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Asymmetric Transport Costs and Economic Geography

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Abstract

This paper has explored the impacts of the asymmetry of transport costs with regard to the directions of shipments upon economic geography. In doing so, we have focused on the asymmetry arising from the optimizing behavior of transport firms: they charge different prices for the transport services involving shipments in different directions in response to the difference in the marginal costs of shipment and the price elasticities of the demands. Two cases are studied: the case where the economic activities are distributed equally between regions and the case where they are concentrated in one region. It has been shown that whether each of those two patterns is supported as a stable long-run equilibrium pattern depends on the values of several parameters in a complicated way.

Keywords: marginal cost of shipment, joint production, oligopolistic transport sector, substitution between imports and domestic products, import price effect, export price effect

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1 Introduction

We often encounter the situation in which the levels of transport costs differ from each other if the directions of shipments are different, that is to say, the transport cost necessary to ship goods from one region to another is not equal to the cost necessary to ship them in the opposite direction. Casual observation suggests that the levels depend on the sizes of the demands for transport services or shipments: if the demand for the shipment from one region to another is larger than the counterpart in the opposite direction, the former costs more than the latter. This asymmetry of the transport costs is not a thing which is so trivial that one can ignore when explaining the determination of the geographical pattern of economic activities.

Consider, for instance, an economy with two regions, West and East, where more economic activities are located in West than in East but the difference is quite small. Because the amount of the shipment from West to East is larger than that from East to West, we expect that the transport cost to ship goods in the former direction is higher than the cost to ship them in the latter direction. It implies that the import is less expensive in the West than in East up to the difference in the transport cost, other things being equal. This affects the relative sizes of the force that promotes geographical agglomeration of economic activities (a 'centripetal force') and the force that promotes their geographical dispersion (a 'centrifugal force'), which determine the geographical pattern. On the one hand, consumers find themselves more affordable in West, because of the less expensive imports there, than in East, ceteris paribus. This gives them an incentive to locate in West, which works as a centripetal force. On the other hand, firms attempt to exploit the opportunity of the less competition pressure from the imports by locating in East rather than in West; a centrifugal force. Unfortunately, which force, the centripetal or centrifugal force, dominates the other is ambigous. In any case, nonetheless, we can assert that the asymmetry of the transport costs gives a substantial impact on the relative sizes of the two counteracting forces, and thus, greatly affects the spatial distribution of economic activities.

The purpose of this paper is to study how the asymmetry of transport costs affects the economic geography. For that purpose, a two region model with a transport sector is constructed. As in the standard models in the literature, I focus on two distribution patterns, the symmetric distribution pattern, in which mobile factors are distributed equally in the two regions, and the agglomeration pattern, in which they are concentrated in one region; and examine the conditions for each of them to be supported as a stable equilibrium pattern taking into account the possibility that the transport costs become asymmetric.

Here, it is important to emphasize that in this piece of research, the asymmetry is not something to be assumed but something to be generated as a result of the behaviors of transport firms. Although there would be many reasons for the asymmetry, the following two are particularly considered in this paper.

The first reason is related to a technology. Notice that the shipments in one direction and in the opposite direction are usually *joint products* of a transport firm. One consequence is that the marginal cost of shipping in a certain direction depends on the amounts of not only the shipment in that direction but also on the shipment in the opposite direction. This point would be better understood by a simple example. Let us consider a carrier based in Chicago, say, which owns some cargo trucks. It probably keeps them at its sites within or around Chicago when they are idle. This means that the trucks delivering goods to, say, Seattle, will eventually have to come back to Chicago. Suppose that the carrier gets the order to ship 100 units of goods from Chicago to Seattle. To fulfill the order, 10 trucks need to be dispatched. Then, they can carry goods from Seattle to Chicago on their returning trip *with virtually 0 marginal cost* unless the amount of the return cargo exceeds 100 units, because the 10 trucks must return to Chicago anyway. In general, if there is a physical requirement that transport equipments eventually return to the place of their departure, the marginal cost to ship goods in one direction differs in size from that in the opposite direction: it is positive in the direction with a greater demand but 0 in the opposite direction.

The second reason for the asymmetry is a monopolistic industry structure of the transport sector. As long as the price elasticities of the demand for the transport services in the two directions are not equal to each other, carriers with a certain size of monopoly power attempt to price discriminate. If larger demand is associated with lower price elasticities, they charge a higher price for the transport service in the direction with a greater demand.

So as to pay full attention to those two factors, I consider the situation in which transport firms produce the transport services (shipments) in the two directions jointly, and the marginal costs to produce them are inter-dependent as has been explained in the above illustrative example. Furthermore, it is assumed that they compete with each other à la Cournot in an oligopolistic market. Then, I show that whether each of the two patterns is supported as a stable long-run equilibrium depends on several key parameters. For one thing, higher elasticity of substitution in consumers' preference makes the symmetric distribution pattern more likely to occur while the agglomeration pattern less likely to occur. Furthermore, the higher the marginal cost of shipment associated with the binding capacity, the less likely it is that the symmetric distribution pattern and agglomeration pattern are supported as a stable long-run equilibrium patterns.

For all its importance, the asymmetry of the transport costs has seldom attracted attentions of researchers. Even in the field of new economic geography, which underscores the role of the transport costs in the determination of the geographical pattern of economic activities, the possibility of the asymmetry has been ignored (see Fujita et al. (1999); Fujita and Thisse (2000); and Fujita and Thisse (2002)). This disregard is, for the most part, owning to the fact that to give proper regard to the two reasons of the asymmetry mentioned earlier involves embodying the transport sector explicitly in the model and analyzing how each transport firm behaves. It obviously makes the analysis much more complex, which most of the studies have been avoided.[1] There are a few exceptions. Takahashi (2006b) discusses the behaviors of individual transport firms to obtain welfare implications in a general equilibrium setting. Behrens et al. (2006), on the other hand, examine the impacts of the regulation of a transport sector upon the welfare levels paying attention to the incentives of transport firms, though in a very simple way.

The paper is organized as follows. In the next section, I present a basic model. Section 3 formulates the transport sector and solves the problem faced by each transport firm. In the subsequent two sections, the symmetric distribution pattern and the agglomeration pattern are examined in order. We derive the conditions for each pattern to be supported as a stable equilibrium pattern. Finally, Section 6 concludes.

2 Model

As a basic framework, I use the model used in Takahashi (2006a), which is a modified version of the analytically solvable model by Forslid and Ottaviano (2003).

2.1 Basic Framework

There are two regions, denoted by 1 and 2; and two goods, an intermediate and a final good. Total labor force consists of workers and entrepreneurs. The workers are not endowed with the skill necessary to produce the final good and thus work in the intermediate sector. There are 2 units of workers in the economy, who cannot migrate across the regions. They are distributed equally in the two regions: 1 unit lives in each region. The entrepreneurs are, on the other hand, endowed with the skill so that they can engage in the production of the final good. Furthermore, they can freely move across the two regions. Their number is fixed at n, of which $\lambda_1 n$ live in region 1 and $\lambda_2 n$ live in region 2 ($\lambda_1 + \lambda_2 = 1$). Instead of λ_1 and λ_2, a parameter $\lambda \in [0, 1]$ with $\lambda \equiv \lambda_1$ ($\lambda_2 = 1 - \lambda$) will be often used when doing so is more convenient.

[1] Some works discuss the endogenous determination of the transport costs; but do not deal with the transport sector explicitly. Mori and Nishikimi (2002), for instance, examine the effect of the economy of density on transport costs. Bougheas et al. (1999), and Mun and Nakagawa (2005) study the impacts of infrastructure investment. Finally, Takahashi (2006a) examines the selection of the transport technology, which affects the levels of transport costs.

The intermediate is a homogeneous product and produced in a competitive sector. Each worker produces 1 unit of the intermediate, whose price, therefore, equals his wage rate in each region. The cost to ship the intermediate from one region to the other is assumed to be 0 so that the prices of the intermediate and, consequently, the wage rates of workers are equalized in the two regions. The wage rate is taken as a numeraire.

On the contrary, the final good is a differentiated product and produced by a monopolistically competitive sector. Each entrepreneur owns a firm which produces a variety in the region of her residence. Thus, there are n varieties in the economy of which $\lambda_i n$ are produced in region i $(i = 1, 2)$. Taking a unit appropriately, we can consider that each firm produces 1 unit of a variety from 1 unit of the intermediate using the skill of an entrepreneur. All the revenue left after the payment for the intermediate is taken by the entrepreneur. In other words, the profit of a firm is equal to 0:

$$pq - 1 \cdot q - w = 0, \tag{1}$$

where p and q denote a price and an amount of a variety produced, and w a wage rate received by an entrepreneur.

The workers and entrepreneurs have the same preference represented by a utility function, $U = [\int_0^n x(k)^\rho \, \mathrm{d}k]^{1/\rho}$, where $\rho \in (0, 1)$, and $x(k)$ denotes the amount of the kth variety.

The entrepreneurs sell their varieties at the price that maximizes their own wages. Since not only they are subject to the same technological condition but also all the varieties enter the utility function in a symmetric manner, they charge the same price as long as they are located in the same region. Thus, I denote the prices of a variant produced in respective regions by p_1 and p_2.

Next, I introduce transport costs. It takes no cost to carry the final good within the same region. However, in order to carry it across the regions, consumers need to buy transport services, which are provided by transport firms or 'carriers'. The prices of the services are assumed to be proportional to the values of the good to be shipped. Let $\tau_i \in (0, 1)$ be the proportional coefficient for the transport service that carriers the good from region i to region j $(i = 1, 2$ and $j \neq i)$. Here, remember that the subscript of τ refers to the region of the *origin* of the shipment. Then, in order to have 1 unit of a variety shipped from region i to region j, consumers pay $\tau_i p_i$, which is a price of the transport service, or, in a more ordinary expression, a transport cost, in that direction. The delivered price in region j of the k-th variety produced in region i is, therefore, equal to $(1 + \tau_i)p_i$. Here, we are taking into account the possibility that *the transport costs may differ from each other depending on the direction of shipment*, that is, τ_1 is not necessarily equal to τ_2. This point distinguishes the model from the conventional ones. In what follows, it is more convenient to use notation $t_i \equiv 1 + \tau_i$ as a measure of the prices of the transport services or transport costs rather than

τ_i $(i = 1, 2)$.

Let X_{ii} and X_{ij} be the total demands for a variety produced in region i, by the region i consumers and region j consumers, respectively. Applying the standard procedure to derive consumers' demand functions yields

$$X_{ii} = p_i^{-\sigma} P_i^{\sigma-1} Y_i \qquad (i = 1, 2) \tag{2}$$

and

$$X_{ij} = t_i^{-\sigma} p_i^{-\sigma} P_j^{\sigma-1} Y_j. \qquad (i = 1, 2;\ j \neq i) \tag{3}$$

Here, $\sigma \equiv 1/(1-\rho) > 1$ is an elasticity of substitution, Y_i is an aggregate income and

$$P_i = \left[n \left(\lambda_i p_i^{1-\sigma} + \lambda_j t_j^{1-\sigma} p_j^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}} \qquad (i = 1, 2;\ j \neq i) \tag{4}$$

is a price index in region i. Since the amount of each variety produced in region i, q_i, must be equal to its demand, we have

$$q_i = X_{ii} + X_{ij}. \qquad (i = 1, 2;\ j \neq i) \tag{5}$$

An entrepreneur in region i maximizes her wage given by $w_i = p_i q_i - 1 \cdot q_i$ $(i = 1, 2)$ (see (1)), setting the price at

$$p_1 = p_2 = p \equiv \frac{\sigma}{\sigma - 1}. \tag{6}$$

The wage received by an entrepreneur is reduced to

$$w_i = \frac{1}{\sigma - 1} q_i. \qquad (i = 1, 2) \tag{7}$$

Moreover, because transport firms earn no profit, as will be explained later, the aggregate income consists of workers' and entrepreneurs' earnings:

$$Y_i = 1 + \lambda_i n w_i. \qquad (i = 1, 2) \tag{8}$$

This completes a description of a basic framework.

