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Rolling-Time-Dummy House Price Indexes: Window Length, Linking and Options for Dealing with the Covid-19 Shutdown

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Abstract:
The rolling-time-dummy (RTD) method is used by a number of countries to compute their official house price indexes (HPIs), since it requires less data and is more flexible than other hedonic methods. These features also make it well suited for computing higher frequency HPIs (e.g., monthly or weekly). In this paper we address three key issues relating to the RTD hedonic method. First, we develop a method for determining the optimal length of the rolling window. Second, we consider variants on the standard way of linking the current period with earlier periods, and show how the optimal linking method can be determined. Third, we propose three ways of modifying the RTD method to make it robust to the distorting effects of the covid-19 shutdown. These modifications could prove useful for countries using the RTD method in their official HPIs. (JEL: C43, E31, R31)

Keywords: House price index; Hedonic quality adjustment; Optimal window length; Optimal chain linking; Higher frequency indexes; Covid-19 shutdown

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1 Introduction

The housing market and the broader economy are closely connected. While it is true that economic booms and recessions can trigger booms and busts in the housing market, the causation can also run in the opposite direction. The global financial crisis of 2007-2010 was a case in point. For central banks to effectively maintain financial stability it is therefore important to have reliable and timely house price indexes (HPIs).

To effectively distinguish between genuine price changes and compositional differences, HPIs are typically computed using hedonic methods. The hedonic approach entails estimating shadow prices on the characteristics of properties (such as floor area, age, and location) so as to ensure that quality is held fixed when measuring price changes from one period to the next. For example, Eurostat recommends that countries in Europe should compute their official HPIs using hedonic methods (Eurostat, 2016).

A number of hedonic methods for constructing HPIs have been proposed in the literature (see Hill, 2013). One hedonic method that has been attracting increased attention in recent years is the rolling-time-dummy (RTD) method. It was first proposed by Shimizu in 1998 as part of a project entitled *Construction of Property Price Indexes using Big Data* 1998-2002 at Reitaku University (see Shimizu et al., 2003, 2010).

The RTD method has three desirable properties. First, it is relatively simple to compute and interpret.

Second, it requires less data than some other hedonic methods, such as the hedonic imputation or average characteristics methods. This means that it is particularly useful for smaller countries that have less data. In Europe it is used by Croatia, Cyprus, France, Ireland and Portugal to construct their official HPIs (see Hill et al. 2018). Japan has recently decided to compute its official residential and commercial property price indexes using the RTD method (see Shimizu and Diewert, 2019). Also, Brunei Darussalam (see `https://www.ambd.gov.bn/Site%20Assets%20%20News/RPPI-Technical-Notes.pdf`), Peru and Thailand (see `https://www.bot.or.th/App/BTWS_STAT/statistics/DownloadFile.aspx?file=EC_EI_008_S2_ENG.PDF`) are using RTD, and Indonesia is about to start using it (see Rachman, 2019). RTD's effectiveness with smaller datasets means that it is also a good candidate for computing higher frequency indexes, such as monthly or weekly.

Third, an index provider using the RTD method can choose the length of the rolling window. This makes the method very flexible. A longer window increases the robustness of the index, which can be important when the dataset is small, while a shorter window increases the current market relevance of the index. Index providers can trade off these two aspects when choosing the window length.

In this paper we address three key issues relating to the RTD hedonic method. First, there is no clear consensus regarding window length. For example, France and Portugal use a two-quarter rolling window, Cyprus and Croatia a four-quarter window, and Ireland a five-quarter window (see Hill et al. 2018). The question is, how can one determine the optimal window length for any given dataset? We develop an approach for answering this question and use it to compute the optimal window length for weekly HPIs in Sydney and Tokyo.

Second, the standard version of the RTD method links the current period to the period directly preceding it. It turns out this is just one of many ways that the HPI can be computed from the estimated hedonic model. We compare a number of different ways of linking in the current period, and develop an approach for determining which linking method is optimal. Our approach is then again tested on Sydney and Tokyo data.

Third, the covid-19 shutdown can generate weak links in the RTD HPI, which can undermine the integrity of the whole time series. We propose three ways of modifying the RTD method to make it robust to the covid-19 shutdown. We illustrate the problem using Sydney data. These modifications could prove useful for countries that are already using the RTD method in their official HPIs.

2 The Rolling Time Dummy (RTD) Method

2.1 The standard version

Consider the standard version of the RTD method with a window length of $k+1$ periods, as defined in Shimizu et al. (2010) and O'Hanlon (2011). Supposing that the first period in the window is period t, the first step is to estimate a semilog hedonic model as follows:

$$\ln p_h = \sum_{c=1}^{C} \beta_c z_{hc} + \sum_{s=t+1}^{t+k} \delta_s d_{hs} + \varepsilon_{hs}, \tag{1}$$

where h indexes the housing transactions that fall in the rolling window, p_h the transaction price, c indexes the set of available characteristics of the transacted dwellings, and ε is an identically, independently distributed error term with mean zero. The characteristics of the dwellings are given by the z_{hc}, while d_{hs} is a dummy variables that equals 1 when s is the period in which the dwelling sold, and zero otherwise.

Estimating this model using ordinary least squares, the change in the price index from period $t+k-1$ to period $t+k$ is then calculated as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-1})}, \tag{2}$$

where $\hat{\delta}$ denotes the least squares estimate of δ. A superscript t is included on the estimated δ coefficients to indicate that they are obtained from the hedonic model with period t as the base (i.e., $P_t = 1$). As can be seen from (2), the hedonic model with period t as the base is only used to compute the change in house prices from period $t+k-1$ to period $t+k$. The window is then rolled forward one period and the hedonic model is re-estimated. The change in house prices from period $t+k$ to period $t+k+1$ is now computed as follows:

$$\frac{P_{t+k+1}}{P_{t+k}} = \frac{\exp(\hat{\delta}^{t+1}_{t+k+1})}{\exp(\hat{\delta}^{t+1}_{t+k})}, \tag{3}$$

where now the base period in the hedonic model is period $t+1$. The price index over multiple periods is computed by chaining these bilateral comparisons together as follows:

$$\frac{P_{t+k+1}}{P_t} = \left[\frac{\exp(\hat{\delta}^{t-k}_{t+1})}{\exp(\hat{\delta}^{t-k}_{t})}\right]\left[\frac{\exp(\hat{\delta}^{t-k+1}_{t+2})}{\exp(\hat{\delta}^{t-k+1}_{t+1})}\right] \times \cdots \times \left[\frac{\exp(\hat{\delta}^{t+1}_{t+k+1})}{\exp(\hat{\delta}^{t+1}_{t+k})}\right]. \tag{4}$$

An important feature of the RTD method is that once a price change P_{t+k}/P_{t+k-1} has been computed, it is never revised. Hence when data for a new period $t+k+1$ becomes available, the price indexes P_t, P_{t+1}, ..., P_{t+k} are already fixed. The sole objective when re-estimating the hedonic model to include period $t+k+1$ is to compute P_{t+k+1}/P_{t+k}.

2.2 Linking variants on the RTD method

Instead of always focusing on the last two estimated δ coefficients in each hedonic model, an alternative would be to focus on the last and third last coefficients. In this case the price change from period $t+k-1$ to period $t+k$ could be calculated as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left(\frac{P_{t+k-2}}{P_{t+k-1}}\right) \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-2})}, \tag{5}$$

where as has been noted above both P_{t+k-1} and P_{t+k-2} are already fixed by the time the data for period $t+k$ becomes available. Another alternative is the following:

$$\frac{P_{t+k}}{P_{t+k-1}} = \left(\frac{P_{t+k-3}}{P_{t+k-1}}\right) \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-3})}, \tag{6}$$

and more generally,

$$\frac{P_{t+k}}{P_{t+k-1}} = \left(\frac{P_{t+k-j}}{P_{t+k-1}}\right) \frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-j})}, \tag{7}$$

where $j \leq k$. In (eq:dvg1), the hedonic model is used to link each new period with two periods earlier. In (eq:dvg2), each new period with three periods earlier, while in (eq:dvg3), each new period with j periods earlier. In other words, given a window length of $k+1$ periods, there are k distinct ways of linking period $t+k$ with the earlier periods. Each will give a different answer, and one cannot say ex ante that one is better than another.

Another possibility is to compute the geometric mean of these k sets of results as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \prod_{j=1}^{k} \left[\left(\frac{P_{t+k-j}}{P_{t+k-1}}\right) \left(\frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-j})}\right) \right]^{1/k}. \tag{8}$$

This method uses each single-period link in turn to generate k distinct estimates of P_{t+k}/P_{t+k-1}, and then takes the geometric mean of these estimates.

A weighted geometric mean could also be computed, with more recent periods being given more weight. For example, the weights could decline geometrically as follows:

$$\frac{P_{t+k}}{P_{t+k-1}} = \prod_{j=1}^{k} \left\{ \left[\left(\frac{P_{t+k-j}}{P_{t+k-1}}\right) \left(\frac{\exp(\hat{\delta}^t_{t+k})}{\exp(\hat{\delta}^t_{t+k-j})}\right) \right]^{\frac{(1-\lambda)\lambda^{j-1}}{1-\lambda^k}} \right\}, \tag{9}$$

where $0 < \lambda < 1$.

There are close similarities here with the literature on constructing monthly or weekly price indexes for consumer goods using scanner data. Focusing on the case of monthly indexes, price indexes in this literature are often computed using a 13 month rolling window. The price index itself in each window is not necessarily computed using a hedonic method. Nevertheless, the same issue arises regarding how the current period should be linked to earlier periods. In this literature, the standard linking method used by the RTD method in (2) is sometimes referred to as a *movement* splice (see for example de Haan 2015, and Chessa, Verburg and Willenborg 2017). Other possibilities considered are a *window* splice which links the new month in by comparing it with the corresponding month one year earlier (see Krsinich 2014), and a mean splice, which is analogous to the geometric mean in (8) (see Diewert and Fox 2017). These methods are all discussed in de Haan, Hendriks and Scholz (2020). In addition, Melser (2018) proposes a weighted mean splice. Melser's method is similar in spirit to our weighted mean described in (9), although the context is rather different. His weighted mean splice is a solution to a logarithmic weighted least squares problem focused on transitivizing bilateral price indexes. The weights are derived from the product overlaps between adjacent periods. No such equivalent weights exist in our context.

2.3 The covid-19 shutdown (and other periods of low transactions)

The covid-19 shutdown has created a situation where there is a lack of data for some periods. This could create problems when estimating the price index for these periods. However, it can also cause a level shift or drift in a RTD price index.[1] Consider the case of a monthly index. Suppose the shutdown drastically reduces the number of transactions in 2020, months 4, 5, and 6. The price index for these months will therefore contain an unusually high level of noise. For example, suppose the price index for 2020 month 5 contains a large positive random error. This will have a permanent impact on the RTD price index for all subsequent periods, which will now be subject to upward drift.

[1] We thank Niall O'Hanlon for alerting us to this issue.

We consider three ways of mitigating the effect of covid-19 on RTD house price indexes. Our starting point is that the desired window length is known, and that special action is deemed necessary for any period that has less than N transactions.

Covid 19-Method 1

When computing the price index for period $t+k$, if any of earlier periods that are supposed to be in the window have less than N transactions, then these periods are deleted and replaced by the most recent available earlier period that has at least N transactions. If period $t+k$ has less than N transactions, the RTD method is still used to compute P_{t+k}/P_{t+k-1} (or P_{t+k}/P_{t+k-2} if period $t+k-1$ has less than N transactions). But period $t+k$ is then not used to compute the price indexes of later periods.

An example should help clarify the rule. Suppose the window length is set at 3 months, and that 2020, months 4, 5, and 6 have less than N transactions. This means that the periods in the rolling window are as follows:

Current period	**Periods included in the rolling window**
2020m3:	(2020m1, 2020m2, 2020m3)
2020m4:	(2020m2, 2020m3, 2020m4)
2020m5:	(2020m2, 2020m3, 2020m5)
2020m6:	(2020m2, 2020m3, 2020m6)
2020m7:	(2020m2, 2020m3, 2020m7)
2020m8:	(2020m3, 2020m7, 2020m8)
2020m9:	(2020m7, 2020m8, 2020m9)

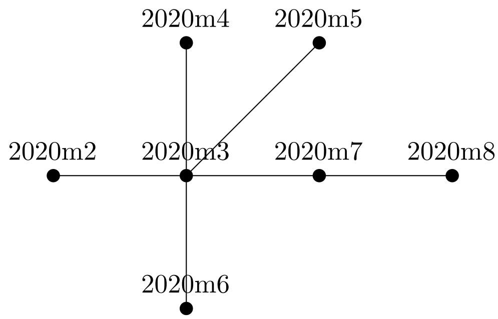
The linking structure in this example is graphed in Figure 1. 2020m4, 2020m5, 2020m6 and 2020m7 are all linked into the price index via 2020m3. From then on normal chronological chaining as described in section 2.1 resumes.

