

Cost-Benefit Analysis for Transport Networks
-Theory and Application-

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Abstract: In this paper, practical methods for estimating benefits corresponding to the second-best situation are derived by modeling a congestion-prone transport network explicitly. In the second-best situation, a change in total benefit of an investment in transport infrastructure can be calculated in three ways: (a) the sum of the changes in consumers' and producers' surpluses in all routes; (b) the sum of the changes in consumers' and producers' surpluses in the invested routes and the change in the deadweight loss in all other routes; and (c) the sum of a change in the total benefits in the first-best case and a change in the deadweight loss in all routes. Applying method (c), we demonstrate that the final benefits of distortion-relieving policies, such as introduction of congestion tolls on the congested routes of a network, are the sum of a change in the deadweight loss in all routes. Theoretical results are derived in practically useful forms, and then illustrated with examples that reveal typical errors in benefit estimation for transport projects.

Keywords: cost-benefit analysis, investment in transport infrastructure, network, externality

* I would like to thank Richard Arnott, Ryuji Fukushima, Tatsuo Hatta, Yoshitsugu Kanemoto, Se-il Mun, Kenneth A. Small, and Takayuki Ueda for helpful suggestions and discussions. I also thank two anonymous referees and editors for encouraging suggestions and comments. The final version of this paper was written while I was visiting the Department of Economics, at the University of California, Irvine. I thank the faculty members and staff there for their hospitality. Of course, I am solely responsible for any remaining errors and omissions.

1 Introduction

Transport projects often require huge public funds. Thus, a cost-benefit analysis is very important to judge if the project in question is worthy of implementing. However, it is rather difficult to carry out cost-benefit analysis for transport projects accurately, because benefit-estimation for transport projects has the following features, which are often less organized in the standard textbooks¹;

Transport Networks: Many transportation methods actually co-exist, and they form complex transport networks. How do we consider transport networks in benefit-estimation for transport projects?

Congestion: Congestion often arises in some parts of transport networks. Investments in some parts of transport networks to relieve the congestion change the degree of congestion in all parts of transport networks. How do we address such a congestion problem in transport networks?

Divergence between Private User Price and Social Marginal Cost: Due to political and technical difficulties, the congestion tax is rarely adopted, and consequently, the private user price of a trip differs from its social marginal cost in transport networks with congestion. In such a second-best situation, how do we accurately estimate the final benefit of transport projects?

The purpose of this paper is to propose a consistent benefit-estimation method for transport projects in a practically useful manner, including all the points stated above. Firstly, transport networks are modeled as an economy with multiple goods, applying a general equilibrium approach, shown in Boadway and Bruce (1984), for instance. This approach enables us to represent any kind of transport networks, because the relationship between routes, which may be substitutes or

¹ See, for example, Lesourne (1975), Sassone and Schaffer (1978), Pearce and Nash (1981), Sugden and Williams (1978), Gramlich (1990), Layard and Glaister (1994), Brent (1996), Nas (1996), and Boardman et al. (2000).

complements, need not to be specified. Secondly, the degree of congestion is explicitly taken into account in all the parts of transport networks. Thus, we can focus on the benefit-estimation in the case where a transport project not only changes the degree of congestion in the invested area but also changes it subsequently in all the other areas of a transport network. Thirdly, our analysis is presented within a consistent framework from the first-best case, in which the private user prices of trips equal their social marginal costs in all the parts of a transport network, to the second-best case, in which the private user prices of trips differ from their social marginal costs at least in a part of a transport network. The approach in this paper highlights how to calculate the total benefits in the second-best case, which is less focused but practically more important than in the first-best case.

The main results of the paper are as follows. In the first-best case, the total benefits from an investment in transport infrastructures in a route result in the sum of changes in the consumers' and producers' surpluses in the invested route. In the first-best case, we do not have to consider the accompanying changes in the other routes that have not received investment.

In the second best case, we need more elaborate calculations. The total benefits in the second-best case can be written in three forms. The first form shows that the total benefits equal the sum of changes in the consumers' and producers' surpluses in all routes. This method is a natural extension of the benefit estimation method in the first-best case; that is, the total benefits in the first-best case is indeed the sum of changes in the consumers' and producers' surpluses in all routes. However, in the first-best case, a change in consumers' surpluses and a change in producers' surpluses cancel each other out in the non-invested routes, and consequently, the total benefits to be calculated reduces to the sum of changes in the consumers' and producers' surplus in the invested route only. In the second-best case, this cancellation does not apply, and we have to calculate changes in the consumers' and producers' surpluses in all routes.