2.2 Short-Run Equilibrium

Before deriving the short-run equilibrium, it is useful to introduce one additional parameter, $\theta_i \equiv \lambda_i + \lambda_j t_j^{1-\sigma}$ $(i = 1, 2; j \neq i)$. Notice that $\theta_i^{-1} = pX_{ii}/(Y_i/n)$, where the numerator represents the actual level of the spending on each *home* variety by region i consumers while the denominator represents the hypothetical level that would prevail if they consume the home varieties and the imports equally. Because the imports are in fact more expensive than the home products up to the transport cost, they spend more money on each home variety than each foreign variety. Thus, $\theta_i^{-1} > 1$ as long as the transport cost from region

j to region i is positive ($t_j > 1$). The inverse of θ_i measures this *home product bias effect*. With this interpretation, it is straightforward to explain how θ_i depends on t_j and λ_i. First, $\mathrm{d}\theta_i/\mathrm{d}t_j < 0$: as the transport cost of the import declines, θ_i^{-1} decreases in correspondence with the abatement of the home product bias effect. When the transport cost becomes 0 ($t_j = 1$) at one extreme, θ_i^{-1} reaches 1: consumers spend their income equally among the home products and the imports. Second, $\mathrm{d}\theta_i/\mathrm{d}\lambda_i > 0$: as more varieties come to be produced in the home region, the competition among the home varieties becomes severer, which relatively reduces the spending on each home variety.

A system of equations (2) to (8) determines a short-run equilibrium, in which entrepreneurs' distribution, λ, is given. Substituting (2) to (7) into the two equations in (8) and solving these equations simultaneously, we can obtain the equilibrium values of Y_1 and Y_2:

$$Y_i = \frac{\sigma\theta_i\big(\sigma\theta_j - \lambda_j + \lambda_i t_i^{-\sigma}\big)}{(\sigma\theta_1 - \lambda_1)(\sigma\theta_2 - \lambda_2) - \lambda_1\lambda_2 t_1^{-\sigma} t_2^{-\sigma}}. \ (i = 1, 2;\ j \neq i) \tag{9}$$

Y_1 and Y_2, in turn, give the equilibrium values of the demand variables, X_{ii}'s and X_{ij}'s, by (2) and (3):

$$\begin{aligned} X_{ii} &= \frac{(\sigma-1)Y_i}{\sigma n\theta_i} \qquad (i = 1, 2) \\ X_{ij} &= \frac{(\sigma-1)t_i^{-\sigma}Y_j}{\sigma n\theta_j}. \qquad (i = 1, 2;\ j \neq i) \end{aligned} \tag{10}$$

This would be a good place to note that the transport costs affect the wage rates through *substitution effect between imports and domestic products*. Suppose that t_i rises, which implies that the region i products become more expensive in region j ($j \neq i$). Then, the entrepreneurs in region i will be disappointed to find that the demand for their export decline, which causes the fall in their wage rate (an *export price effect*), ceteris paribus. At the same time, those in region j will be happy because the domestic demand for their products expands due to the rise in the import price and therefore their wage rate rise (an *import price effect*). To sum up, the rise in t_i is followed by a fall in w_i by the export price effect and a rise in w_j by the import price effect, other things being constant.

However, this is not the end of the story: in addition to these direct effects, there are indirect effects through the changes in the regional incomes. The fall in w_i and the rise in w_j bring about a decrease in Y_i and an increase in Y_j, respectively. For w_i, the decrease in Y_i works adversely but the increase in Y_j works favorably. The overall direction of the change depends on the sizes of the changes in respective regional incomes and the sensitivities of the wage rate to these changes. When the two regions are sufficiently 'alike', the decrease in Y_i gives a greater impact on w_i than the increase in Y_j because of the home product bias effect. In that case, w_i tends to fall: the indirect *regional income effect* reinforces the direct effects. However, if region j is much 'bigger' than region i, that is, if it has a quite large population and/or a big advantage in the transport costs (the transport cost to ship the goods from that region to the other is much less expensive than the counterpart in the opposite direction), the effect of the increase in Y_j dominates that of the decrease in Y_i; and therefore the changes in the regional incomes result in the rise in w_i.

2.3 Long-Run Equilibrium

In the long run, entrepreneurs move freely across the regions. The long-run equilibrium is the pair of prices and distribution of entrepreneurs for which they have no incentive to change their locations. Let $v_i(\lambda)$ be the indirect utility for an entrepreneur living in region i:

$$v_i(\lambda) = \frac{w_i}{P_i} = \frac{(\sigma-1)(n\theta_i)^{\frac{1}{\sigma-1}}\left[\sigma\theta_j - \lambda_j + t_i^{-\sigma}\left(\sigma\theta_i + \lambda_j t_j^{-\sigma}\right)\right]}{\sigma n\left[(\sigma\theta_1 - \lambda_1)(\sigma\theta_2 - \lambda_2) - \lambda_1\lambda_2 t_1^{1-\sigma}t_2^{1-\sigma}\right]}. \qquad (i = 1, 2;\ j \neq i) \qquad (11)$$

Then, an interior distribution pattern $\lambda \in (0, 1)$, for which at least some entrepreneurs are located in each region, is a long-run equilibrium pattern if $v_1(\lambda) = v_2(\lambda)$. The agglomeration pattern with $\lambda_i = 1$ is so if $v_i(1) \geq v_j(1)$ $(i = 1, 2; j \neq i)$.

In addition, suppose that the equilibrium distribution pattern is perturbed by the relocation of infinitesimally small numbers of entrepreneurs. If they cannot become better off, the equilibrium pattern is considered to be stable. Formally, the equilibrium pattern λ is stable if and only if

$$\begin{cases} v_1(\lambda) \geq v_2(\lambda - \varepsilon) \text{ and } v_2(\lambda) \geq v_1(\lambda + \varepsilon) & \text{when } \lambda \in (0, 1) \\ v_1(1) \geq v_2(1 - \varepsilon) & \text{when } \lambda = 1 \\ v_2(0) \geq v_1(\varepsilon) & \text{when } \lambda = 0, \end{cases}$$

where $\varepsilon > 0$ is a number arbitrarily close to 0.

This condition can be expressed in a more convenient manner. First, recall that $v_1(\lambda) = v_2(\lambda)$ for any equilibrium pattern with $\lambda \in (0, 1)$. Therefore, $v_1(\lambda) \geq v_2(\lambda - \varepsilon)$ in the first line is equivalent to $v_2(\lambda) \geq v_2(\lambda - \varepsilon)$ at the equilibrium pattern. The similar argument applies to the second inequality. Second, for the equilibrium pattern with $\lambda = 1$ (an agglomeration pattern), we can distinguish two cases, the case with $v_1(1) = v_2(1)$ and the case with $v_1(1) > v_2(1)$. In the first case, the same logic applies: $v_1(1) \geq v_2(1 - \varepsilon)$ is equivalent to $v_2(1) \geq v_2(1 - \varepsilon)$. In the second case, the requirement for the stability is automatically satisfied as long as $v_2(\lambda)$ is continuous at $\lambda = 1$, which, it will be shown, holds true. The similar argument can be used for the equilibrium pattern with $\lambda = 0$ (another agglomeration pattern). Putting these results altogether, the stability condition for the equilibrium pattern

λ can be rewritten as follows:

$$\begin{cases} \text{both } \dfrac{\mathrm{d}v_1(\lambda)}{\mathrm{d}\lambda} \leq 0 \text{ and } \dfrac{\mathrm{d}v_2(\lambda)}{\mathrm{d}\lambda} \geq 0 \text{ holds when } \lambda \in (0,1), \\ \text{at least either } \dfrac{\mathrm{d}v_2(1)}{\mathrm{d}\lambda} \geq 0 \text{ or } v_1(1) > v_2(1) \text{ holds when } \lambda = 1, \text{ and} \\ \text{at least either } \dfrac{\mathrm{d}v_1(0)}{\mathrm{d}\lambda} \leq 0 \text{ or } v_1(0) < v_2(0) \text{ holds when } \lambda = 0. \end{cases} \tag{12}$$

3 Transport Sector

There are $m \geq 1$ transport firms or carriers, which produce transport services. The unit is normalized so that 1 unit of the services is necessary to ship 1 unit of the final good from one region to the other. This implies that the demand for the transport service from region i to region $j \neq i$ is equal to that for the import of the final good in region j from region i, that is, $\lambda_i nX_{ij}$, which is expressed as a function of t_1, t_2 and λ:

$$Z_i(t_1, t_2 : \lambda) \equiv \lambda_i nX_{ij} = \frac{\lambda_i t_i^{-\sigma}(\sigma - 1)\big(\sigma\theta_i - \lambda_i + \lambda_j t_j^{-\sigma}\big)}{(\sigma\theta_1 - \lambda_1)(\sigma\theta_2 - \lambda_2) - \lambda_1\lambda_2 t_1^{-\sigma} t_2^{-\sigma}}, \qquad (i = 1, 2, j \neq i) \tag{13}$$

where (10) is used. On the other hand, the total supply of the service from region i to region $j \neq i$ is equal to $\sum_{k=1}^{m} z_i^k$, where z_i^k is the amount of the service in that direction provided by the k-th carrier. The prices of the transport services, t_1 and t_2, are determined so that the demand for each service becomes equal to its supply. In other words, they are given as solutions to the following system:

$$\begin{cases} Z_1(t_1, t_2 : \lambda) &= \displaystyle\sum_{k=1}^{m} z_1^k \\ Z_2(t_1, t_2 : \lambda) &= \displaystyle\sum_{k=1}^{m} z_2^k. \end{cases} \tag{14}$$

Furthermore, the transport services are produced from the intermediate. It is convenient to distinguish two components. One is the intermediate used as a fixed input: each carrier needs to use F units of the intermediate no matter what the production levels are. Thus, it bears the fixed cost whose amount is equal to F. The other is the intermediate used as a variable input. The number or the scale of the equipments necessary to ship the final good, such as cargo ships, freight cars and cargo trucks, evidently depends on the amount of the shipment. The payment for them comprises a variable cost.

For this variable cost, furthermore, I impose an important assumption: there is a physical requirement that transport equipments eventually return to the place of their departure. Then, as has been explained in the introduction by a simple illustrative example, the variable cost of carriers depends on the amount of shipment in one of the two directions, the amount that is larger than the other, namely, $\max[z_1^k, z_2^k]$. In addition, it is assumed that the variable cost is proportional to z_i^k when $z_i^k \geq z_j^k (j \neq i)$ with the proportionality constant being equal to c.