In the scenario described above, the three-month rolling window price indexes are calculated as follows:

$$\frac{P_{2020m4}}{P_{2020m3}} = \frac{\exp(\hat{\delta}^{2020m2}_{2020m4})}{\exp(\hat{\delta}^{2020m2}_{2020m3})}, \quad \frac{P_{2020m5}}{P_{2020m3}} = \frac{\exp(\hat{\delta}^{2020m2}_{2020m5})}{\exp(\hat{\delta}^{2020m2}_{2020m3})}, \quad \frac{P_{2020m6}}{P_{2020m3}} = \frac{\exp(\hat{\delta}^{2020m2}_{2020m6})}{\exp(\hat{\delta}^{2020m2}_{2020m3})}$$

$$\frac{P_{2020m7}}{P_{2020m3}} = \frac{\exp(\hat{\delta}^{2020m2}_{2020m7})}{\exp(\hat{\delta}^{2020m2}_{2020m3})}, \quad \frac{P_{2020m8}}{P_{2020m7}} = \frac{\exp(\hat{\delta}^{2020m3}_{2020m8})}{\exp(\hat{\delta}^{2020m3}_{2020m7})}, \quad \frac{P_{2020m9}}{P_{2020m8}} = \frac{\exp(\hat{\delta}^{2020m7}_{2020m9})}{\exp(\hat{\delta}^{2020m7}_{2020m8})}.$$

Figure 1: The Linking Structure of Covid-19 Method 1 During a Shutdown in 2020m4, 2020m5 and 2020m6



In these equations, the superscript denotes the base month in each hedonic model and the subscript denotes the month of the estimated δ parameter.

The overall price index with 2020m3 normalized to 1 is then constructed as follows:

$$1, \frac{P_{2020m4}}{P_{2020m3}}, \frac{P_{2020m5}}{P_{2020m3}}, \frac{P_{2020m6}}{P_{2020m3}}, \frac{P_{2020m7}}{P_{2020m3}}, \frac{P_{2020m7}}{P_{2020m3}} \times \frac{P_{2020m8}}{P_{2020m7}}, \frac{P_{2020m7}}{P_{2020m3}} \times \frac{P_{2020m8}}{P_{2020m7}} \times \frac{P_{2020m9}}{P_{2020m8}}, \ldots$$

As can be seen the lack of data for 2020m4, 2020m5 and 2020m6 does not contaminate the longer time series RTD price index after 2020m6. However, the price indexes for periods 2020m4, 2020m5 and 2020m6 may be unreliable.

Covid 19-Method 2

When a period has less than N transactions, more transactions are drawn from the preceding period, until the threshold N is reached. The chronologically closest transactions from the previous period are used first. If say there are enough transactions in the last 10 days of the preceding period to reach the N threshold, then only these last 10 days are added to the current period, when computing the current period price index. These 10 days are still also included in the previous period.

As an example, suppose that each of 2020m4, 2020m5 and 2020m6 has more than $N/2$ and less than N transactions. The RTD rolling window would now be constructed as follows:

Current period	**Periods included in the rolling window**
2020m3:	(2020m1, 2020m2, 2020m3)

2020m4:	(2020m2, 2020m3, 2020m4$^+$)
2020m5:	(2020m3, 2020m4$^+$, 2020m5$^+$)
2020m6:	(2020m4$^+$, 2020m5$^+$, 2020m6$^+$)
2020m7:	(2020m5$^+$, 2020m6$^+$, 2020m7)
2020m8:	(2020m6$^+$, 2020m7, 2020m8)
2020m9:	(2020m7, 2020m8, 2020m9)

The + superscripts above denote that the month is being supplemented with data from the previous month. Once each month with less than N transactions has been supplemented with transactions from the previous month, the RTD price index is computed in the standard way described in section 2.1.

Covid 19-Method 3

Our third alternative approach is to not compute an index for a period with less than N transactions. Instead it is merged with the next period. If together these two periods reach the N transaction threshold, then they are treated as a single period. If together they still do not reach the N transaction threshold, then again no index is computed until the next period becomes available, etc.

As an example, again suppose that each of 2020m4, 2020m5 and 2020m6 has more than $N/2$ and less than N transactions. The RTD rolling window would now be constructed as follows:

Current period	**Periods included in the rolling window**
2020m3:	(2020m1, 2020m2, 2020m3)
2020m4:	NA
2020m4-m5:	(2020m2, 2020m3, 2020m4-m5)
2020m6:	NA
2020m6-m7:	(2020m3, 2020m4-m5, 2020m6-m7)
2020m8:	(2020m4-m5, 2020m6-m7, 2020m8)
2020m9:	(2020m6-m7, 2020m8, 2020m9)
2020m10:	(2020m8, 2020m9, 2020m10)

All three methods ensure that the covid-19 shutdown does not contaminate the index in later periods. They differ in their treatment of the covid-19 period. Method 1 computes price indexes for each shutdown month using whatever transaction data are actually available. Method 2 supplements the data for shutdown months with data from the previous month (or more if required), while method 3 merges shutdown months and treats them as a single period if each individually does not contain enough transactions. Which method is best depends on the needs of users. If it is important that a price index is computed for every period (here months) then either method 1 or 2 should be used. Method 1 will better capture actual price movements during the shutdown unless it becomes too distorted by noise arising from small sample sizes.

While transaction data are not available yet for 2020m4, 2020m5 and 2020m6, we can illustrate these covid-19 methods and their impact on the resulting price index using data for Sydney. The number of transactions in Sydney are much lower in the second half of December through to late February (corresponding to the Australian summer holiday). The summer period each year therefore acts like a mini shutdown.

3 Quarterly Benchmarks

3.1 The hedonic imputation method

The hedonic imputation method is an alternative to the RTD method (see Diewert 2011 and Hill 2013). We use the hedonic imputation method here as a reference index for assessing the performance of different versions of the RTD method.

The hedonic imputation approach estimates a separate hedonic model for each period:

$$\ln p_{t,h} = \beta_t \cdot z_{t,h} + \varepsilon_{t,h}, \tag{10}$$

where for convenience β_t and $z_{t,h}$ now both denote vectors. The hedonic model is then used to impute prices for individual houses. For example, let $\hat{p}_{t+1,h}(z_{t,h})$ denote the imputed price in period $t+1$ of a house with the characteristic vector $z_{t,h}$ sold in period t. This price is imputed by substituting the characteristics $z_{t,h}$, into the estimated hedonic model of period $t+1$ as follows:[2]

$$\hat{p}_{t+1,h}(z_{t,h}) = \exp\left(\sum_{c=1}^{C} \hat{\beta}_{c,t+1} z_{c,t,h}\right). \tag{11}$$

With these imputed prices it is now possible to construct a matched sample, thus allowing standard price index formulas to be used.

$$\text{Paasche} - \text{Type Imputation}: \; P_{t,t+1}^{PI} = \prod_{h=1}^{H_{t+1}} \left[\left(\frac{\hat{p}_{t+1,h}}{\hat{p}_{t,h}(z_{t+1,h})}\right)^{1/H_{t+1}}\right] \tag{12}$$

$$\text{Laspeyres} - \text{Type Imputation}: \; P_{t,t+1}^{LI} = \prod_{h=1}^{H_{t}} \left[\left(\frac{\hat{p}_{t+1,h}(z_{t,h})}{\hat{p}_{t,h}}\right)^{1/H_{t}}\right] \tag{13}$$

$$\text{Törnqvist} - \text{Type Imputation}: \; P_{t,t+1}^{TI} = \sqrt{P_{t,t+1}^{PI} \times P_{t,t+1}^{LI}} \tag{14}$$

In a comparison between periods t and $t+1$, the Laspeyres-type index focuses on the H_t houses that sold in the earlier period t. Similarly the Paasche-type index focuses on the H_{t+1} houses that sold in the later period $t+1$. These price indexes give equal weight to each house sold.[3] By taking the geometric mean of Paasche and Laspeyres, the Törnqvist-type index gives equal weight to both periods. The Paasche, Laspeyres and Törnqvist-type indexes above are of the double imputation variety, meaning that both prices in each price relative are imputed. A single imputation approach by contrast imputes only one price in each pair (since the actual price is always available for one of the two periods being compared). There has been some discussion in the literature over the relative merits of the two approaches (see for example de Haan 2004, and Hill and Melser 2008). Empirically we try both approaches. The resulting price indexes are virtually indistinguishable. Hence to simplify the presentation we focus here only on double imputation price indexes. The hedonic imputation method is flexible in that it allows the characteristic shadow prices to update each period.

[2]See Silver and Heravi (2007), Diewert, Heravi, and Silver (2009), and Rambaldi and Rao (2013) for a discussion of some of the advantages of the hedonic imputation method.

[3]This democratic weighting structure is in our opinion more appropriate in a housing context than weighting each house by its expenditure share. See de Haan (2010) for a discussion on alternative weighting schemes.

In the context of weekly indexes, the hedonic imputation method is unlikely to work well since the sample sizes in many weeks may be too small to justify estimating a separate hedonic model each week. However, in our context, quarterly hedonic imputation will provide a useful benchmark for weekly RTD indexes.

3.2 The time-dummy method

We also use the time-dummy index as a reference for assessing the performance of RTD weekly indexes. The time-dummy method is the limiting case of the RTD method where the window length is the same as the number of periods in the comparison.

$$\ln p_h = \sum_{c=1}^{C} \beta_c z_{hc} + \sum_{t=2}^{T} \delta_t d_{ht} + \varepsilon_{ht}. \tag{15}$$

The price index for period t relative to period 1 is then calculated as follows:

$$\frac{P_t}{P_1} = \exp(\hat{\delta}_t). \tag{16}$$

3.3 A performance criterion for weekly indexes derived from quarterly indexes

We propose a criterion here for determining the optimal window length and linking method for weekly RTD indexes, by comparing them with reference quarterly hedonic indexes.[4] We consider two reference quarterly hedonic indexes: these are the hedonic imputation method and the time-dummy method described above. We focus on these two methods because they are quite different (i.e., one re-estimates the hedonic model every quarter while the other does not re-estimate at all). Using these quarterly indexes as benchmarks should avoid biasing the results towards any particular window length.

Empirically we find that the quarterly hedonic imputation and time-dummy methods approximate each other closely. By contrast for weekly RTD indexes, if we allow the window length to vary between 2 and 53 weeks, the range of possible results becomes much larger (see section 5).

[4] With slight modifications, the same method can be used to determine the optimal window length for monthly indexes as well.

The greater sensitivity of weekly indexes to the choice of hedonic method makes them a more interesting focus of analysis than quarterly indexes. The choice of window length really matters for weekly RTD indexes. Furthermore, the greater robustness of quarterly indexes is a property we can exploit to discriminate between competing weekly RTD indexes.

The first step of our criterion for assessing the performance of alternative weekly RTD indexes is to construct a quarterly index from each weekly index. This can be done in the following way. Let $t = 1, \ldots, T$ index the quarters in the data set, and $v = 1, \ldots, V$ the 13 weeks in a quarter. A quarterly price index $P^w_{t,t+1}$ is obtained from a weekly price index as follows:

$$P^w_{t,t+1} = \prod_{v=1}^{13} \left(\frac{P_{t+1,v}}{P_{t,v}} \right)^{1/13}, \tag{17}$$

where $P_{t,v}$ denotes the level of the weekly price index in quarter t, week v. Each element $P_{t+1,v}/P_{t,v}$ in (17) is a price index comparing a particular week with another week one quarter later. In other words, each of these elements is a price index calculated at a quarterly frequency. A total of 13 such indexes can be computed in each quarter.[5] By taking the geometric mean of these 13 quarterly frequency price indexes, we obtain an overall quarterly price index, which can be interpreted as the quarterly equivalent of the original weekly index.

The index above in (17) is unweighted. It gives equal weight to all weeks irrespective of how the observations are distributed across weeks. A weighted variant that weights according to the number of observations each week is the following:

$$P^w_{t,t+1} = \prod_{v=1}^{13} \left(\frac{P_{t+1,v}}{P_{t,v}} \right)^{\frac{s_{t,v}+s_{t+1,v}}{2}}, \tag{18}$$

where $s_{t,v} = S_{t,v}/S_t$, with $S_{t,v}$ denoting the number of observations in week (t, v), and S_t the total number of observations in quarter t. Empirically we find that the weighted and unweighted results are very similar. Hence in what follows we focus only on the unweighted case represented in (17).

[5] The weeks do not overlap exactly with the quarters. So occasionally it is necessary to include 14 weeks in (17).