The second form shows that the total benefits consist of changes in the consumers' and

producers' surpluses in the invested route and a change in the dead weight loss in all the other routes. This second form highlights that total benefits to be calculated in non-invested routes are a change in the dead weight loss. The second form is also related to the benefit estimation in the first-best case. In the first-best case, no dead weight loss exists. Thus, a change in the dead weight loss are zero in all routes, which again shows that final benefits in the first best case are the sum of changes in the consumers' and producers' surplus in the invested route only.

The third form shows that the total benefits in the second-best case equal the sum of total benefits in the first-best case and a change in the dead weight loss in all routes. The third form highlights the relationship between the total benefits in the first-best case and those in the second-best case, and demonstrates that their difference stems from a change in the dead weight loss in all routes.

We actually live in a second-best world, and consequently, it is often difficult to know the total benefits under the hypothetical first-best situation. Thus, practical benefit estimation would rely on the first and second methods. However, this does not mean that the third method is not practically useful. The third method is not only intuitively easy to understand, but also important in conducting benefit-estimation in the case where congestion tolls are introduced. Applying the third form in the second-best analysis, we can easily show that the total benefits from an introduction of congestion tolls are the sum of a change in the dead weight loss in all routes.

Before proceeding, we will briefly relate our study to the existing literature. Firstly, Kanemoto and Mera (1985) and Jara-Diaz (1986) show that under the first-best situation with no price distortion, a change in the benefits in transportation sectors includes all the economic benefits from a transport project. Since we ignore the price distortion in non-transport sectors, we can also obtain final benefits from a transport investment, by calculating changes in the benefits in the transport sector only. However, their analysis neglects the inner structure of the transport sectors, and does not focus on the important issues to be addressed regarding transport sectors, e.g., transport networks and congestion.

In this paper, we formally model transport networks with congestion and analyse the benefit-estimation method for transport projects, explicitly considering the second-best nature in the real world.

Secondly, benefit-estimation for actual transport networks with congestion, which are neglected in Kanemoto and Mera (1985) and Jara-Diaz (1986), are dealt with in older literature (e.g., Harrison (1974), Williams (1976), Jones (1977), and Jara-Diaz and Friesz (1982)). However, these analyses are less organized or incomplete, as they do not illustrate the benefit-estimation method under the second-best situation in detail or the relationship between the benefit-estimation in the first-best and that in the second-best. Such deficiencies also apply to more recent literature that includes analyses of the benefit-estimation (e.g., Button (1993), Oppenheim (1995), and Williams et al. (1991, 2001)). Our analysis offers a practical benefit-estimation method both in the first-best case and in the second-best case and clarifies the link between them in a consistent framework.

Thirdly, the approach in this paper is most related to Harberger (1972), Mohring (1976), and Kanemoto (1996) in that benefit-estimation in the first-best case is explicitly distinguished from that in the second-best case. These studies deal with the benefit estimation both in the first-best case and in the second best case, focusing on two-parallel congestion-prone roads. However, they do not model general transport networks and, consequently, do not fully investigate how to estimate the total benefits from transport investments in the case of actual complex transport networks. They also do not show that the total benefits in the second-best case can be expressed in the three forms, each of which has practical implication. Our analysis can also be considered as a practical extension of these studies, explicitly taking into account congestion-prone transport networks, which often have very complex structures, especially in urban areas of large cities.

The structure of this paper is as follows. In Section 2, we set up the model. In Section 3, we derive a benefit-estimation method for transport networks in the first-best case, in which private

prices of trips equal their social marginal costs in all routes. In Section 4, we focus on the second-best case, in which private prices of trips differ from their social marginal costs, at least with respect to one route. In Section 5, by applying our results to actual benefit estimation, we use examples to illustrate typical errors in benefit-estimation methods for transport networks. Section 6 concludes our analysis.

2 The Model

Consider a transport network in which N nodes are linked to one another. Between node i ($1 \leq i \leq N$) and node j ($1 \leq j \leq N$), M means of transportation, which we call “routes” hereafter, are available. One may think that M routes correspond to M kinds of transport modes or M links in a single transport mode. The k th ($1 \leq k \leq M$) route from node i to node j is called route ijk . The number of trips in the route ijk is denoted by x^{ijk} . We assume that $x^{ijk} > 0$ always holds. We also assume that the number of trips from node i to node i is zero; i.e., $x^{iik} = 0$. Figure 1 illustrates the example of a network in which $N = 2$ and $M = 2$.