Recalling that carriers receive $p\tau_i \equiv p(t_i - 1)$ for 1 unit of the transport service from region i to region $j \neq i$, we can write down the k-th carrier's profit as

$$\pi^k \equiv p(t_1 - 1)z_1^k + p(t_2 - 1)z_2^k - c\max[z_1^k, z_2^k] - F. \tag{15}$$

Each carrier chooses its level of production given the other carriers' levels à la Cournot, that is, it maximizes its profit subject to

$$z_i^k + Z_i^{-k} = Z_i(t_1, t_2 : \lambda) \qquad (i = 1, 2), \tag{16}$$

where Z_i^{-k} is the amount of the transport services from region i to region $j \neq i$ produced by the other firms, i.e., $Z_i^{-k} \equiv \sum_{l \neq k} z_i^l$.

In its decision making, each carrier anticipates how its production levels affect the transport prices. Totally differentiating each of the two equations in (14) and solving the derived system simultaneously, we obtain

$$\begin{pmatrix} \dfrac{\partial t_1}{\partial z_1^k} & \dfrac{\partial t_2}{\partial z_1^k} \\ \dfrac{\partial t_1}{\partial z_2^k} & \dfrac{\partial t_2}{\partial z_2^k} \end{pmatrix} = \frac{1}{Z_{00}} \begin{pmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{pmatrix}, \tag{17}$$

where $Z_{ij} \equiv \partial Z_i(t_1, t_2 : \lambda)/\partial t_j$ and $Z_{00} \equiv Z_{11}Z_{22} - Z_{12}Z_{21}$ $(i = 1, 2; j \neq i)$. Here, take a look at Z_{ij} more closely. Since $Z_i(t_1, t_2 : \lambda) = \lambda_i n X_{ij}$, changes in the transport prices are transmitted to $Z_i(t_1, t_2 : \lambda)$ only through X_{ij}. As is clear from (3), t_1 and t_2 affect X_{ij} via three channels: first, directly; second, through Y_j; and finally, through P_j. It is not plausible, however, that the carriers take into account such far-flung effects as those of the changes in regional incomes and price indices. More appropriate description would be that they consider only the direct effects, or, in other words, that they are 'regional income takers' and 'price index takers'. Under this assumption, they evaluate Z_{ii} at $-\sigma Z_i(t_1, t_2 : \lambda)/t_i$ while Z_{ij} at 0 $(i = 1, 2; j \neq i)$. If this is the case, we have

$$\begin{cases} \dfrac{\partial t_i}{\partial z_i^k} = -\dfrac{t_i}{\sigma Z_i(t_1, t_2 : \lambda)} \\ \dfrac{\partial t_i}{\partial z_j^k} = 0 \end{cases} \qquad (i = 1, 2, j \neq i) \tag{18}$$

by (17). The regional income and price index taker assumption is not only appropriate for a description of the real world but also helpful in keeping the model simple.

In the rest of the paper, furthermore, we will focus on the symmetric equilibria in which all the carriers produce the same amounts of the transport services. Let us use the asterisk to denote the variables at the symmetric equilibrium. By definition, we have

$$m^* z_i^* = Z_i(t_1^*, t_2^* : \lambda) \qquad (i = 1, 2) \tag{19}$$

for given λ.

Finally, the number of carriers is determined by free entry and exit. To keep the model tractable, I ignore the integer constraint. Then, the 0-profit condition prescribes

$$p\big(t_1^*-1\big)Z_1(t_1^*,t_2^*:\lambda)+p\big(t_2^*-1\big)Z_2(t_1^*,t_2^*:\lambda)-c\max\big[Z_1(t_1^*,t_2^*:\lambda),Z_2(t_1^*,t_2^*:\lambda)\big]-m^*F=0. \tag{20}$$

4 Symmetric Distribution Pattern

In this section, we examine the equilibrium at the symmetric distribution with $\lambda = 1/2$ and the condition for a symmetry breaking.

4.1 Short-Run Equilibrium

First of all, let us derive the short-run equilibrium realized when the distribution of entrepreneurs is symmetric or almost symmetric. Without loss of generality, suppose that $\lambda \geq 1/2$. The analysis is made up of two steps. First, we consider the situation in which all the competitors of one carrier, the k-th carrier, are producing the same amounts of the transport services in the following two senses; the same amounts among carriers ($z_i^l = z_i^h, h \neq l$) and the same amounts with respect to the two directions of shipment ($z_i^l = z_j^l, j \neq i$). It will be shown that the best strategy of the k-th carrier is to do the same as the competitors do. Second, because this result gives a justification for us to limit our attention to the case in which the amounts of the services produced by each firm are equal with respect to the directions of shipment, we solve the k-th carrier's profit maximization problem with the constraint of $z_i^k = z_j^k$. The solutions are then evaluated under the supposition that all the carriers are symmetric.

Let us begin with the first step. All the carriers except the k-th produce z^0 units of z_1 and z^0 units of z_2. Suppose that the shipment of the k-th carrier is not balanced: without loss of generality, we assume that $z_1^k < z_2^k$. Then, can it raise the profit by increasing z_1^k? Using (18), we obtain

$$\begin{aligned}\frac{\partial \pi^k}{\partial z_1^k} &= p\left[t_1'\left\{1-\frac{z_1^k}{\sigma Z_1(t_1',t_2':\lambda)}\right\}-1\right]\\ \frac{\partial \pi^k}{\partial z_2^k} &= p\left[t_2'\left\{1-\frac{z_2^k}{\sigma Z_2(t_1',t_2':\lambda)}\right\}-1\right]-c,\end{aligned} \tag{21}$$

where t_1' and t_2' represent particular levels of transport prices corresponding to this unbalanced situation. Because $z_1^k < z_2^k$, z_2^k becomes a capacity constraint and thus increasing z_2^k involves an additional marginal cost, c. This is why $\partial \pi^k/\partial z_2^k$ contains the negative term of $-c$ while $\partial \pi^k/\partial z_1^k$ does not.

The market clearing prescribes

$$Z_i(t_1', t_2' : \lambda) = z_i^k + (m-1)z^0 \tag{22}$$

because each of $m-1$ carriers produces z^0 units (see (16)). Using this, we can show that $\partial\pi^k/\partial z_1^k$ always exceeds $\partial\pi^k/\partial z_2^k$ as long as λ is sufficiently close to 1/2. Thus, the carrier can raise its profit by either enhancing the production of z_1^k or reducing that of z_2^k. This implies that it chooses the production levels with $z_1^k = z_2^k$.

Lemma 1. *Suppose that all the carriers except the k-th produce $z_1 = z_2 = z^0$ units of the transport services. Then, there is $\varepsilon > 0$ such that the k-th carrier chooses the production levels with $z_1^k = z_2^k$ for any $\lambda \in [1/2, 1/2+\varepsilon]$.*

The proof is relegated to the appendix.

Because the k-th carrier chooses z_1^k and z_2^k with $z_1^k = z_2^k$, its problem is to maximize the profit given by (15) with the constraint that $z_1^k = z_2^k$. It is straightforward to derive the first order condition. Furthermore, recall that we focus on the symmetric equilibrium in which all the carriers produce the same amounts of the transport services, that is, z_1^* units of z_1 and z_2^* units of z_2. Applying (19) and $z_1^* = z_2^*$ to the first condition yields

$$t_1^S + t_2^S = \frac{\big[2\sigma + c(\sigma-1)\big]m^S}{\sigma m^S - 1}, \tag{23}$$

where the superscript S refers to the equilibrium at the symmetric or almost symmetric distribution pattern. What is more, because $Z_1\big(t_1^S, t_2^S : \lambda\big) = Z_2\big(t_1^S, t_2^S : \lambda\big)$ (see (19)), (13) implies that

$$(\sigma-1)\Big[(1-\lambda)^2\big(t_1^S\big)^\sigma - \lambda^2\big(t_2^S\big)^\sigma\Big] + \sigma\lambda(1-\lambda)\big(t_1^S - t_2^S\big) = 0. \tag{24}$$

Finally, the 0-profit condition, (20), is reduced to

$$\big[2\sigma + c(\sigma-1)\big]Z_1\big(t_1^S, t_2^S : \lambda\big) - Fm^S(\sigma-1)(\sigma m^S - 1) = 0, \tag{25}$$

where $Z_1\big(t_1^S, t_2^S : \lambda\big) = Z_2\big(t_1^S, t_2^S : \lambda\big)$ is used again. Those three equations give the equilibrium prices and the equilibrium number of the carriers. Thus, we have established the following proposition.

Proposition 1. *There is $\varepsilon > 0$ such that there exists an equilibrium with $z_1^k = z_2^k = z^S$ for all k and for any $\lambda \in [1/2, 1/2+\varepsilon]$. The equilibrium prices and the equilibrium number of the carriers are given as the solutions of the system consisting of (23) to (25).*

Next, let us examine the special case of the symmetric distribution pattern, $\lambda = 1/2$. First of all, it immediately follows from Proposition 1 and (23) that there exists a unique equilibrium with

$$\bar{t}^S = \frac{\left[2\sigma + c(\sigma-1)\right]\bar{m}^S}{2\left(\sigma\bar{m}^S - 1\right)} > 1, \tag{26}$$

where $\bar{t}^S$ and $\bar{m}^S$ represent the values of t^S and m^S when $\lambda = 1/2$, respectively. It is straightforward to see that $\bar{t}^S$ decreases with $\bar{m}^S$: when the sector is more competitive, the equilibrium price becomes lower. This is a standard result of the Cournot oligopoly model. Furthermore, (25) gives $\bar{m}^S$ as a solution to

$$\frac{2\left[2\sigma + c(\sigma-1)\right]}{2 + \sigma\bar{m}^S(\sigma-1)(2+c) + 2\alpha(\sigma-1)(\sigma\bar{m}^S - 1)} - F\bar{m}^S = 0, \tag{27}$$

where

$$\alpha \equiv \left[\frac{\bar{m}^S\left\{2\sigma + c(\sigma-1)\right\}}{2(\sigma\bar{m}^S - 1)}\right]^{\sigma}.$$

Thus, we have established the following result:

Corollary 1. *When $\lambda = 1/2$, there is a unique equilibrium with $t_1 = t_2 = \bar{t}^S$ and $m = \bar{m}^S$. The two variables are implicitly determined by (26) and (27).*

It is easily verified that

$$\frac{\mathrm{d}\bar{m}^S}{\mathrm{d}c} < 0, \quad \frac{\mathrm{d}\bar{m}^S}{\mathrm{d}F} < 0 \quad \text{and} \quad \frac{\mathrm{d}\bar{m}^S}{\mathrm{d}\sigma} < 0. \tag{28}$$

Notice that $\bar{t}^S$ decreases with $\bar{m}^S$, that $\partial\bar{t}^S/\partial c > 0$ and that $\partial\bar{t}^S/\partial F = 0$. Therefore, it immediately follows from (28) that

$$\frac{\mathrm{d}\bar{t}^S}{\mathrm{d}c} > 0 \quad \text{and} \quad \frac{\mathrm{d}\bar{t}^S}{\mathrm{d}F} > 0. \tag{29}$$

The direction of the change in $\bar{t}^S$ with respect to σ is ambiguous. Numerical simulation, however, suggests the following. When c is relatively large, $\bar{t}^S$ increases with σ (see Fig. 1 (a), which describes the case with $c = 1$ and $F = 0.01$). When c is relatively small, $\bar{t}^S$ first increases and then decreases (see Fig. 1 (b), which depicts the case with $c = 0.1$ and $F = 0.01$).