Once the quarterly version of the weekly index has been constructed, its performance can be measured by comparing it with a reference quarterly index. Here we make the comparison using two alternative metrics proposed by Diewert (2002, 2009).

$$X_1 = \frac{1}{T-1}\sum_{t=1}^{T-1}\left[\left(\frac{P^{w}_{t,t+1}}{P^{quart}_{t,t+1}}\right) + \left(\frac{P^{quart}_{t,t+1}}{P^{w}_{t,t+1}}\right) - 2\right],$$

$$X_2 = \frac{1}{T-1}\sum_{t=1}^{T-1}\left\{\left[\left(\frac{P^{w}_{t,t+1}}{P^{quart}_{t,t+1}}\right) - 1\right]^2 + \left[\left(\frac{P^{quart}_{t,t+1}}{P^{w}_{t,t+1}}\right) - 1\right]^2\right\}.$$

The smaller the value of the X metric, the more similar are the two indexes. The ordinal rankings generated by the two X metrics are almost identical. Hence in our empirical analysis we only present results for X_1.

Given a reference quarterly index, we can then vary the length of the RTD rolling window and observe how it affects the X metric. We prefer whichever window length generates the smallest X metric. An important question then is how robust is the optimal window length to the choice of reference quarterly index? If it is reasonably robust, then the selected window length is optimal in the sense that it generates a weekly RTD index that is the most consistent with our reference quarterly indexes. Similarly, holding the window length fixed at 53 weeks, we can observe how changing the RTD linking method affects the X metric. Again, we prefer the linking method with the smallest X metric.

4 The Data Sets

4.1 The Sydney data set and hedonic model

We use a data set obtained from Australian Property Monitors that consists of prices and characteristics of houses sold in Sydney (Australia) for the years 2003–2014. For each house we have the following characteristics: the actual sale price, time of sale, postcode, property type (i.e., detached or semi), number of bedrooms, number of bathrooms, land area, exact address, longitude and latitude. (We exclude all townhouses from our analysis since the corresponding land area is for the whole strata and not for the individual townhouse itself.)

For a robust analysis it was necessary to remove some outliers. This is because there is a concentration of data entry errors in the tails of the distribution, caused for example by the inclusion of erroneous extra zeroes. These extreme observations can distort the results. Complete data on all our hedonic characteristics are available for 433 202 observations. To simplify the computations we also merged the number of bathrooms and number of bedrooms to broader groups (one, two, and three or more bathrooms; one or two, three, four, five or more bedrooms).

Using weekly periods, the hedonic model for Sydney is estimated with a rolling window ranging between 2 weeks and 53 weeks. The window is then rolled forward one period and the hedonic model re-estimated. Hence in the case of the 2 week window, a total of 711 hedonic models are estimated covering the time interval from January 2003 to December 2014.

The hedonic model estimated for Sydney is semilog, and contains the following five characteristics:
(i) number of bedrooms,
(ii) number of bathrooms,
(iii) log of land area,
(iv) house type (detached, or semi),
(v) postcode.
All these variables with the exception of land area take the form of dummy variables.

4.2 The Tokyo data set and hedonic model

The Tokyo data set consists of 23 wards of the Tokyo metropolitan area (621 square kilometers), and the analysis period is approximately 30 years between January 1986 and June 2016. The data set covers previously-owned condominiums published in Residential Information Weekly (or Shukan Jyutaku Joho in Japanese) published by RECRUIT, Co. This magazine provides information on the characteristics and asking prices of listed properties on a weekly basis. Moreover, Shukan Jutaku Joho provides time-series data on housing prices from the week they were first posted until the week they were removed as a result of successful transactions. We only use the price in the final week because this can be safely regarded as sufficiently close to the contract price.

The available housing characteristics include floor space and age. The convenience of public transportation from each housing location is represented by travel time to the central business district (CBD), and time to the nearest station. Ward dummies (i.e., city codes) and a railway dummy to indicate along which railway/subway line a housing property is located are also available.

The hedonic model for Tokyo is estimated over 242 233 observations. The functional form is semilog. The explanatory variables used here are:

(i) log of floor area,

(ii) age,

(iii) time to nearest station,

(iv) time to Tokyo central station (included as a quadratic),

(v) city code,

(vi) ward dummy.

5 Results

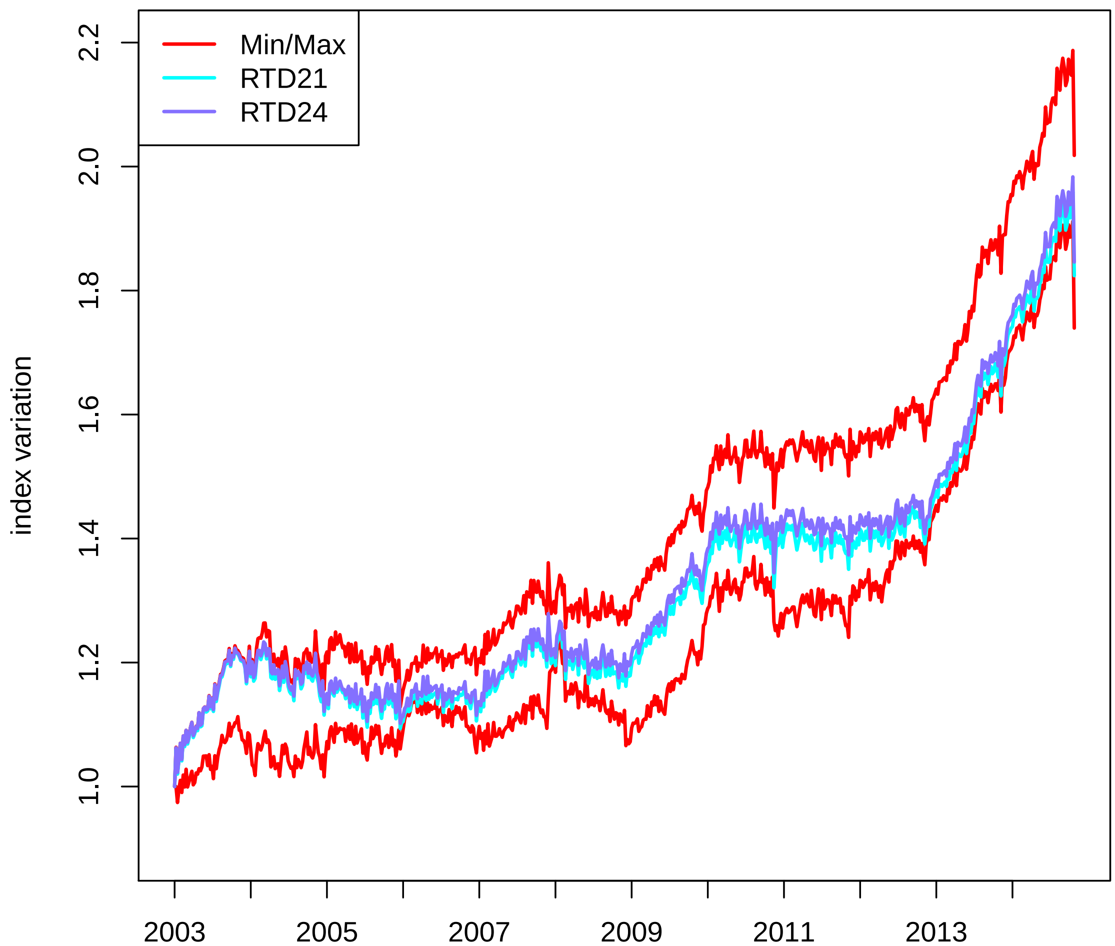
5.1 The sensitivity of the results to the choice of window length

The spreads of the weekly RTD hedonic price indexes for Sydney and Tokyo as the window length is varied between 2 and 53 weeks are shown in Figures 2 and 3. It can be seen that the weekly indexes are quite sensitive to the choice of window length.

5.2 The sensitivity of the results to the choice of linking method

Holding the window length fixed at 53 weeks, the sensitivity of a weekly RTD method to the choice of linking method is shown for Sydney and Tokyo in Figures 4 and 5. It can be seen that the variation in the RTD price indexes from varying the linking method is smaller than the variation resulting from changing the window length. However, the spread is still significant.

Figure 2: The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Sydney

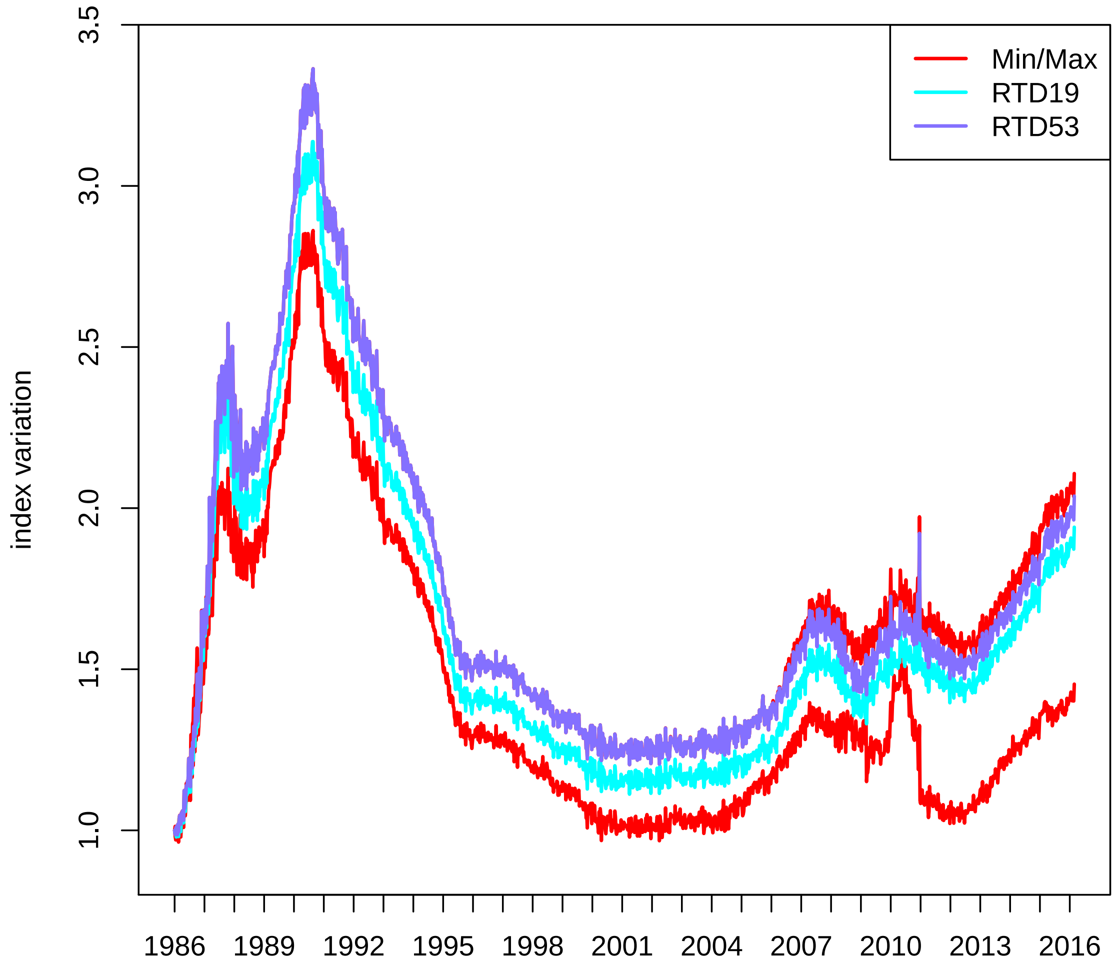


Note: RTD21 and RTD24 denote the 21 and 24 week RTD price indexes for Sydney. Min and max denote the lower and upper bounds on all the RTD price indexes with windows ranging between 2 and 53 weeks.

