transportation in Barcelona," *Journal of Urban Economics*, 72, 176–190.

Garcia-López, M.-À. (2019) "All roads lead to Rome... and to sprawl? Evidence from European cities," *Regional Science and Urban Economics*, 79, 103467.

Glaeser, E. L., and M. E. Kahn. (2004) "Sprawl and urban growth," in *Handbook of Regional and Urban Economics* vol. 4 ed. by J. H. Henderson, and J.-F. Thisse, Elsevier, Amsterdam, Netherlands, 2481–2527.

Glaeser, E. L., J. Kolko, and A. Saiz (2001) "Consumer city," *Journal of Economic Geography*, 1, 27–50.

Gonzalez-Navarro, M., and M. A. Turner. (2018) "Subways and urban growth: Evidence from earth," *Journal of Urban Economics*, 108, 85–106.

Heblich, S., S. J. Redding, and D. M. Sturm. (2020) "The making of the modern metropolis: Evidence from London," *The Quarterly Journal of Economics*, 135, 2059–2133.

Kahn, M. E. (2000) "The environmental impact of suburbanization," *Journal of Policy Analysis and Management*, 19, 569–586.

Koziatek, O., and S. Dragićević. (2019) "A local and regional spatial index for measuring three-dimensional urban compactness growth," *Environment and Planning B*, 46, 143–164.

Lee, J., K. Kurisu, K. An, and K. Hanaki. (2015) "Development of the ompact city index and its application to Japanese cities," *Urban Studies*, 52, 1054–1070.

LeRoy S. F., and J. Sonstelie. (1983) "Paradise lost and regained: Transportation innovation, income, and residential location," *Journal of Urban Economics*, 13, 67–89.

Lucas, R., and E. Rossi-Hansberg. (2002) "On the internal structure of cities," *Econometrica*, 70, 1445–1476.

Mayer, T., and C. Trevien. (2017) "The impact of urban public transportation evidence from the Paris region," *Journal of Urban Economics*, 102, 1–21.

Mills, E. S. (1967) "An aggregative model of resource allocation in a metropolitan area," *American Economic Review*, 57, 197–210.

Mossay, P., P. M. Picard, and T. Tabuchi. (2020) "Urban structures with forward and backward linkages," *Regional Science and Urban Economics*, 83, 103522.

Muth, R. F. (1969) *Cities and Housing*, University of Chicago Press, Chicago, IL, US.

Murata, Y., and J.-F. Thisse. (2005) "A simple model of economic geography à la Helpman–Tabuchi," *Journal of Urban Economics*, 58, 137–155.

OECD. (2012) *Compact city policies: A comparative assessment*, OECD Green Growth Studies, OECD Publishing, Paris, France.

OECD. (2018) *Rethinking urban sprawl: Moving towards sustainable cities*, OECD Publishing, Paris, France.

Picard, P. M., and T. Tabuchi. (2013) "On microfoundations of the city," *Journal of Economic Theory*, 148, 2561–2582.

Redding, S. J., and Turner, M. A. (2015) "Transportation costs and the spatial organization of economic activity," in *Handbook of Regional and Urban Economics*, vol. 5 ed. by G. Duranton, J. V. Henderson, and W. C. Strange, Elsevier, Amsterdam, Netherlands, 1339–1398.

Sasaki, K. (1989) "Transportation system change and urban structure in two-transport mode setting," *Journal of Urban Economics*, 25, 346–367.