4.2 Long-Run equilibrium

As has been mentioned, a distribution pattern is supported by a long-run equilibrium if $v_1(\lambda) = v_2(\lambda)$, provided that $\lambda \in (0, 1)$. It is straightforward to see that this is satisfied at the symmetric distribution pattern, $\lambda = 1/2$, and therefore, it is a long-run equilibrium pattern. Thus, the rest of this section is devoted to the examination of the stability of the symmetric distribution pattern. Henceforth, the argument λ of v_i's is often suppressed for shortness' sake as long as it yields no confusion, and, in addition, all the derivatives are evaluated at $\lambda = 1/2$ unless otherwise mentioned.

Before proceeding to the analysis of stability, we need to introduce one additional assumption. Recall that the stability is determined by whether an entrepreneur (or more preciously, an infinitesimally small mass of entrepreneurs) becomes better off when she relocates. Theoretically, her relocation (change in λ) affects the equilibrium number of the carriers operating in the transport sector, m^S (through (25)), which in turn influences the prices of the transport services, t_1^S and t_2^S (through (23) and (24)). Notwithstanding, I treat m^S as a constant *as far as the stability of the equilibrium distribution pattern is concerned.* There are three justifications. First, the entry to and exit from the transport sector often take a long period of time: they will take place after all the adjustments of consumers' locations have finished. Therefore, to judge the stability of the equilibrium pattern, we can focus on the period of time that is not too long for the carriers' entry and exit to occur but is long enough for the entrepreneurs' location changes to be completed. Second, it is not likely that an entrepreneur, when making the decision about the relocation, takes into account the fact that the number of the active carriers changes if she relocates. Rather, she would take it for granted that her action gives no impact on it. Third, the assumption makes the analysis much more tractable.

Applying this assumption, we can easily determine the directions of changes in the transport prices at or near the symmetric distribution. Totally differentiating (23) and (24) provided that m^S is fixed yields

$$\begin{aligned} \frac{\mathrm{d}t_1^S}{\mathrm{d}\lambda} &= \frac{4(\sigma-1)\big(t^S\big)^{\sigma}}{\sigma\Big[1+(\sigma-1)\big(t^S\big)^{\sigma-1}\Big]} > 0 \qquad \text{and} \\ \frac{\mathrm{d}t_2^S}{\mathrm{d}\lambda} &= -\frac{\mathrm{d}t_1^S}{\mathrm{d}\lambda} < 0 \end{aligned} \tag{30}$$

at $\lambda = 1/2$. The result is because as a region gets bigger, the demand for the shipment from that region to the other swells while that in the opposite direction shrinks.

Now, we can show that the condition for the stability represented in the first line of (12) is rewritten in terms of the relative levels of indirect utilities, $\hat{v}(\lambda) \equiv v_1(\lambda)/v_2(\lambda)$ (in this paper, the circumflex always refers to the ratios of the variables with respect to region 1 to those with respect to region 2):

Lemma 2. *The symmetric distribution pattern is a stable equilibrium pattern if and only if* $d\hat{v}(1/2)/d\lambda \leq 0$.

The proof is relegated to the appendix.

Now, we are ready to examine the stability condition, $\mathrm{d}\hat{v}(1/2)/\mathrm{d}\lambda \leq 0$. Let $\hat{w} \equiv w_1/w_2$ and $\hat{P} \equiv P_1/P_2$. Then, (11) implies that

$$\frac{\mathrm{d}\hat{v}}{\mathrm{d}\lambda} = \frac{\mathrm{d}\hat{w}}{\mathrm{d}\lambda} - \frac{\mathrm{d}\hat{P}}{\mathrm{d}\lambda}, \tag{31}$$

because $\hat{v} = \hat{w} = \hat{P} = 1$ at $\lambda = 1/2$. Since our main concern is on the question of how the possibility of the asymmetric transport costs affects the stability, it would be worth separating the effects of the induced changes in the transport costs. To do so, it is convenient to look at the change in the relative wage rate and the relative price index one by one.

Let us begin with the change in the wage rates. Equations (2) - (7), along with the facts that $\hat{Y} \equiv Y_1/Y_2 = 1$, that $\hat{\theta} \equiv \theta_1/\theta_2 = 1$ and that $\mathrm{d}t_2^S/\mathrm{d}\lambda = -\mathrm{d}t_1^S/\mathrm{d}\lambda$ (see (30)) at $\lambda = 1/2$, imply that

$$\frac{\mathrm{d}\hat{w}}{\mathrm{d}\lambda} = -\frac{2\sigma\left[t^S + \left(t^S\right)^\sigma\{\sigma + (\sigma-1)t^S\}\right]}{\beta\left(t^S\right)t^S\left[1 + \left(t^S\right)^\sigma\right]}\frac{\mathrm{d}t_1^S}{\mathrm{d}\lambda} + \frac{\left(t^S\right)^\sigma - 1}{1 + \left(t^S\right)^\sigma}\left(\left.\frac{\mathrm{d}\hat{Y}}{\mathrm{d}\lambda}\right|_{t_1^S const} - \left.\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}\lambda}\right|_{t_1^S const}\right), \tag{32}$$

with

$$\begin{aligned}
\left.\frac{\mathrm{d}\hat{Y}}{\mathrm{d}\lambda}\right|_{t_1^S const} &= \frac{8\left(t^S\right)^\sigma(1+t^S)}{\beta\left(t^S\right)\left[t^S + \left(t^S\right)^\sigma\right]} > 0, \\
\left.\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}\lambda}\right|_{t_1^S const} &= \frac{4\left[\left(t^S\right)^\sigma - 1\right]}{t^S + \left(t^S\right)^\sigma} > 0,
\end{aligned}$$

where all the derivatives and t^S are evaluated at $\lambda = 1/2$ and

$$\beta\left(t^S\right) \equiv 1 + \sigma t^S + (\sigma-1)\left(t^S\right)^\sigma > 0.$$

Equation (32) indicates that the impact of the change in the distribution of entrepreneurs on the relative wage rate is decomposed into three parts.

First, it affects the relative wage rate through the transport costs. Suppose that λ rises. As we have seen in (30), t_1 rises and t_2 falls, which provokes the substitution between imports and domestic products. The rise in t_1, on the one hand, induces the region 2 consumers to substitute the domestic products for the imports from region 1, which results in the fall in w_1 (the export price effect in terms of region 1) and the rise in w_2 (the import price effect in terms of region 2). The fall in t_2, on the other hand, promotes the substitution of the imports for the domestic products in region 1, which again lowers w_1 (the import price effect in terms of region 1) and enhances w_2 (the export price effect in terms of region 2). Consequently, the relative wage rate declines. As we have explained before, however, there is also an indirect effect through the changes in the regional incomes (regional income effect). The two regions are sufficiently 'alike' in the sense explained in Section 2.2, the regional income effect reinforces the direct effects just mentioned. Hence, the total effect on the relative wage rate is negative, which is captured by the first term in (32).

Second, the distribution of entrepreneurs affects the regional incomes even if the transport cost remained the same. The relative regional income rises as a result of the increase in λ. This is because of the *demand linkage effect*: when a region has more population, the demand for the varieties produced in that region increases. This effect is represented by the term $\mathrm{d}\hat{Y}/\mathrm{d}\lambda\big|_{t_1^S const}$.

Third and finally, the change in the distribution of entrepreneurs is transmitted to the wage rates through the change in the size of the home product bias effect even if the transport costs remain unchanged. As more varieties come to be produced in a region, competition among them becomes intensified. This counteracts the home product bias effect and raises θ in that region. Therefore, the increase in λ results in the rise in $\hat{\theta}$. This effect is represented by $\mathrm{d}\hat{\theta}/\mathrm{d}\lambda\big|_{t_1^S const}$.

Next, we turn our eyes to the change in the price index. It is straightforward to see

$$\frac{\mathrm{d}\hat{P}}{\mathrm{d}\lambda} = -\frac{2}{(\sigma-1)\Big[t^S + \big(t^S\big)^\sigma\Big]}\left[2\Big\{\big(t^S\big)^\sigma - t^S\Big\} + (\sigma-1)\frac{\mathrm{d}t_1^S}{\mathrm{d}\lambda}\right] < 0, \tag{33}$$

where again all the derivatives are evaluated at $\lambda = 1/2$. Two factors affect the ratio of the price index. The first is a direct effect: as more entrepreneurs choose to locate in region 1, the mass of varieties produced there increases, which lowers the relative price index. The second is the effect through the change in the transport costs. When λ increases, t_1 rises and t_2 falls. As a result, the region 1's import from region 2 relatively falls compared to that of the region 2's import from region 1; and, consequently, the ratio of the price indices falls. Both effects are negative and therefore the total effect is also negative.

Substituting the relevant expressions to (31) and arranging it yields

$$\frac{\mathrm{d}\hat{v}}{\mathrm{d}\lambda} = \frac{2}{\beta\big(t^S\big)(\sigma-1)(1+t^S)\Big[t + \big(t^S\big)^\sigma\Big]}\left[2\gamma\big(t^S\big) + (\sigma-1)\delta\big(t^S\big)\frac{\mathrm{d}t_1^S}{\mathrm{d}\lambda}\right] \tag{34}$$

where all the derivatives are evaluated at $\lambda = 1/2$, and

$$\begin{aligned}
\gamma\big(t^S\big) &\equiv 2\big(t^S\big)^\sigma(\sigma-1)(1+t^S)\Big[\big(t^S\big)^\sigma - 1\Big] + \beta\big(t^S\big)\Big[\big(t^S\big)^\sigma - t^S\Big]\Big[\sigma + (2-\sigma)\big(t^S\big)^\sigma\Big],\\
\delta\big(t^S\big) &\equiv 1 - \sigma\big(t^S\big)^\sigma\left[\sigma + (\sigma-2)t^S\right] - 2\big(t^S\big)^{2\sigma-1}\left[\sigma^2 + (\sigma-1)^2 t^S\right].
\end{aligned}$$

If $\mathrm{d}t_1^S/\mathrm{d}\lambda$ were 0, the equation $\mathrm{d}\hat{v}/\mathrm{d}\lambda$ would give a conventional break point for the symmetric distribution pattern. In that case, the pattern would be stable if $\gamma\big(t^S\big) \le 0$ and unstable otherwise. Notice that $\gamma\big(t^S\big) > 0$ for t^S that is sufficiently close to 1. This is because $\gamma(1) = 0$ and $\mathrm{d}\gamma(1)/\mathrm{d}t^S = 8\sigma(\sigma-1) > 0$. On the other hand, we note that $\gamma\big(t^S\big) < 0$ for sufficiently high t^S, because $\gamma\big(t^S\big)$ approaches negative infinity as t^S goes to infinity. Thus, the symmetric pattern is, for a given value of σ, stable if t^S given by (26) becomes sufficiently high while it is unstable if t^S becomes sufficiently small. The conclusion that the high (low, resp.) transport cost tends to make the symmetric pattern stable (unstable, resp.) is along the line of the conventional literature of the new economic geography.

What is missing in the literature is, however, the term containing $\mathrm{d}t_1/\mathrm{d}\lambda$ at the square brackets in (34). This term represents the effect through the change in the transport cost due to the profit maximizing behaviors of the carriers. Since $\mathrm{d}t_1/\mathrm{d}\lambda > 0$, this effect works toward the direction to stabilize the symmetric pattern if $\delta(t^S) < 0$ and the direction to destabilize it if $\delta(t^S) > 0$. Several observations follow with respect to $\delta(t^S)$. First, $\delta(t^S) = (t^S - 1)^2 > 0$ for any t^S at $\sigma = 1$. By continuity, it implies that $\delta(t^S) > 0$ for σ sufficiently close to 1. Therefore, the transport cost effect works against the stabilization when σ is sufficiently small. Second, $\delta(t^S) < 0$ for any $\sigma > 2$.[2] Thus, the effect stabilizes the symmetric pattern whenever $\sigma > 2$. Third, notice that $\mathrm{d}\delta(t^S)/\mathrm{d}t^S > 0$. We have already seen that as c and/or F rises, the equilibrium $\bar{t}^S$ rises (see (29)) because fewer carriers come to operate in the sector. Consequently, $\delta(t^S)$ increases for given σ: as a result of the rise in c and/or F, it becomes more likely that the transport effect works to destabilize the symmetric pattern. We have established the following.