5.3 A quarterly index as a benchmark

The hedonic imputation and time-dummy methods generate very similar quarterly price indexes. The results are shown in Figures 6 and 7. These results indicate that at a quarterly frequency we have quite a good idea of what the right answer is. Hence these

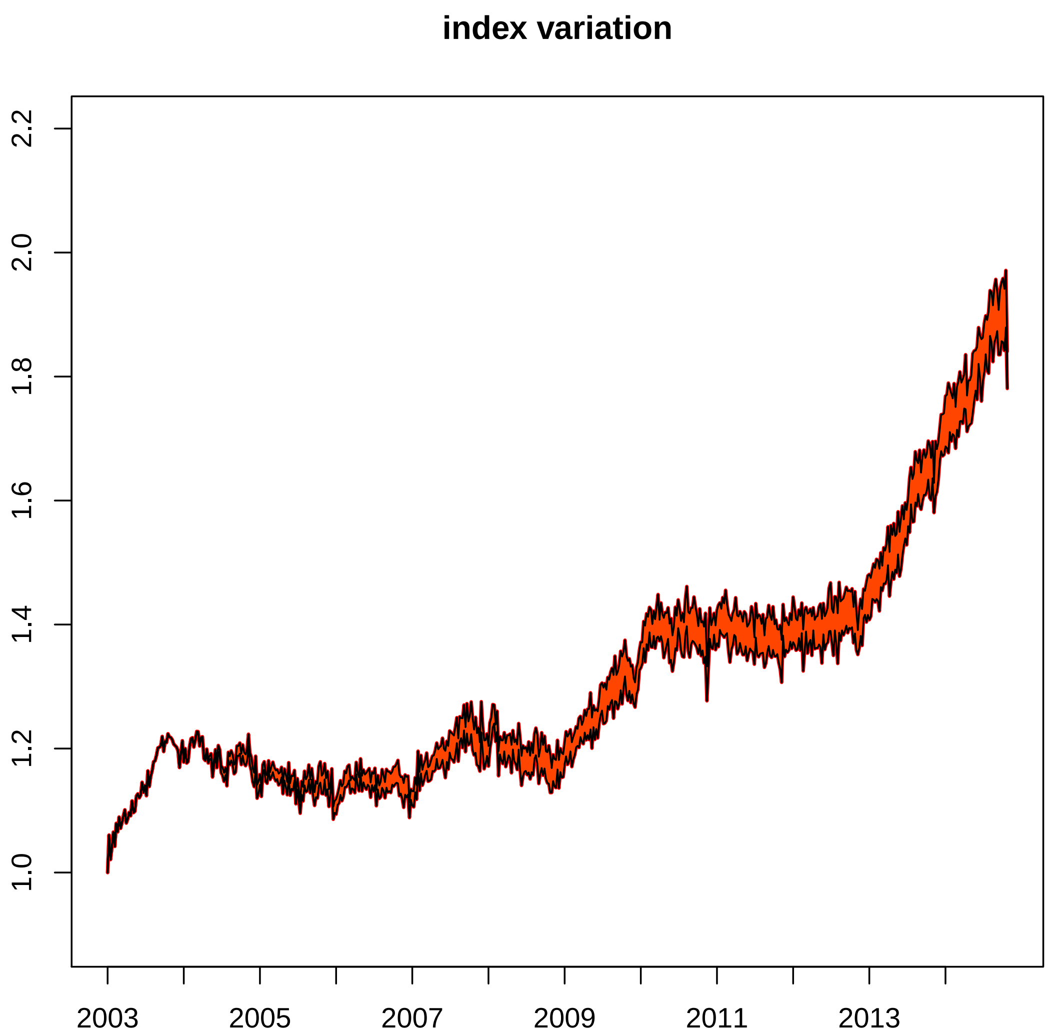
Figure 3: The Impact of Varying the Window Length on Weekly RTD House Price Indexes for Tokyo



Note: RTD19 and RTD53 denote the 19 and 53 week RTD price indexes for Tokyo. Min and max denote the lower and upper bounds on all the RTD price indexes with windows ranging between 2 and 53 weeks.

quarterly indexes can be used as a benchmark for discriminating between competing weekly indexes.

Figure 4: The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Sydney

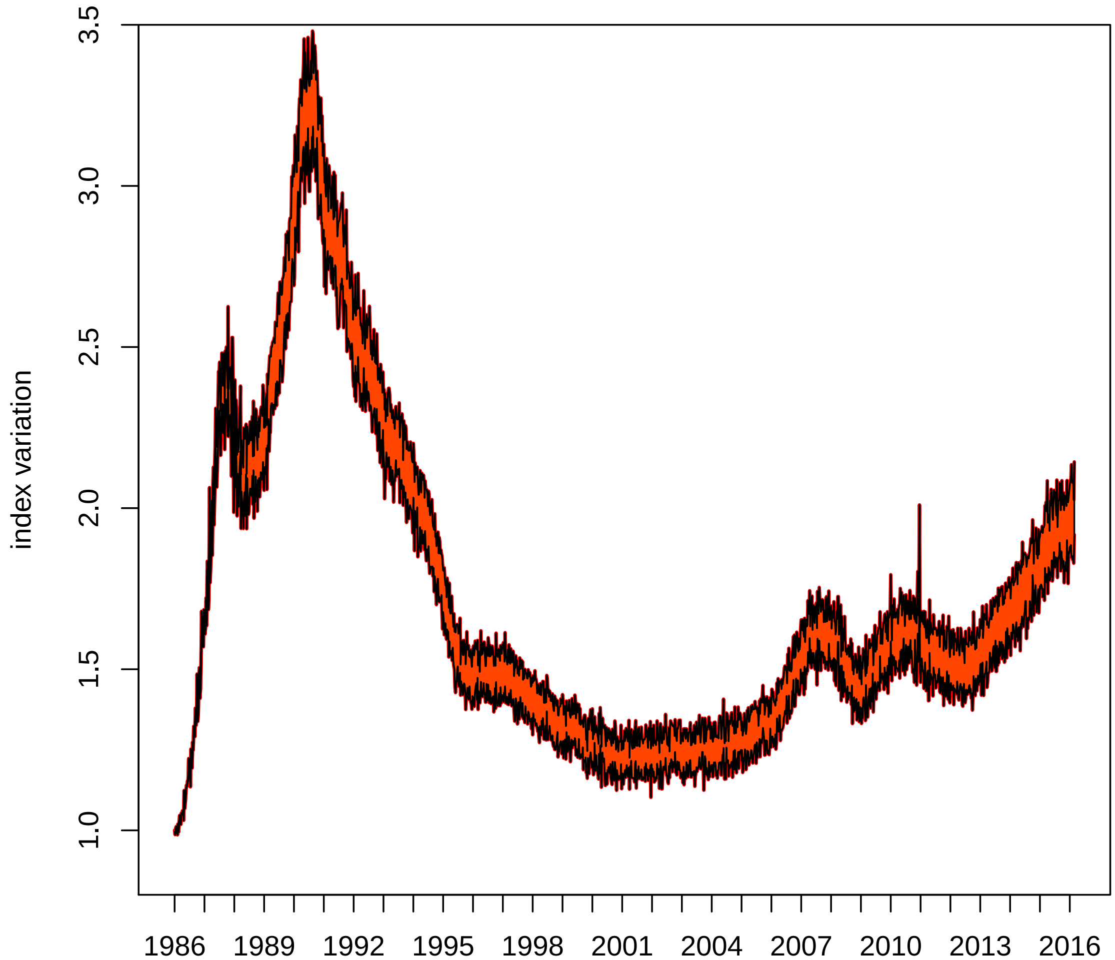


Note: This graph shows the range of RTD price indexes for Sydney resulting from using different single-period linking methods. With the window length fixed at 53 weeks, there are 52 ways of doing single-period linking.

5.4 How RTD index performance depends on window length

Here we focus on the standard RTD linking method described in section 2.1. For this case, the X_1 metric for each RTD window length for Sydney with the hedonic imputation index as the reference quarterly index is shown in Figure 8. The X_1 metric

Figure 5: The Impact of Varying the Linking Method on Weekly RTD House Price Indexes with a 53 Week Window for Tokyo

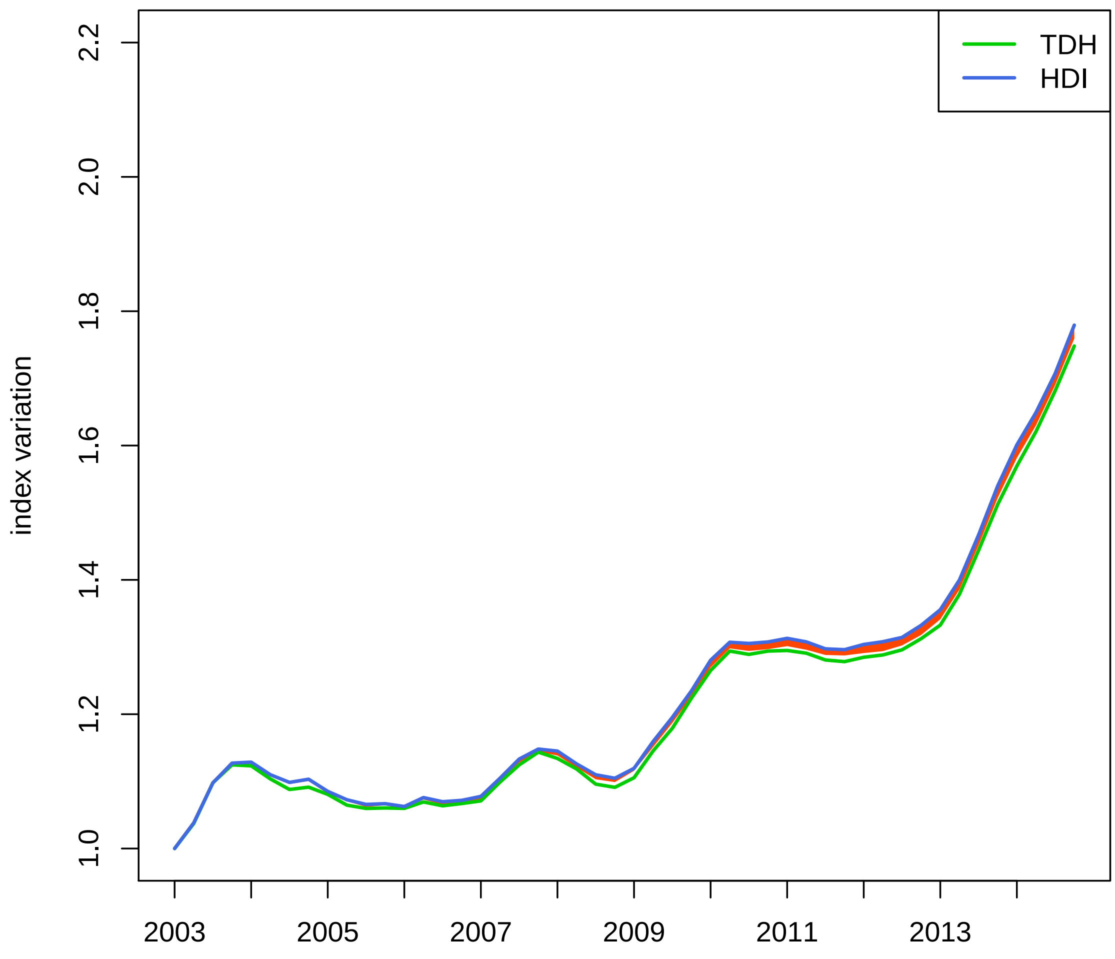


Note: This graph shows the range of RTD price indexes for Tokyo resulting from using different single-period linking methods. With the window length fixed at 53 weeks, there are 52 ways of doing single-period linking.

is minimized when the RTD window length is 21 weeks.[6] The same answer is obtained when the time-dummy index is used as the reference quarterly index as shown in the Appendix in Figure A1.

[6]The red curve in Figure 8 and subsequent Figures is a fitted curve obtained by a local-linear smoother.

Figure 6: Quarterly Hedonic Imputation and Time-Dummy House Price Indexes for Sydney

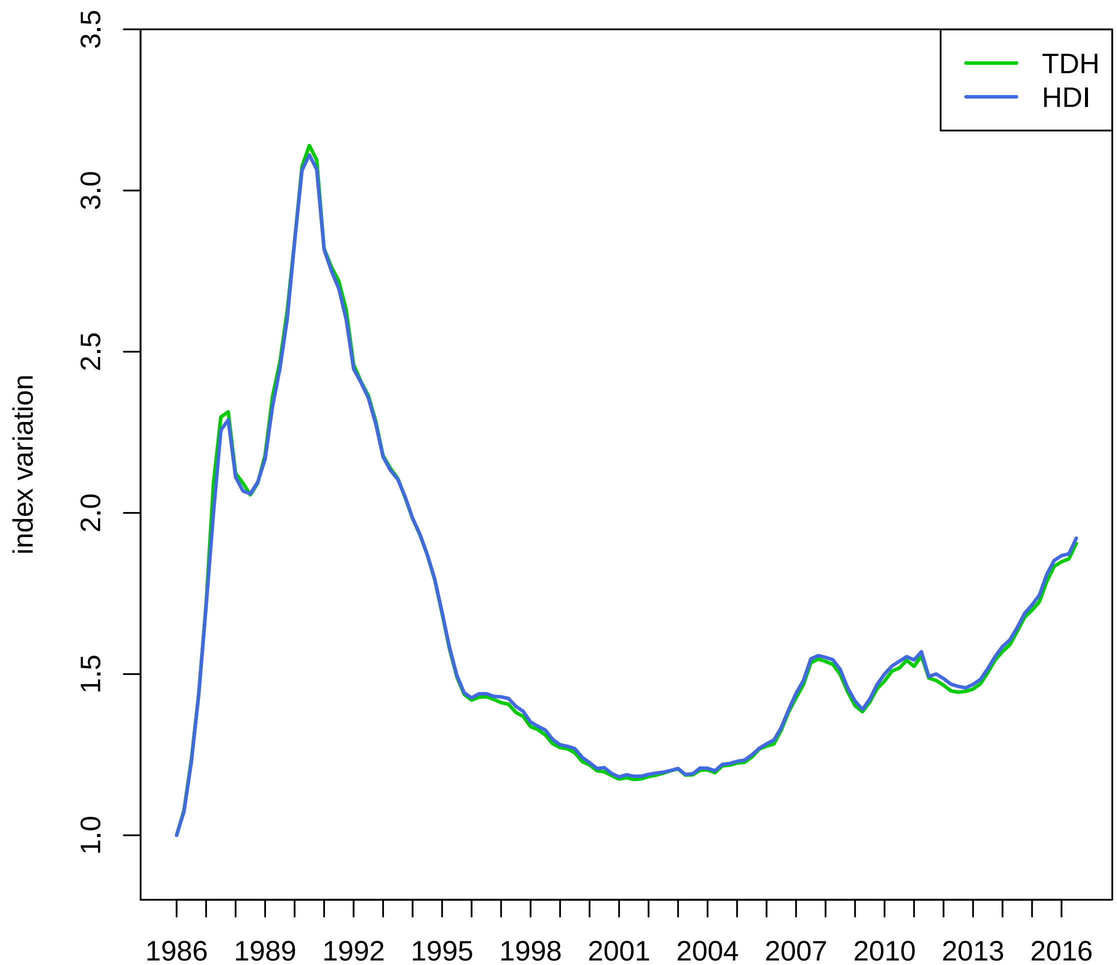


Note: TDH and HDI here denote quarterly time-dummy hedonic and hedonic double imputation price indexes for Sydney. As can be seen, the two indexes closely approximate each other.