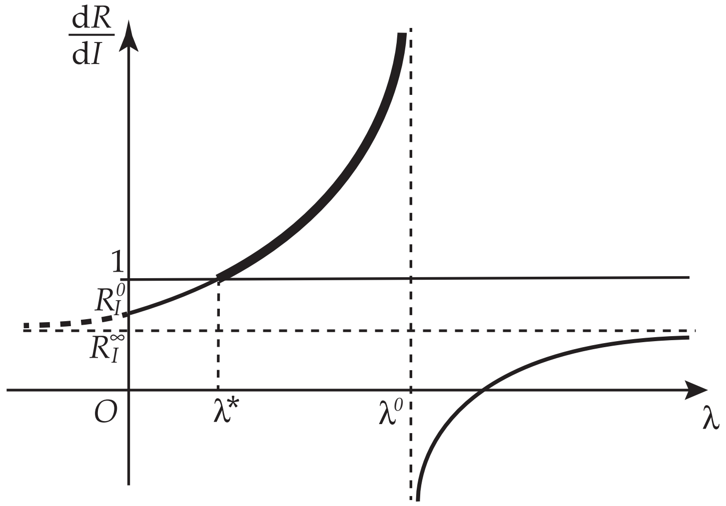
Sasaki, K. (1990) "Income class, modal choice, and urban spatial structure," *Journal of Urban Economics*, 27, 322–343.

Tabuchi, T. (1998) "Urban agglomeration and dispersion: A synthesis of Alonso and Krugman," *Journal of Urban Economics*, 44, 333–351.

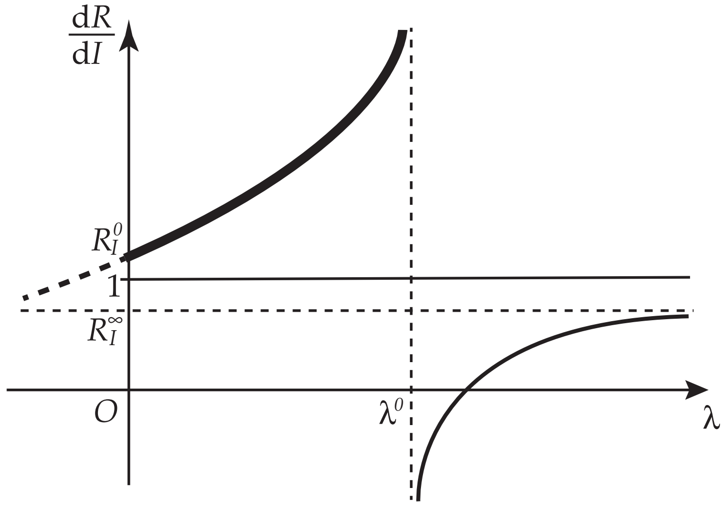
Tsai, Y.-H. (2005) "Quantifying urban form: compactness versus 'sprawl'," *Urban Studies*, 42, 141-161.

Venables, A. J. (2007) "Evaluating urban transport improvements: Cost-benefit analysis in the presence of agglomeration and income taxation," *Journal of Transport Economics and Policy*, 41, 173–188.

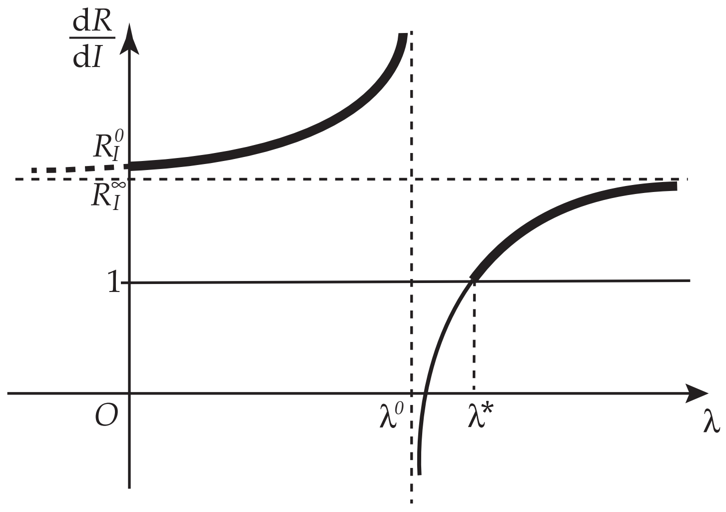
Wheaton, W. C. (1974) "A comparative static analysis of urban spatial structure," *Journal of Economic Theory*, 9, 223–237.



case i) $B < \underline{B}$



case ii) $\underline{B} \leq B \leq \overline{B}$



case iii) $B > \overline{B}$

Fig. 1. Feasibility of investments