Proposition 2. *i) The transport cost effect works to stabilize, rather than to destabilize, the symmetric pattern if $\sigma > 2$; and furthermore, it works to destabilize the pattern if σ is sufficiently small.*
ii) As c and/or F rises, it becomes more likely that the transport cost effect works to destabilize the symmetric pattern.

Finally, what can we say about the overall stability? To examine the stability in terms of the three parameters, c, F and σ, is not straightforward because the key equations are highly nonlinear and some of them give the solution only implicitly. Thus, the best strategy would be to rely on some numerical analysis. Fig. 2 shows how c affects $\mathrm{d}\hat{v}/\mathrm{d}\lambda$ for a given value of $F = 0.01$ and σ. The three panels describe the cases with different values of σ, namely, $\sigma = 1.5$, $\sigma = 2$ and $\sigma = 3$. When σ is low ($\sigma = 1.5$), the symmetric pattern is stable for small c but unstable for large c, that is, it 'breaks' when c is greater than a critical value. When σ is high ($\sigma = 2$ or $\sigma = 3$), the symmetric pattern is stable even for sufficiently small c: it never breaks. Thus, concerning the stability of the symmetric pattern, the size of parameter c matters only when σ is low. The numerical simulation suggests that the same conclusion applies to parameter F: its size matters only when σ is low; and in that case, the higher F is, the more likely it is that the symmetric pattern becomes unstable.

[2]This is because $1 < \sigma(t^S)^\sigma[\sigma + (\sigma - 2)t^S]$.

5 Agglomeration

In this section, let us focus on the case in which almost all the entrepreneurs are located in region 1, that is, λ is sufficiently close to 1. The other polar case with λ being sufficiently close to 0 is similarly analyzed. The short-run equilibrium is characterized first; and then its sustainability is discussed.

5.1 Short-Run Equilibrium

When λ is sufficiently close to 1, we expect that carriers produce the transport service from region 1 to region 2 more than that from region 2 to region 1 since most of the varieties are produced in region 1, which implies that almost all the demand for transport services comes from the consumers in region 2: most of the variants are produced in region 1. Indeed, it turns out that there is an equilibrium in which all the carriers produce z_1^* and z_2^* units with $z_1^* > z_2^*$ for given λ.

This is shown through several steps. First, it is shown that no carrier has an incentive to choose z_1 and z_2 with $z_1 < z_2$ when all the other carriers produce z_1^0 and z_2^0 units with $z_1^0 > z_2^0$. Second, the profit maximization problem of a representative carrier, which is subject to the constraint $z_1 > z_2$, is solved. Here, I focus on the case of symmetric carriers in the sense of (19): any of them produces z_1^* and z_2^* units. Third and finally, it is verified that the solution to the maximization problem indeed satisfies $z_1^* > z_2^*$.

Let us begin with the first step by examining the profit maximization of a single carrier, say the k-th carrier, when every other carrier produces z_1^0 and z_2^0 units, given λ. Suppose that the k-th carrier produces z_1^k and z_2^k units with $z_1^k \le z_2^k$. Its marginal profits are given by (21), because z_2^k becomes the capacity constraint. Furthermore, $Z_i(t_1', t_2' : \lambda) = z_i^k + (m-1)z_i^0$, where t_1' and t_2' are the prices realized in this specific situation. In addition, let t_1^0 and t_2^0 be the prices when all the carriers including the k-th produce z_1^0 and z_2^0 units. If $\partial\pi^k/\partial z_1^k > \partial\pi^k/\partial z_2^k$ for any z_1^k and z_2^k with $z_1^k \le z_2^k$, then the k-th carrier can raise its profit by relatively increasing the production of z_1 compared to that of z_2. In this case, therefore, the carrier is inclined to choose z_1^k and z_2^k with $z_1^k > z_2^k$, which I prove in the following lemma.

Lemma 3. *Suppose that all the carriers except the k-th produces z_1^0 and z_2^0 units of the transport services with $z_1^0 > z_2^0$. Then, as long as $t_1^0 > t_2^0$, the k-th carrier chooses the production levels with $z_1^k > z_2^k$.*

The proof is tedious and relegated to the appendix.

Now, let us proceed to the second step. Because Lemma **??** guarantees, as long as $t_1^0 > t_2^0$, that the k-th carrier has no incentive to choose z_1^k and z_2^k with $z_1^k \leq z_2^k$, it suffices to solve the profit maximization problem with the constraint $z_1^k > z_2^k$. The necessary condition is given as

$$\begin{aligned} \frac{\partial \pi^k}{\partial z_1^k} &= p\left[t_1\left\{1 - \frac{z_1^k}{\sigma Z_1(t_1, t_2 : \lambda)}\right\} - 1\right] - c &= 0 \\ \frac{\partial \pi^k}{\partial z_2^k} &= p\left[t_2\left\{1 - \frac{z_2^k}{\sigma Z_2(t_1, t_2 : \lambda)}\right\} - 1\right] &= 0. \end{aligned} \tag{35}$$

Assuming the symmetry among carriers, we can obtain the following equations:

$$\begin{aligned} t_1^* = t_1^A &\equiv \frac{\big[\sigma + c(\sigma - 1)\big]m^A}{\sigma m^A - 1} \\ t_2^* = t_2^A &\equiv \frac{\sigma m^A}{\sigma m^A - 1}. \end{aligned} \tag{36}$$

Therefore, we have indeed $t_1^* > t_2^*$, which is assumed in Lemma **??** as $t_1^0 > t_2^0$. Here, it is worth noting that parameter c, the marginal cost of the binding transport service, plays a critical role in the determination of the transport prices. At a limit case with $c = 0$, the two transport prices become equal. As it rises, the divergence between them increases. Thus, c corresponds to the degree of the asymmetry in the transport prices.

The last step is to verify that the solution is consistent with the supposition that $z_1^* > z_2^*$. Take a look at the limit case with $\lambda = 1$. The total amounts of the transport services are given as

$$Z_1(t_1^*, t_2^* : 1) = \frac{\sigma - 1}{\sigma t_1^*}, \qquad Z_2(t_1^*, t_2^* : 1) = 0. \tag{37}$$

Since $Z_1(t_1^*, t_2^* : 1)/m^* > Z_2(t_1^*, t_2^* : 1)/m^*$, we have $z_1^* > z_2^*$ for $\lambda = 1$. Because no regime change occurs, as has been discussed in Lemma **??**, the solution to the maximization problem is continuous. Consequently, for λ sufficiently close to 1, it must still hold that $z_1^* > z_2^*$.

These steps establish the following result:

Proposition 3. *There is $\varepsilon > 0$ such that for any $\lambda \in [1 - \varepsilon, 1]$, there exists an equilibrium with $z_1^k = z_1^A$ and $z_2^k = z_2^A$ for all k with $z_1^A > z_2^A$. The equilibrium prices satisfy (36).*

Finally, let us examine the special case of the agglomeration, namely, the case with $\lambda = 1$. Substituting (36) and (37) into (20) yields $m = \bar{m}^A \equiv 1/\mu$, where $\mu \equiv \sqrt{F\sigma}$ is a parameter which measures the size of the fixed cost. The transport prices are, consequently, given by

$$\begin{aligned} t_1 &= \bar{t}_1^A \equiv \frac{\sigma + c(\sigma - 1)}{\sigma - \mu} \\ t_2 &= \bar{t}_2^A \equiv \frac{\sigma}{\sigma - \mu}. \end{aligned} \tag{38}$$

Corollary 2. *When $\lambda = 1$, the equilibrium prices and the number of carriers are given by $\bar{t}_1^A$ and $\bar{t}_2^A$, and $\bar{m}^A$, respectively.*

5.2 Long-Run Equilibrium

As has been mentioned earlier, the agglomeration pattern with $\lambda = 1$ is supported by the long-run equilibrium if $v_1(1) \geq v_2(1)$, and if so, it is stable if at least either $\mathrm{d}v_2(1)/\mathrm{d}\lambda \geq 0$ or $v_1(1) > v_2(1)$ holds. Recall that the condition $\mathrm{d}v_2(1)/\mathrm{d}\lambda \geq 0$ matters only when v_1 and v_2 *happen* to become equal to each other at the agglomeration pattern. Because there is no reason to believe that such is an important and often observed situation in the real world, I rather focus on the case with $v_1(1) > v_2(1)$. In short, the case with $v_1(1) = v_2(1)$ is disregarded as a singular situation. According to the terminology in the literature, furthermore, I say that the agglomeration pattern is *sustainable* when it is supported as a stable equilibrium pattern, that is, when $v_1(1) > v_2(1)$, and unsustainable when $v_1(1) < v_2(1)$. To put it another way, it is sustainable when $\hat{v}(1) > 1$ and unsustainable when $\hat{v}(1) < 1$, where $\hat{v}(1)$ turns out to be equal to

$$\hat{v}(1) = \frac{\sigma \bar{t}_1^A \left(\bar{t}_2^A\right)^\sigma (1 + \bar{t}_1^A)}{1 + \sigma \bar{t}_1^A + \left(\bar{t}_1^A\right)^\sigma \left(\bar{t}_2^A\right)^\sigma (\sigma - 1)}. \tag{39}$$

Our goal is to find how this sustainability is affected by parameters, c, σ and μ. To begin with, the next result immediately follows from (39) (the proof is again relegated to the appendix):

Proposition 4. *i) When $\sigma \leq 2$, the agglomeration pattern is sustainable for any c and μ. ii) When $\sigma > 2$, it is unsustainable for sufficiently large c.*

The reason why sufficiently large c makes the agglomeration pattern unsustainable is not obvious. Indeed, c is related to the sustainability in a quite complicated way. The first thing to note is that it affects the sustainability only through $\bar{t}_1^A$ (as c rises, $\bar{t}_1^A$ rises as (38) indicates): (39) shows that $\hat{v}(1)$ does not depend on c directly. Therefore, in order to answer the question of how c affects the sustainability, it suffices to study the impact of the change in $\bar{t}_1^A$ on $\hat{v}(1)$, given that $\bar{t}_2^A$ is fixed.