For Tokyo, when the hedonic imputation index is used as the reference quarterly index, the X_1 metric is minimized by an RTD window length of 18 weeks, as shown in Figure 9. When the time-dummy index is used as the reference quarterly index, the results for Tokyo are not so clear, as shown in the Appendix in Figure A2.

The optimal window length may also depend on the linking method. To illustrate this point, we recompute the optimal window length for Sydney, where now the linking is

Figure 7: Quarterly Hedonic Imputation and Time-Dummy House Price Indexes for Tokyo

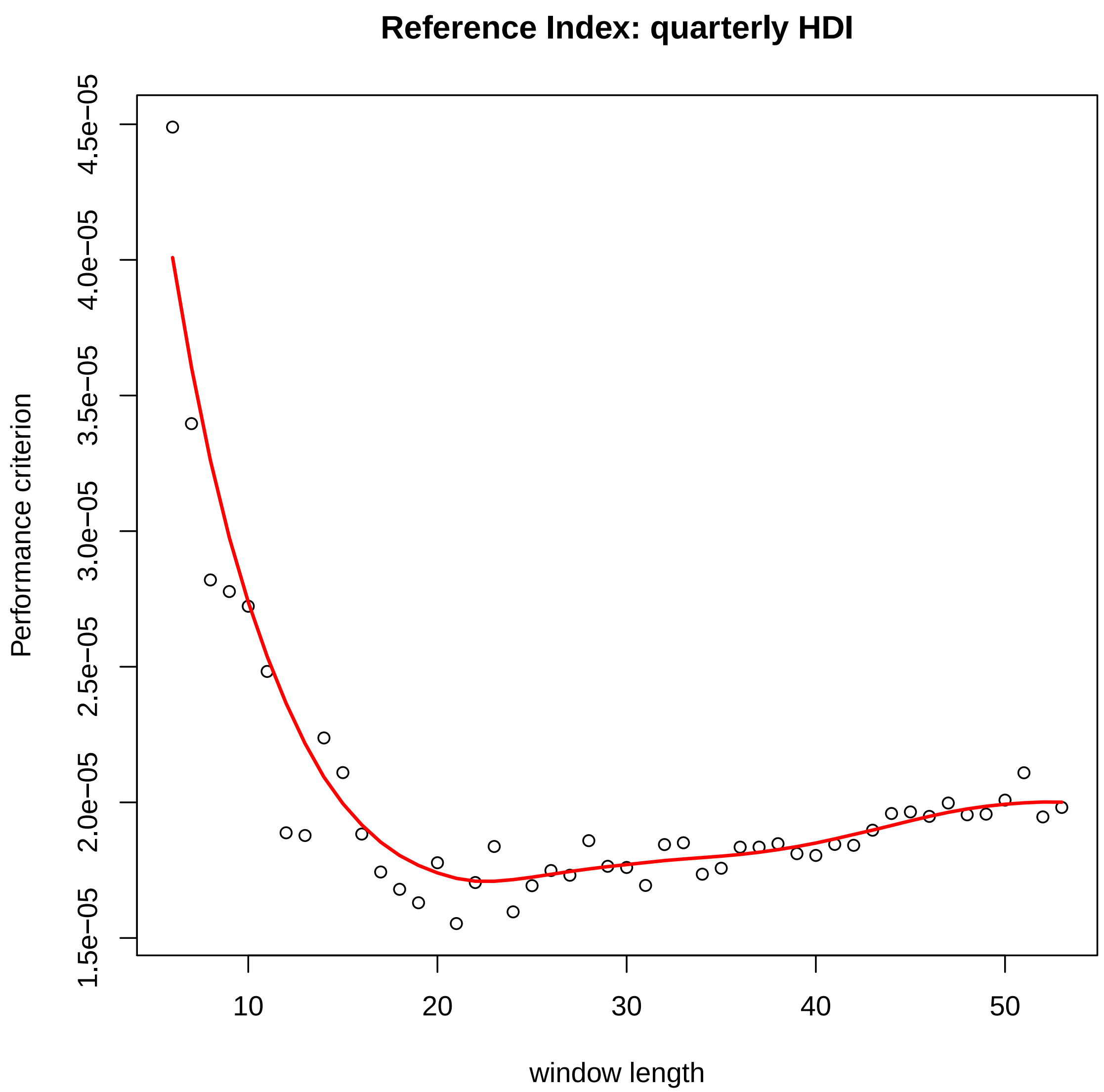


Note: TDH and HDI here denote quarterly time-dummy hedonic and hedonic double imputation price indexes for Tokyo. As can be seen, the two indexes closely approximate each other.

done using the geometric mean linking method as described in (8). Using the quarterly hedonic imputation method as the benchmark, the optimal window length is now 19 weeks (see Figure 10). This is quite similar to the optimal window length of 21 weeks obtained using single-period linking.

In summary, we find that for Sydney the optimal RTD window length is between 19 and 21 weeks depending on the linking method used. For Tokyo, according to the

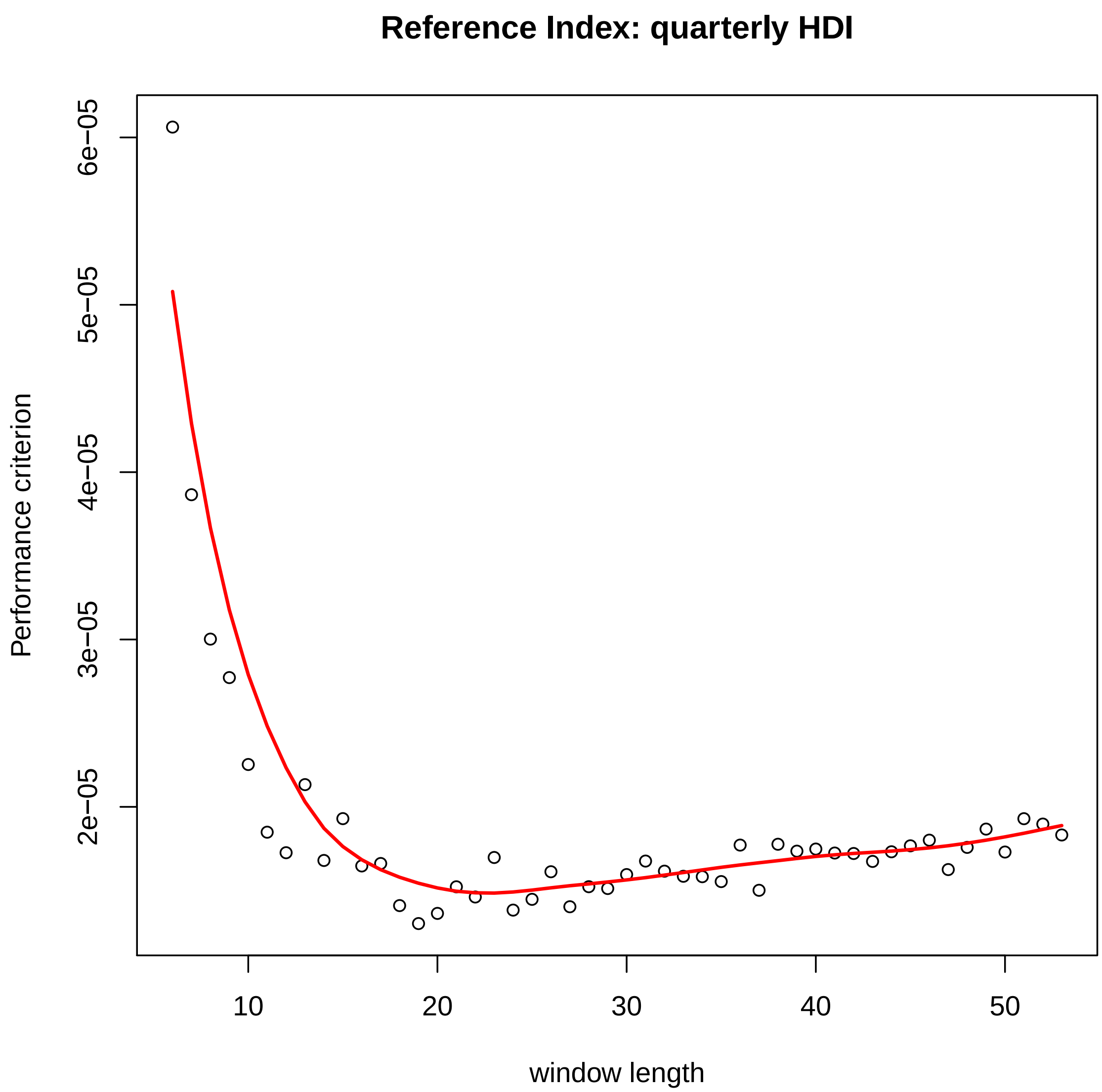
Figure 8: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Sydney



Note: This graph shows which window length generates a weekly RTD price index that most closely approximates a quarterly hedonic imputation index. For Sydney the optimal window length here is 21 weeks.

quarterly hedonic imputation benchmark, the optimal window length is 18 weeks.

Figure 9: Performance of Alternative Window Lengths with the Quarterly Hedonic Imputation Price Index as the Reference: Tokyo

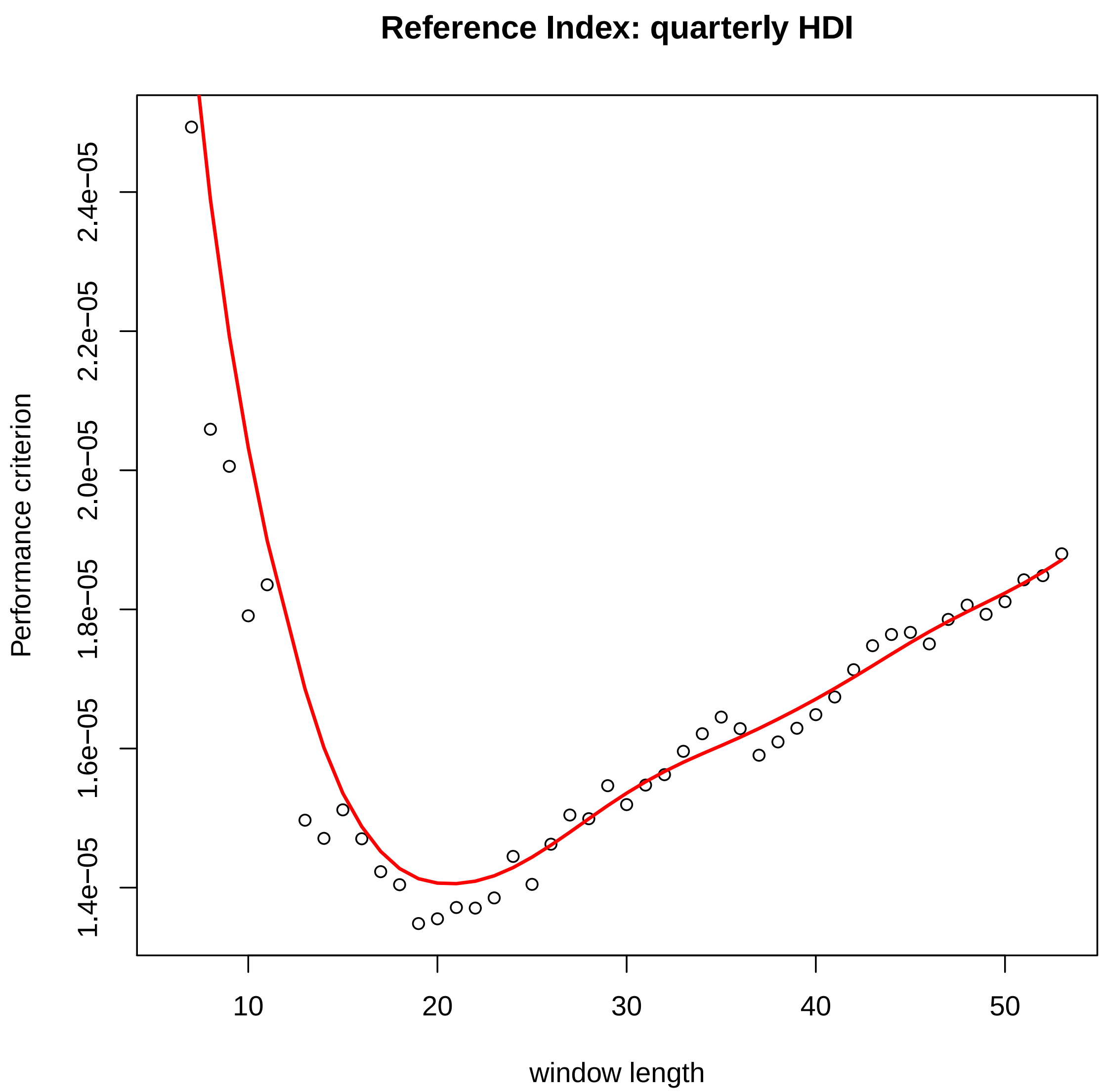


Note: This graph shows which window length generates a weekly RTD price index that most closely approximates a quarterly hedonic imputation index. For Tokyo the optimal window length here is 18 weeks.