The impact is decomposed to that on the wage rates and that on the price indices, as before. On the one hand, the relative wage rate, $\hat{w} \equiv w_1/w_2$, decreases with $\bar{t}_1^A$. Too see this, note that

$$\begin{aligned} w_1 &= \frac{1}{n\sigma}\left(Y_1 + \frac{Y_2}{\bar{t}_1^A}\right) = \frac{1 + \bar{t}_1^A}{n\bar{t}_1^A(\sigma - 1)} \\ w_2 &= \frac{1}{n\sigma}\left[\left(\bar{t}_1^A\right)^{\sigma-1} Y_2 + \frac{Y_1}{(\bar{t}_2^A)^\sigma}\right] = \frac{\left(\bar{t}_1^A\right)^{\sigma-1}\left[\sigma - 1 + \left(\bar{t}_1^A\right)^{-\sigma}\left(\bar{t}_2^A\right)^{-\sigma}(1 + \sigma\bar{t}_1^A)\right]}{n\sigma(\sigma - 1)}. \end{aligned} \tag{40}$$

Now, we suppose that $\bar{t}_1^A$ rises. First, this directly affects the wage rates through the substitution between the import and the domestic products, namely, through the export price effects and the import price effects: w_1 falls by the export price effect while w_2 rises by the import price effect. Second, these changes alter the regional incomes: Y_1 declines and Y_2 increases. These effects are shown by the terms between the two equality signs in each equation of (40): the direct effects (the export price effect for w_1 and the import price effect for w_2) come through the change in $\bar{t}_1^A$ and the indirect effects (the regional income effects) come through the changes in Y_1 and Y_2. For w_1, the sum of the export price effect and the effect of the decline in Y_1 dominates the effect of the rise in Y_2 and therefore the total effect is negative: w_1 decreases with $\bar{t}_1^A$. For w_2, however, the direction of the overall change is ambiguous. When $\sigma > 2$, the substitution is so large that the sum of the import price effect and the effect of the rise in Y_2 dominates the effect of the decline in Y_1, and as a result, w_2 increases. When σ is sufficiently close to 1, to the contrary, the decline in Y_1 gives a severe impact and consequently, w_2 declines. Yet even if w_2 declines, the scale of the decline is relatively small compared to that of the decline in w_1. Therefore, $\hat{w} \equiv w_1/w_2$ decreases.

On the other hand, the relative price index, $\hat{P} \equiv P_1/P_2$, also decreases with $\bar{t}_1^A$. As $\bar{t}_1^A$ rises, varieties, all of which are imported from region 1, become more expensive in region 2. Consequently, the price index in region 2, $P_2 = \sigma\bar{t}_1^A n^{\frac{1}{\sigma-1}}/(\sigma-1)$, rises. Because $\bar{t}_1^A$ does not affect the price index in region 1, $P_1 = \sigma n^{\frac{1}{\sigma-1}}/(\sigma-1)$, $\hat{P}$ falls.

To sum up, as a result of the rise in $\bar{t}_1^A$, both the relative wage rate and the relative price index fall. Because the two forces work on the relative indirect utility, $\hat{v}(1)$, in the opposite directions, therefore, this information is not sufficient to determine whether $\hat{v}(1)$ increase or not. However, it can be verified that when $\bar{t}_1^A$ is large, the relative wage rate declines more rapidly than the relative price index, which results in the decline of $\hat{v}(1)$. Then, for sufficiently large $\bar{t}_1^A$, that is, for sufficiently large c, $\hat{v}(1)$ becomes lower than 1 and the agglomeration pattern becomes unsustainable.

Some numerical simulation analyses may be helpful for us to obtain a clearer image of the impacts of c on the sustainability. Fig. 3 shows how $\hat{v}(1)$ changes as c increases, for given values of σ and $F = 0.1$. The four panels describe the cases with different values of σ. First, when $\sigma < 2$ (the first panel shows the case with $\sigma = 1.8$), $\hat{v}(1)$ increases with c and approaches positive infinity. Because $\hat{v}(1) > 1$ at $c = 0$, the agglomeration pattern is sustainable for any $c \geq 0$. Second, when σ is of an intermediate size (the second panel describes the case with $\sigma = 2.2$), $\hat{v}(1) > 1$ at $c = 0$ and $\hat{v}(1)$ first increases and then decreases. Because it eventually goes to 0, there is a critical value c^* such that $\hat{v}(1) \gtreqless 1$ if $c \lesseqgtr c^*$. Consequently, the agglomeration pattern is sustainable for $c < c^*$ while unsustainable for $c > c^*$. Finally, when σ is large, $\hat{v}(1)$ decreases with c. If σ is moderately large (the case with $\sigma = 3$ is described by the third panel), $\hat{v}(1) > 1$ at $c = 0$. Therefore, there is a critical value c^* explained above. If σ is too large (the case with $\sigma = 7$ is depicted by the last panel), however, $\hat{v}(1) < 1$ at $c = 0$. Thus, $\hat{v}(1) < 1$ for any $c \geq 0$: the agglomeration pattern is unsustainable for any $c \geq 0$.

We can obtain similar results for the change in F. Fig. 4 shows that, given $c = 5$, the agglomeration pattern is sustainable for any $F \geq 0$ when $\sigma < 2$ (see the first panel with $\sigma = 1.8$), that it is sustainable for $F < F^*$ but unsustainable for $F > F^*$ when σ is of an intermediate size (see the second panel with $\sigma = 2.3$), and that it is unsustainable for any F when σ is large (see the last panel with $\sigma = 3$).

Finally, Fig. 5 describes the loci of (F, c) that makes $\hat{v}(1)$ equal to 1 for given values of σ. Because $\partial\hat{v}(1)/\partial c < 1$ at $\hat{v}(1) = 1$, $\hat{v}(1) < 1$ at the area above each locus while $\hat{v}(1) > 1$ at the area below it. Therefore, the agglomeration pattern is more likely to be sustainable when c is lower. This result is consistent with our earlier finding. Furthermore, the figure also suggests that lower σ is more likely to be associated with the sustainability. Lastly, the effect of F is not necessarily monotone. To see this, consider the case with $\sigma = 3$ and $c = 0.65$, for instance. For small F ($F < 0.0211$) and large F ($F > 0.171$), we have $\hat{v}(1) > 1$: the agglomeration pattern is unsustainable. For the intermediate values of F, however, $\hat{v}(1) < 1$ and therefore, it is sustainable. Nonetheless, this non-monotonicity occurs only when c is sufficiently small: when c is large enough, $\hat{v}(1) < 1$ for any F.

The next observation summarizes these simulation results.

Observation 1. *As the varieties become more substitutable (σ becomes higher) and/or the transport costs become more asymmetric (c becomes higher), one find sustaining the agglomeration pattern become more difficult.*

6 Concluding Remarks

This paper has explored the impacts of the asymmetry of transport costs with regard to the directions of shipments upon economic geography. In doing so, we have focused on the asymmetry arising from the optimizing behavior of transport firms: they charge different prices for the transport services involving shipments in different directions in response to the difference in the marginal costs of shipment and the price elasticities of the demands. Two cases are studied: the case where the economic activities are distributed equally between regions and the case where they are concentrated in one region. It has been shown that higher elasticity of substitution in consumers' preference makes the symmetric distribution pattern more likely to occur while the agglomeration pattern less likely to occur. Furthermore, the higher the marginal cost of shipment associated with the binding capacity, the less likely it is that the symmetric distribution pattern and agglomeration pattern are supported as a stable long-run equilibrium patterns.

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Appendix

Proof of Lemma 1.
Suppose that $z_1^k < z_2^k$. This immediately implies that $Z_1(t_1', t_2' : \lambda) < Z_2(t_1', t_2' : \lambda)$ (see (22)). However, (13) implies that

$$\frac{Z_1(t_1', t_2' : 1/2)}{Z_2(t_1', t_2' : 1/2)} = \frac{1 + (\sigma - 1)(t_2')^\sigma + \sigma t_2'}{1 + (\sigma - 1)(t_1')^\sigma + \sigma t_1'}.$$

Therefore, $t_1' > t_2'$ in a sufficiently small neighborhood of $\lambda = 1/2$, that is to say, there exists ε for which $t_1' > t_2'$ for any $\lambda \in [1/2, 1/2 + \varepsilon]$. On the other hand, $z_1^k < z_2^k$ together with (22) implies that $1 - z_1^k/\sigma Z_1(t_1', t_2' : \lambda) > 1 - z_2^k/\sigma Z_2(t_1', t_2' : \lambda)$. These findings establish that $\partial\pi^k/\partial z_1^k > \partial\pi^k/\partial z_2^k$ for any $\lambda \in [1/2, 1/2 + \varepsilon]$. Therefore, the carrier can always raise its profit either by augmenting z_1^k or by reducing z_2^k. Consequently, the profit is maximized at (z_1^k, z_2^k) with $z_1^k \geq z_2^k$. By a similar argument, however, we can prove that the profit maximization prescribes $z_1^k \leq z_2^k$. Hence, $z_1^k = z_2^k$ at the equilibrium. QED

Derivation of (28).
Notice that totally differentiating (27) yields

$$k_m \mathrm{d}\bar{m}^S + k_c \mathrm{d}c + k_F \mathrm{d}F + k_\sigma \mathrm{d}\sigma = 0,$$

where

$$\begin{aligned}
k_m &\equiv -F - 2\zeta_1\sigma(\bar{m}^S)^{-1}(\sigma - 1)\left[2\sigma + c(\sigma - 1)\right]\Big[2\alpha(\bar{m}^S - 1) + \bar{m}^S(2 + c)\Big] < 0, \\
k_c &\equiv -4\zeta_1(\sigma - 1)(\sigma\bar{m}^S - 1)\left[1 + \alpha(\sigma - 1)^2\right] < 0, \\
k_F &\equiv -\bar{m}^S < 0, \quad \text{and} \\
k_\sigma &\equiv -\zeta_1(\zeta_2 + \zeta_3 + \zeta_4) < 0,
\end{aligned}$$

with

$$\begin{aligned}
\zeta_1 &\equiv \Big[2 - 2\alpha(\sigma - 1) + \sigma\bar{m}^S(\sigma - 1)(2 + c + 2\alpha)\Big]^{-2} > 0 \\
\zeta_2 &\equiv 2(2 + c)\left[2(\bar{m}^S\sigma^2 - 1) + c\bar{m}^S(\sigma - 1)^2\right] > 0, \\
\zeta_3 &\equiv 4\alpha\Big[2(\sigma - 1) + 2\sigma^2(\bar{m}^S - 1) + c(\sigma - 1)\left\{\bar{m}^S(\sigma - 1) + \sigma(\bar{m}^S - 1)\right\}\Big] > 0, \quad \text{and} \\
\zeta_4 &\equiv 4\alpha(\sigma - 1)(\sigma\bar{m}^S - 1)\left[2\sigma + c(\sigma - 1)\right] \ln\frac{\bar{m}^S\left[2\sigma + c(\sigma - 1)\right]}{2(\sigma\bar{m}^S - 1)} > 0.
\end{aligned}$$

Here, the inequalities hold because $\sigma > 1$ and $\bar{m}^S \geq 1$.

Proof of Lemma 2.
What we have to show is that $\mathrm{d}v_1/\mathrm{d}\lambda \leq 0$ and $\mathrm{d}v_2/\mathrm{d}\lambda \geq 0$ if and only if $\mathrm{d}\hat{v}/\mathrm{d}\lambda \leq 0$. The 'only if' part is self-evident. To prove the 'if' part, I show that $\mathrm{d}v_1/\mathrm{d}\lambda = -\mathrm{d}v_2/\mathrm{d}\lambda$. For that purpose, it is convenient to treat $v_i(\lambda)$ represented by (11) as functions of t_1, t_2 and λ. First, the short-run equilibrium condition (23) implies that $\mathrm{d}t_1^S/\mathrm{d}\lambda = -\mathrm{d}t_2^S/\mathrm{d}\lambda$. Second, v_1 and v_2 are symmetric with respect to t_1 and t_2 in the sense that $\partial v_1/\partial t_1 = \partial v_2/\partial t_2$ and

$\partial v_1/\partial t_2 = \partial v_2/\partial t_1$. Third and finally, v_1 and v_2 are also symmetric with respect to λ_1 and λ_2, that is, $\partial v_1/\partial\lambda = \partial v_2/\partial(1-\lambda) = -\partial v_2/\partial\lambda$. Consequently, we have

$$\frac{\mathrm{d}v_1}{\mathrm{d}\lambda} = \frac{\mathrm{d}t_1}{\mathrm{d}\lambda}\frac{\partial v_1}{\partial t_1} + \frac{\mathrm{d}t_2}{\mathrm{d}\lambda}\frac{\partial v_1}{\partial t_2} + \frac{\partial v_1}{\partial\lambda} = -\left(\frac{\mathrm{d}t_2}{\mathrm{d}\lambda}\frac{\partial v_2}{\partial t_2} + \frac{\mathrm{d}t_1}{\mathrm{d}\lambda}\frac{\partial v_2}{\partial t_1} + \frac{\partial v_2}{\partial\lambda}\right) = -\frac{\mathrm{d}v_2}{\mathrm{d}\lambda}.$$

Then, it is straightforward to see that $\mathrm{d}v_1/\mathrm{d}\lambda$ has the same sign as $\mathrm{d}\hat{v}/\mathrm{d}\lambda$. QED

Proof of Lemma 3.
Suppose that $z_1^k \le z_2^k$. The equations (21) imply that

$$\frac{\partial\pi^k}{\partial z_1^k} - \frac{\partial\pi^k}{\partial z_2^k} = p\left[t_1'\left\{1 - \frac{z_1^k}{\sigma Z_1(t_1', t_2' : \lambda)}\right\} - t_2'\left\{1 - \frac{z_2^k}{\sigma Z_2(t_1', t_2' : \lambda)}\right\}\right] + c.$$

On the one hand, the term in the first braces is greater than that in the second. This is because the two inequalities $z_1^0 > z_2^0$ and $z_1^k \le z_2^k$ imply that

$$\frac{z_1^k}{z_2^k} < \frac{z_1^0}{z_2^0}, \tag{41}$$

which is a necessary and sufficient condition for $z_1^k/\sigma Z_1(t_1', t_2' : \lambda) < z_2^k/\sigma Z_2(t_1', t_2' : \lambda)$. To prove that $t_1' > t_2'$, on the other hand, notice that (41) implies that

$$\frac{Z_1(t_1', t_2' : \lambda)}{Z_2(t_1', t_2' : \lambda)} = \frac{(m-1)z_1^0 + z_1^k}{(m-1)z_2^0 + z_2^k} < \frac{mz_1^0}{mz_2^0} = \frac{Z_1(t_1^0, t_2^0 : \lambda)}{Z_2(t_1^0, t_2^0 : \lambda)}. \tag{42}$$

Now, suppose that $t_1' \le t_2'$. If $t_1^0 > t_2^0$, it turns out that

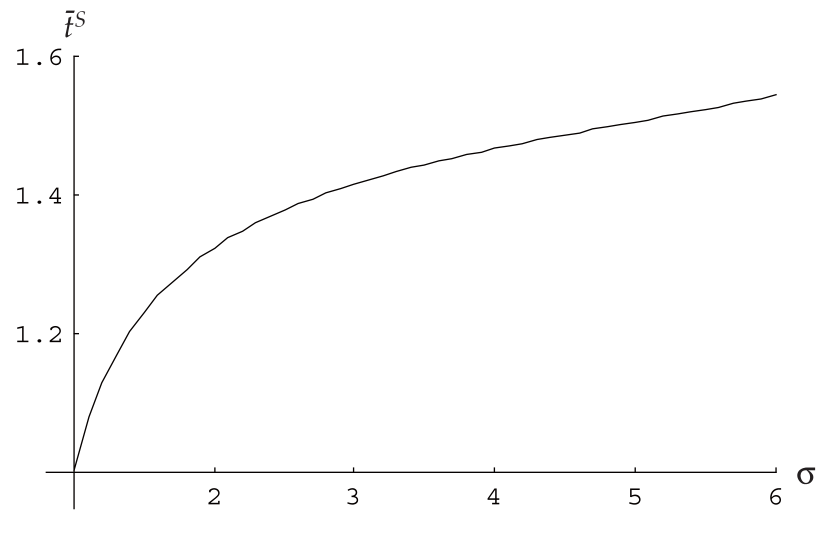
$$\begin{aligned}\frac{Z_1(t_1', t_2' : \lambda)}{Z_2(t_1', t_2' : \lambda)} &= \frac{\lambda\left[(\sigma-1)\lambda(t_2')^\sigma + (1-\lambda)(1+\sigma t_2')\right]}{(1-\lambda)\left[(\sigma-1)(1-\lambda)(t_1')^\sigma + \lambda(1+\sigma t_1')\right]} \\ &\ge \frac{\lambda\left[(\sigma-1)\lambda(t_2^0)^\sigma + (1-\lambda)(1+\sigma t_2^0)\right]}{(1-\lambda)\left[(\sigma-1)(1-\lambda)(t_1^0)^\sigma + \lambda(1+\sigma t_1^0)\right]} = \frac{Z_1(t_1^0, t_2^0 : \lambda)}{Z_2(t_1^0, t_2^0 : \lambda)},\end{aligned}$$

which contradicts (42). Hence, $t_1' > t_2'$. QED

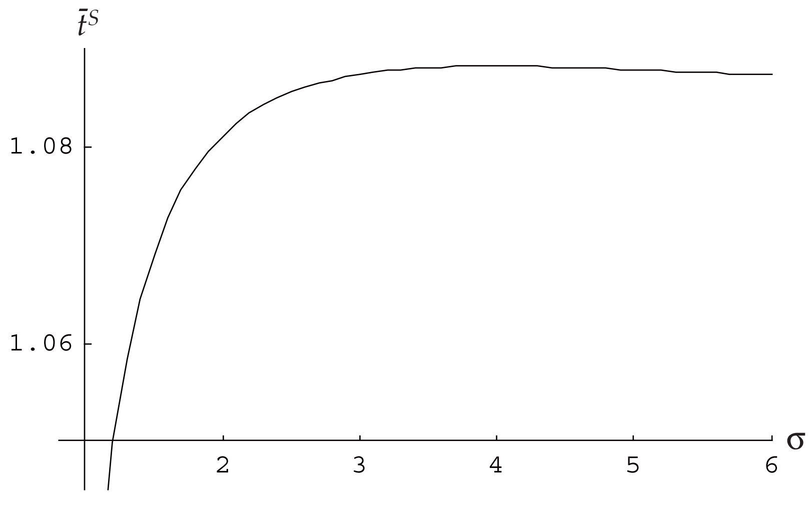
Proof of Proposition 4.
i) If $\sigma \le 2$, we have

$$\sigma\bar{t}_1^A\left[\left(\bar{t}_2^A\right)^\sigma - 1\right] + \sigma\left(\bar{t}_2^A\right)^\sigma\left[\left(\bar{t}_1^A\right)^2 - \left(\bar{t}_1^A\right)^\sigma\right] > 0 > -\left[\left(\bar{t}_1^A\right)^\sigma\left(\bar{t}_2^A\right)^\sigma + 1\right],$$

because $\bar{t}_2^A > 1$ and $\left(\bar{t}_1^A\right)^2 - \left(\bar{t}_1^A\right)^\sigma \ge 0$. The inequality implies that $\hat{v}(1) > 1$.
ii) As c expands unboundedly with the other parameters being kept constant, $\bar{t}_1^A$ approaches positive infinity whereas $\bar{t}_2^A$ remains the same. Then, $\hat{v}(1)$ approaches $\sigma/\left[(\sigma-1)\left(\bar{t}_1^A\right)^{\sigma-2}\right]$, which goes to 0 if $\sigma > 2$. By continuity, therefore, $\hat{v}(1) < 1$ for sufficiently large c. QED