5.5 How RTD index performance depends on the linking method

Now instead we hold the RTD window length fixed at 53 weeks, and compare the impact on the X_1 metric of varying the linking method used by the RTD method. With a 53 week window, there are 52 ways of linking a new period to a single previous period, as

Figure 10: Performance of Geometric Mean Linking for Different Window Lengths: Quarterly Hedonic Imputation Benchmark for Sydney



Note: This graph shows which window length generates a weekly RTD price index that most closely approximates a quarterly hedonic imputation index when the current period is linked in using the geometric linking method in (8). For Sydney the optimal window length here is 19 weeks.

described in (6). For Sydney, the X_1 metric corresponding to each of these 52 ways of linking is shown in Figure 11 for the case where the quarterly hedonic imputation index is used as the reference. Corresponding results with the quarterly time-dummy index as the reference are shown in A3.

In addition to these 52 single period linking methods, we also consider three average linking methods.

(i) **Geomean52** is the geometric mean of the 52 single period linking methods as described in (8).

(ii) **Geomean20** is the geometric mean of the 20 chronologically closest single period linking methods.

(iii) **lambda=0.95** is the weighted geometric mean method with $\lambda = 0.95$ as described in (9).

When the quarterly hedonic imputation method is used as the reference index the optimal linking method is to link week t to week $t-16$ (see Figure 11). Linking through the period 16 weeks earlier even slightly outperforms the average linking methods (i), (ii) and (iii).

When the quarterly time-dummy method is used as the reference index the optimal link for week t is with week $t-13$ (see Figure A3 in the Appendix). In this case, linking through the period 13 weeks earlier slightly outperforms methods (i) and (iii), but is about equivalent to taking the geometric mesn of the chronologically most recent 20 single-week links.

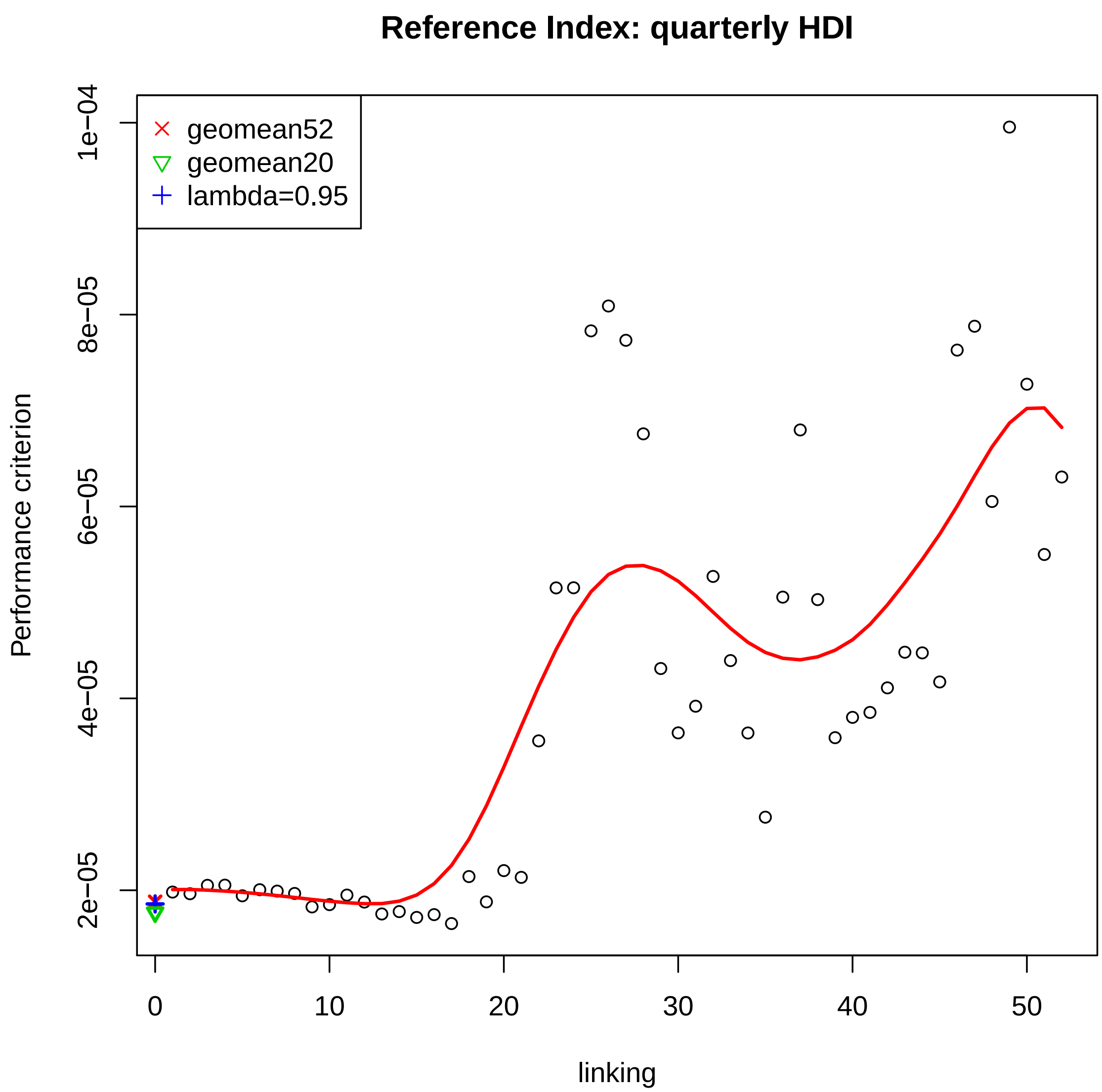
The corresponding results for Tokyo are shown in Figures 12 and A4. For the single-week links, in both Figures 12 (where the quarterly hedonic imputation index is used as a benchmark) and A4 in the Appendix (where the quarterly time-dummy index is used as the benchmark), the X_1 metric is minimized by linking week t with week $t-12$ (i.e., linking the current week with 12 weeks earlier).

For Tokyo in Figure 12 the averaging methods (i), (ii) and (iii) perform equivalently to linking through 12 weeks earlier. In Figure A4, method (i) (i.e., the geometric mean of the 52 single-period lining methods) outperforms all the single-period linking methods.

5.6 The covid-19 shutdown method illustrated using weekly Sydney data

Here we focus specifically on covid-19 shutdown method 1, as described in section 2.3. Setting the minimum number of observations per week to 250, we can see from Figure 13 that every year the last week in December and the first week in January fail to attain

Figure 11: Performance of Alternative Linking Methods: Quarterly Hedonic Imputation Benchmark for Sydney

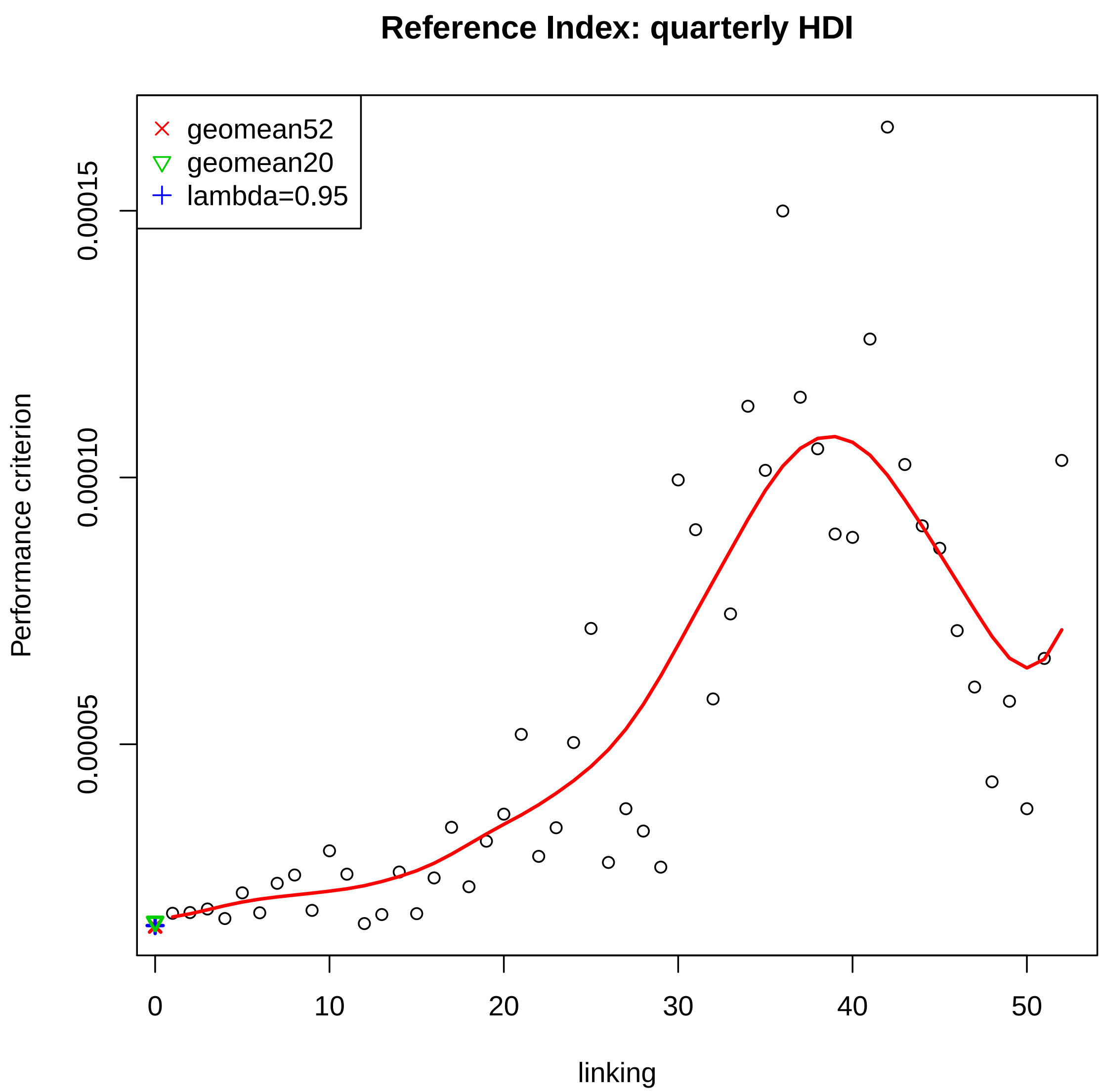


Note: This graph shows the performance for Sydney (relative to a quarterly hedonic imputation index) of different single-period linking methods, Geomean52, Geomean20, and the weighted geomean method in (9) with λ set to 0.95. The best performing method is single-period linking with 16 weeks earlier.

this threshold.

Setting the window length to 7 weeks, the standard RTD method exhibits a slight upward drift compared with covid-19 method 1, as can be seen in Figure 14. Towards the end of our sample, the cumulative magnitude of this upward drift is 8.9 percent. It remains to be seen how big the actual covid-19 shutdown drift will be. Clearly it will

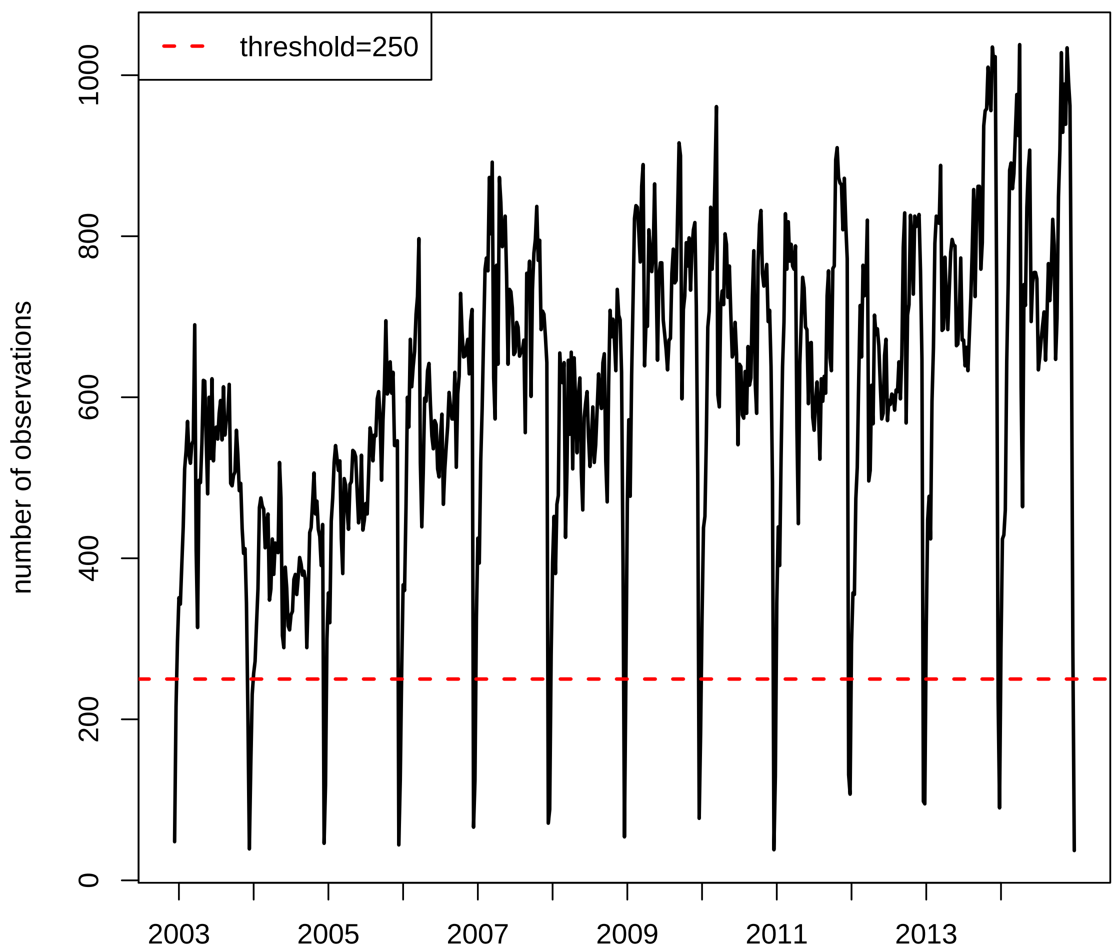
Figure 12: Performance of Alternative Linking Methods: Quarterly Hedonic Imputation Benchmark for Tokyo



Note: This graph shows the performance for Tokyo (relative to a quarterly hedonic imputation index) of different single-period linking methods, Geomean52, Geomean20, and the weighted geomean method in (9) with λ set to 0.95. The best performing method is single-period linking with 12 weeks earlier.

differ depending on the country or city and frequency of the index. Drift is most likely to be a problem for smaller countries without much data.

Figure 13: Weekly Observations Versus the 250 Observation Threshold: Sydney

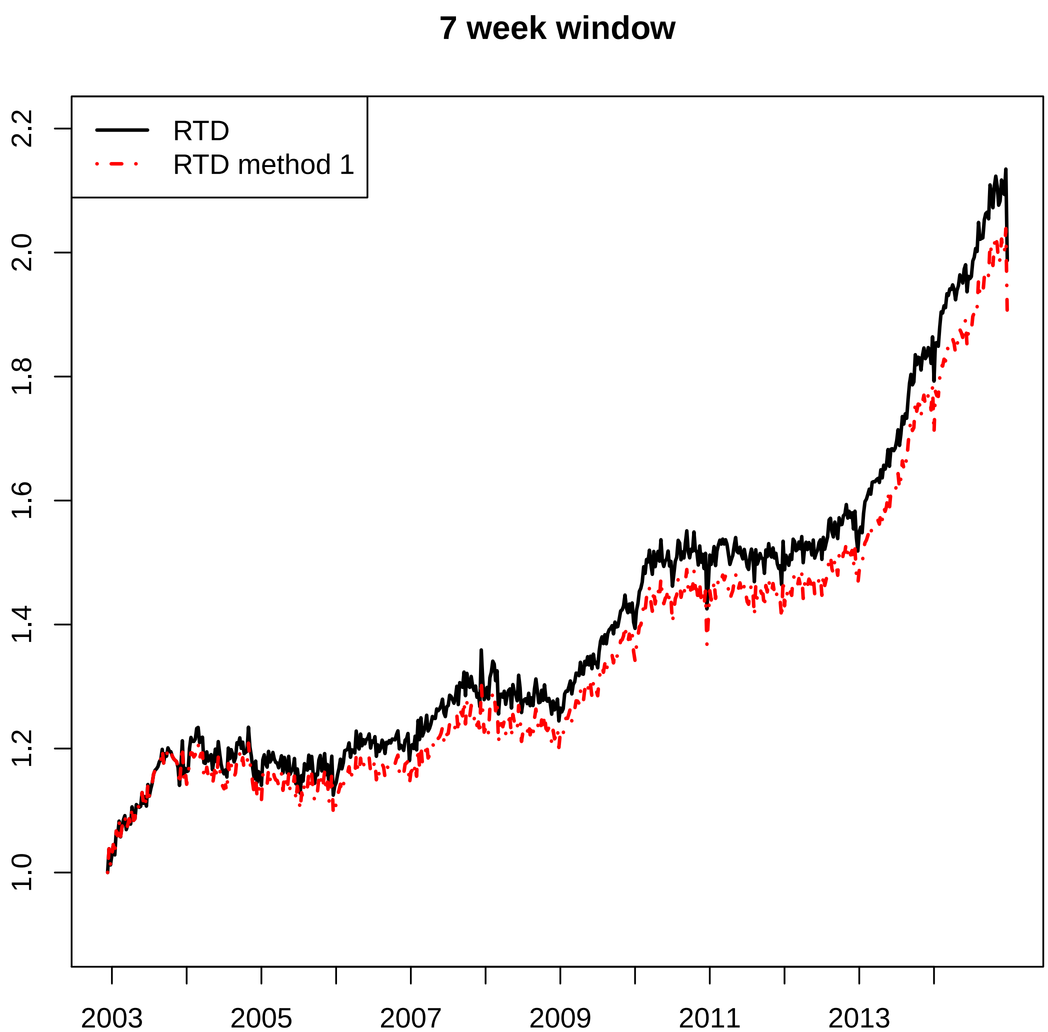


Note: This graph shows the weekly number of transactions in Sydney. The low point each year is the last week in December and the first week in January. When implementing covid-19 Method 1 a threshold of 250 transactions per week is used.

6 Conclusion

We have considered two dimensions over which RTD HPIs can differ. These are the window length and the method used for linking the current period to earlier periods. We have proposed a new criterion for determining the optimal window length and linking method.

Figure 14: Comparing the Standard 7-Week RTD Index with the Covid-19 Method 1 Index



Note: RTD here is a standard weekly RTD price index computed with a 7 week window. RTD method 1 is the modified method where weeks with less than 250 transactions are treated separately as explained in section 2.3 for covid-19 Method 1. Failure to adjust for low transaction weeks seems to cause a slight upward drift in the RTD index.

Focusing on weekly indexes, we find that for Sydney the optimal window length is between 19-21 weeks. For Tokyo the optimal window length is about 18 weeks.

We show that it is possible to improve on the standard linking method used by the RTD method. The linking method that performs best on weekly Sydney data links the current week with a period between 13-16 weeks earlier. For Tokyo, linking the current week with the period 12 weeks earlier performs best. Geometric averages of the single-period linking methods performs about equally well as the best of the single-period linking methods.

We have also considered how the RTD method can be adjusted to mitigate the distorting effects of the covid-19 shutdown on house price indexes. All three of our proposed modifications of RTD are designed to prevent periods with limited data (such as the covid-19 shutdown) from undermining the integrity of the overall index. These methods could prove useful for countries in Europe and the rest of the world that compute their official HPIs using the RTD method.

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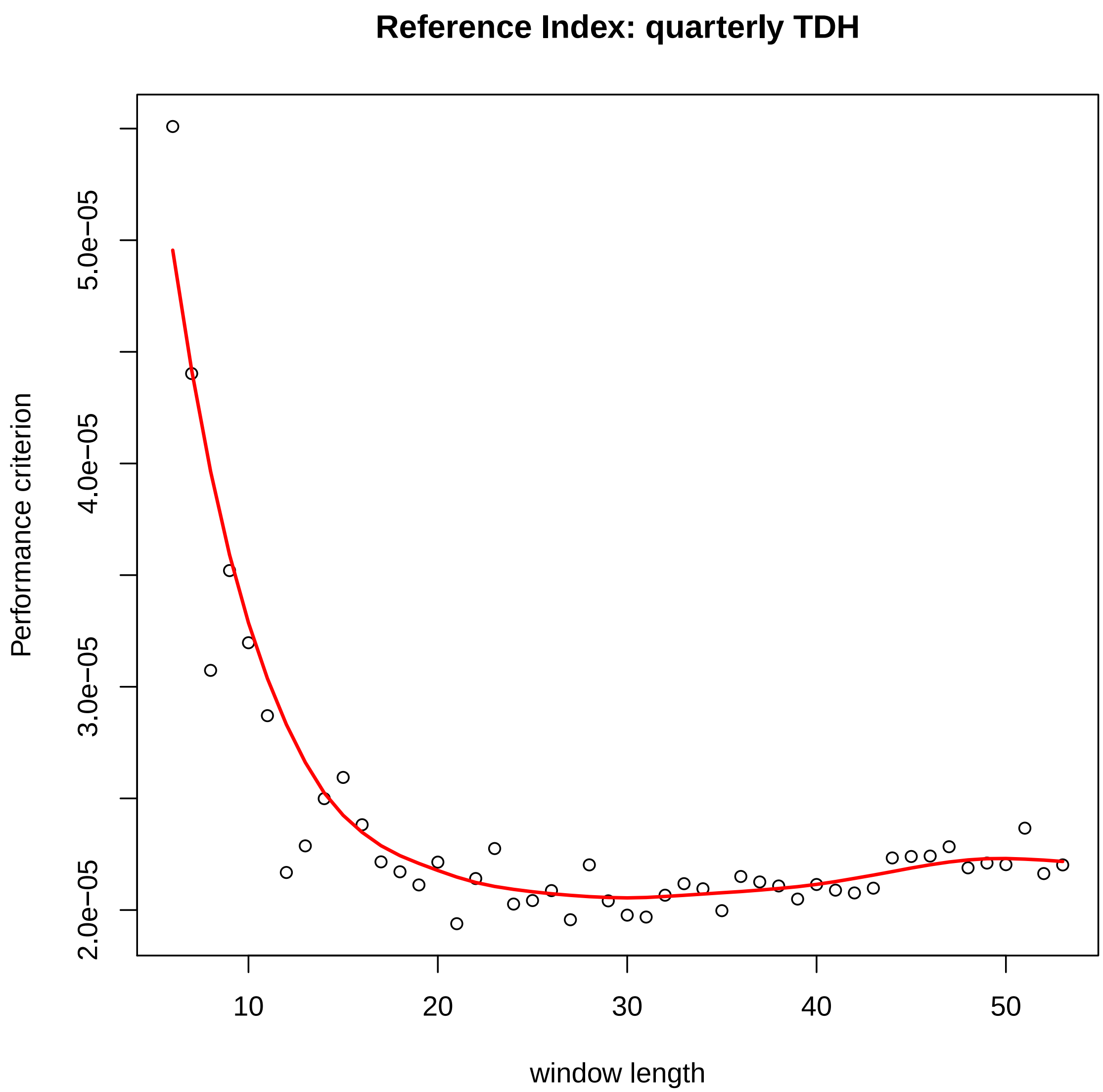
Appendix: Results Obtained Using the Quarterly Time-Dummy Index as the Reference