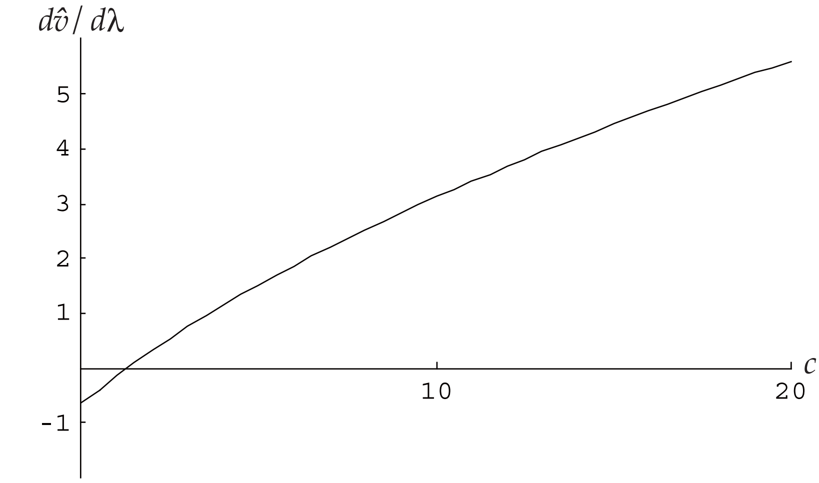


(a) The Case with $c = 1$ and $F = 0.01$

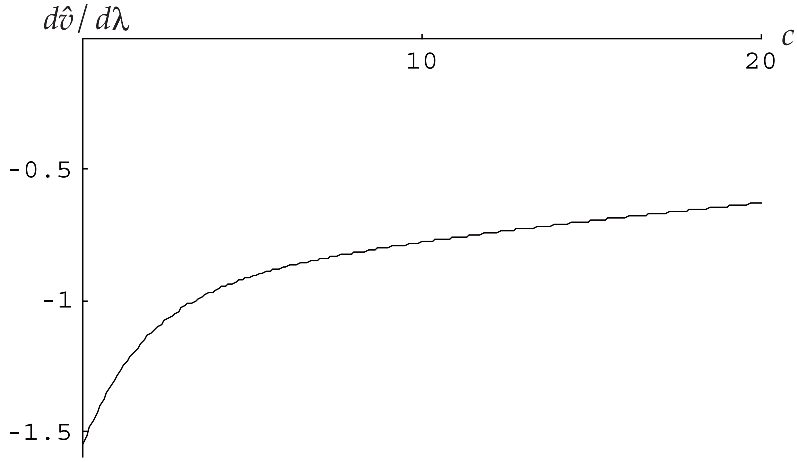


(b) The Case with $c = 0.1$ and $F = 0.01$

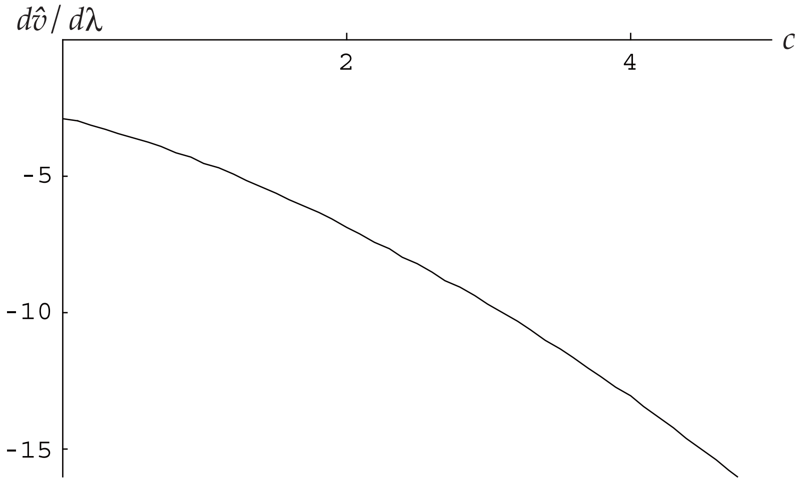
Fig. 1. Effect of σ on $\bar{t}^S$



(a) The Case with $\sigma = 1.5$

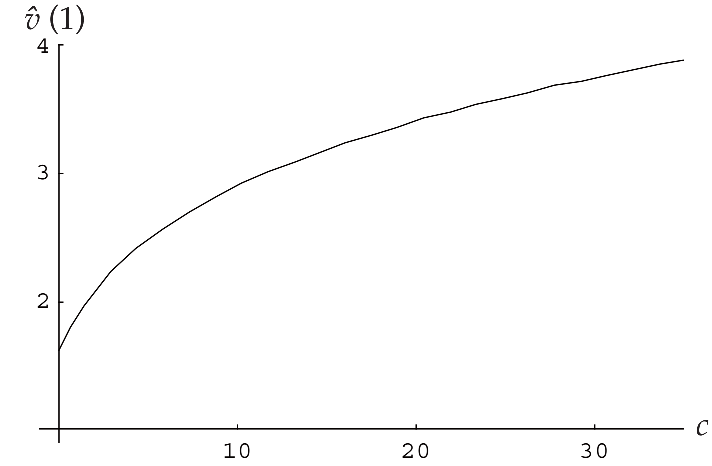


(b) The Case with $\sigma = 2$

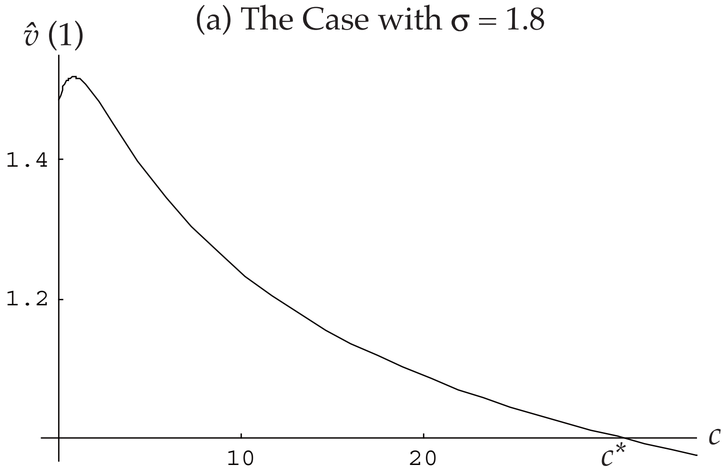


(c) The Case with $\sigma = 3$

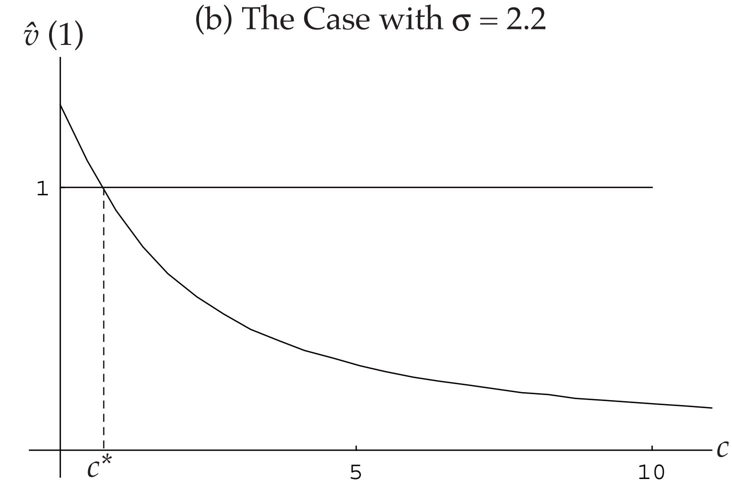
Fig. 2. Effect of c on $d\hat{v}/d\lambda$ when $F = 0.01$



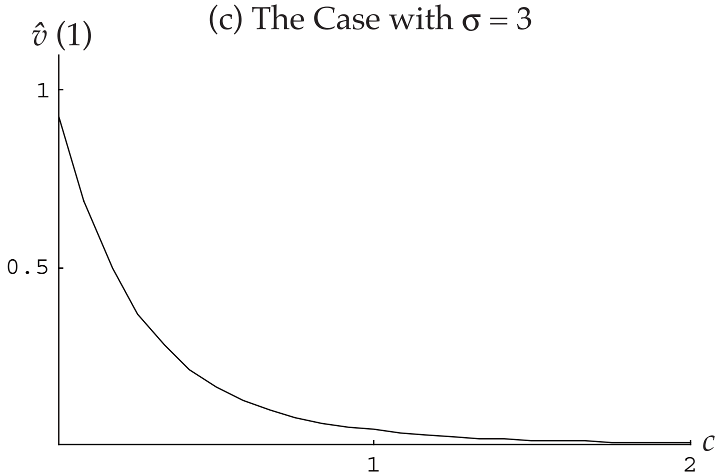
(a) The Case with $\sigma = 1.8$



(b) The Case with $\sigma = 2.2$

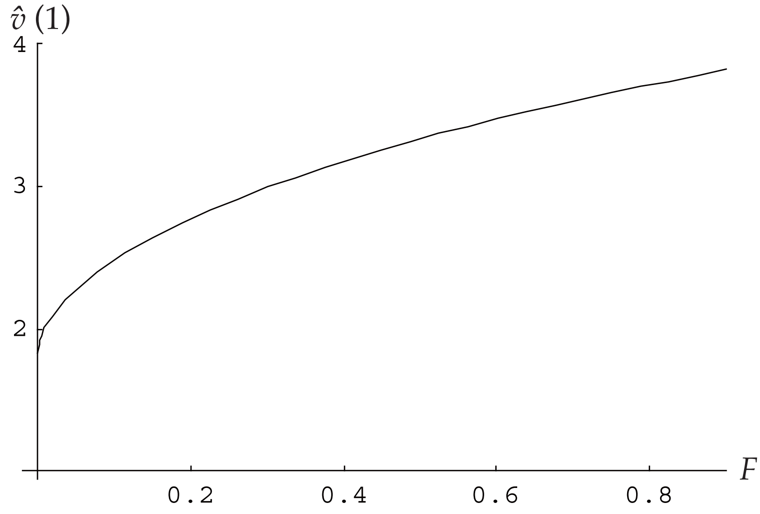


(c) The Case with $\sigma = 3$

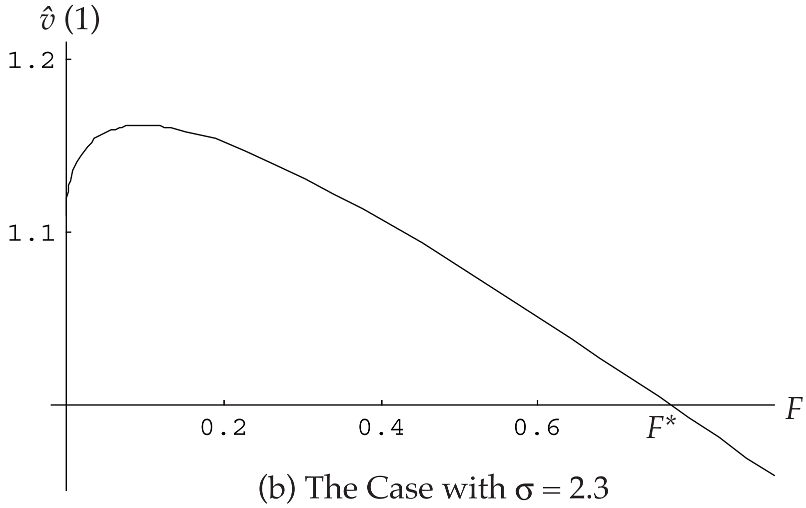


(d) The Case with $\sigma = 7$

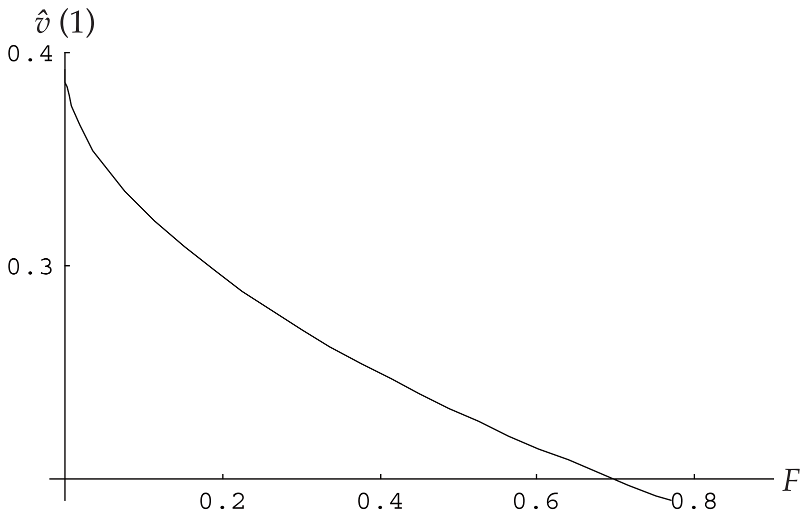
Fig. 3. Effect of c on $\hat{v}$ (1) when $F = 0.1$



(a) The Case with σ = 1.8



(b) The Case with σ = 2.3



(c) The Case with σ = 3

Fig. 4. Effect of F on $\hat{v}$ (1) when $c = 5$

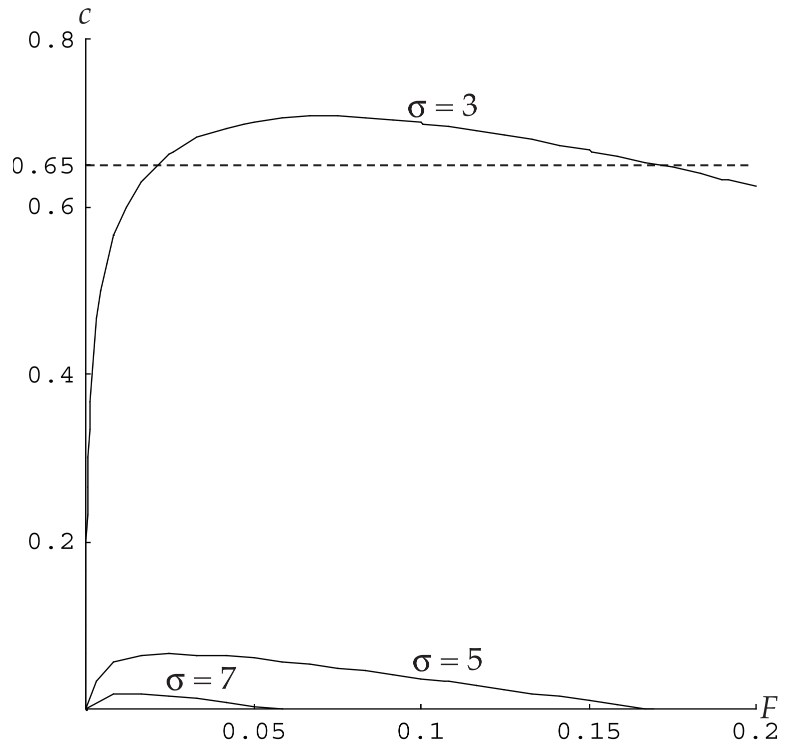


Fig. 5. The Loci of $\hat{v}\,(1) = 1$