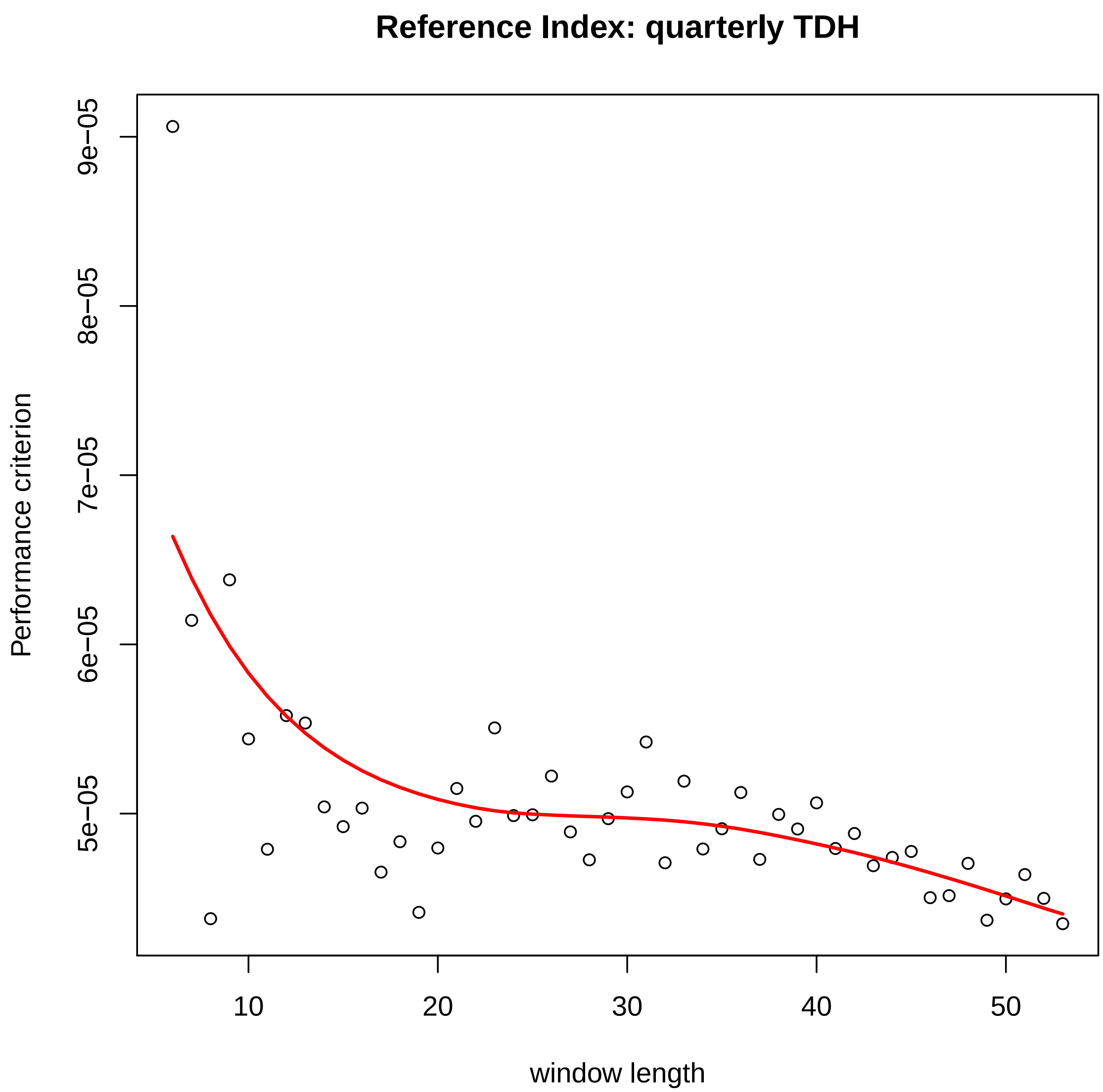
As a robustness check, here we recompute all the results derived using the quarterly hedonic imputation method as a benchmark. Now instead we use the quarterly time-dummy method as the benchmark. In most cases the results are very similar to those obtained using the hedonic imputation method.

Figure A1: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Sydney



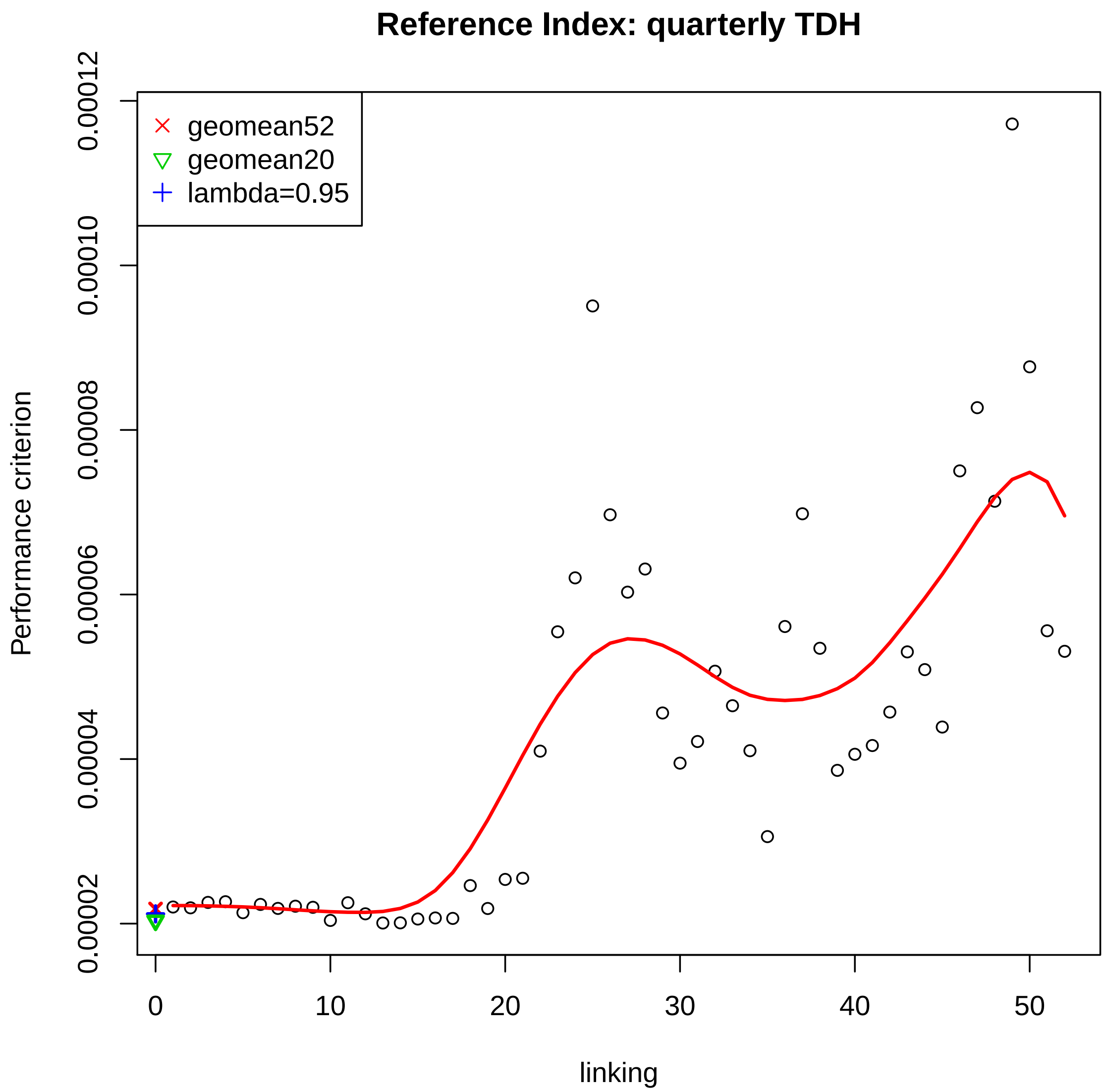
Note: This graph shows which window length generates a weekly RTD price index that most closely approximates a quarterly time-dummy hedonic index. For Sydney the optimal window length here is 21 weeks.

Figure A2: Performance of Alternative Window Lengths with the Quarterly Time-Dummy Price Index as the Reference: Tokyo



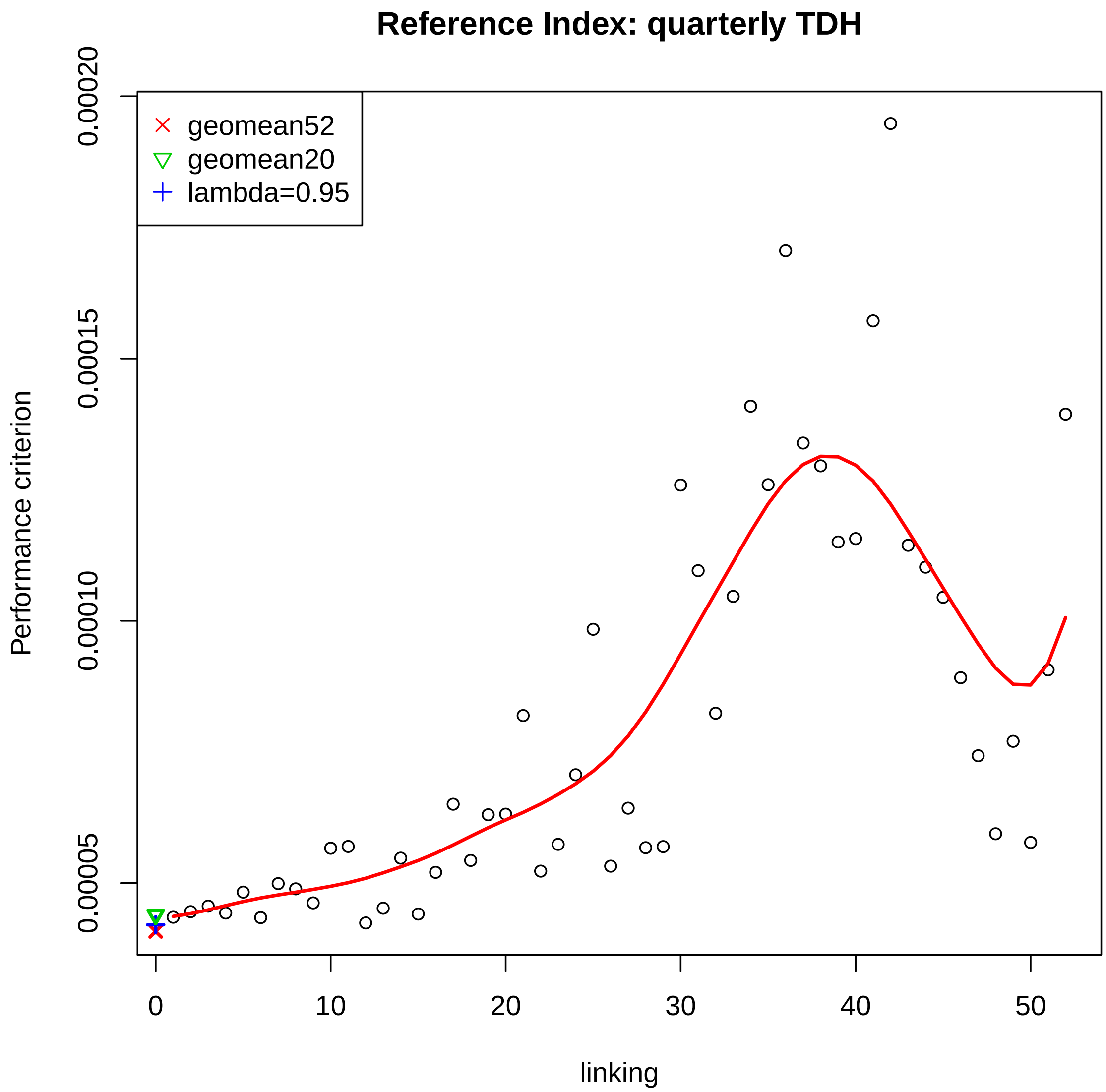
Note: This graph shows which window length generates a weekly RTD price index that most closely approximates a quarterly hedonic imputation index. For Tokyo the optimal window length here is 53 weeks.

Figure A3: Performance of Alternative Linking Methods with Standard RTD Linking: Quarterly Time-Dummy Benchmark for Sydney



Note: This graph shows the performance for Sydney (relative to a quarterly tiem-dummy hedonic index) of different single-period linking methods, Geomean52, Geomean20, and the weighted geomean method in (9) with λ set to 0.95. The best performing method is single-period linking with 13 weeks earlier.

Figure A4: Performance of Alternative Linking Methods with Standard RTD Linking: Time-Dummy Benchmark for Tokyo



Note: This graph shows the performance for Tokyo (relative to a quarterly time-dummy hedonic index) of different single-period linking methods, Geomean52, Geomean20, and the weighted geomean method in (9) with λ set to 0.95. The best performing method is Geomean52.