The representative consumer at node i demands the composite consumer good, z , the price of which is normalized at unity, and his or her trips from node i . Our analysis applies even if the consumer at node i demands the trips $i \rightarrow j \rightarrow i$ or $i \rightarrow j \rightarrow j'$; i.e., if the consumer at node i demands “round” trips or demands trips via another node. We assume that the utility function of the representative consumer at node i has the following quasi-linear form:

$$U^i = z + u^i(\dots, x^{ijk}, \dots), \quad (1)$$

where u^i depends on all directions of the trips from node i . The above quasi-linear utility function implies that we neglect the income effects, and consequently, the consumers’ surplus equals the

equivalent variation and compensating variation.² As Willig (1976) demonstrates, the difference between the consumers' surplus, equivalent variation, and compensating variation is rather small, and hence disregarding the difference is justified in benefit estimation in practice. The quasi-linear utility function also yields the Gorman-type indirect utility function³, which justifies the summation of individual utilities to obtain the total consumers' surplus. In this paper, the utility function is assumed to be concave. Note that we do not make further assumptions about the utility function. Thus, the utility function (1) specifies no particular relationship between routes; for example, one could think that route $ij1$ may be a substitute or complement for route $ij2$. When M routes from node i to node j are perfect substitutes, the utility function (1) becomes

$$U^i = z + u^i(\dots; x^{ij1} + \dots + x^{ijk} + \dots + x^{ijM}; \dots), \quad (2)$$

which is the utility function that is consistent with Wardrop's (1952) analysis.

The generalized price of a trip in route ijk , which includes time costs, is denoted by p^{ijk} . The representative consumer at node i maximizes his or her utility, (1), subject to the budget constraint:

$$z + \sum_j \sum_k p^{ijk} x^{ijk} = y^i, \quad (3)$$

where y^i is the income of the consumer at node i . We assume that the consumer does not take into account the effect of his or her own behaviour on congestion; i.e., we assume that the consumer is a price-taker.

The maximization problem of the representative consumer at node i is then formulated as:

$$\max_{\{z, \dots, x^{ijk}, \dots\}} \left\{ U^i = z + u^i(\dots, x^{ijk}, \dots) : z + \sum_j \sum_k p^{ijk} x^{ijk} = y^i \right\}. \quad (4)$$

² See, for example, Varian (1992).

³ See, for example, Mas-Colell et al. (1995) and Tsuneki (2000).

Solving the maximization problem, (4), yields

$$p^{ijk} = u_{x^{ijk}}^i, \quad (5)$$

from which, we obtain the Marshallian demand function in route ijk :

$$x^{ijk} = x^{ijk}(\dots, p^{ijk}, \dots). \quad (6)$$

For the trips in route ijk , both investments in transport infrastructures, which are measured in monetary terms and denoted by I^{ijk} , and the suppliers of transport services are required. Examples of transport infrastructures are roads, railroad trucks, and airports, and examples of the suppliers of transport services are railway operators and airlines. In the case of road transport, one can think of the drivers themselves as the suppliers of transport services. For the sake of exposition, we assume that the government owns all the transport infrastructures and makes investments in them. This assumption is innocuous for our analysis, because we do not focus on the welfare distribution but only on the total benefits from an investment in transport infrastructures. The government imposes “taxes,” t^{ijk} , per trip in route ijk . This tax, t^{ijk} , can be interpreted in many ways. In the case of road transport, one could think of t^{ijk} as gasoline taxes. t^{ijk} needs not to be positive; for instance, in the case of public transport that is subsidized by the government, one could think of t^{ijk} as unit subsidy per ride, which can be considered as a negative tax.

Let us assume that all the suppliers of transport services in route ijk are homogenous, sufficiently competitive with constant returns to scale, and their unit supply cost is \bar{c}^{ijk} . Since the level of \bar{c}^{ijk} is irrelevant to our analysis, we can set it at zero. Other unit generalized costs of the trips in route ijk , which includes monetized time costs, is $c^{ijk}(x^{ijk}, I^{ijk})$, which is non-decreasing with x^{ijk} and decreasing with I^{ijk} ; i.e.,

$$\frac{\partial c^{ijk}}{\partial x^{ijk}} \equiv c_{x^{ijk}}^{ijk} \geq 0 \quad (7)$$

and

$$\frac{\partial c^{ijk}}{\partial I^{ijk}} \equiv c_{I^{ijk}}^{ijk} < 0. \quad (8)$$

The total cost function of the trips in route ijk , C^{ijk} , is

$$C^{ijk} = c^{ijk}(x^{ijk}, I^{ijk})x^{ijk}, \quad (9)$$

from which we can obtain their marginal cost, MC^{ijk} , as

$$MC^{ijk} = c^{ijk}(x^{ijk}, I^{ijk}) + c_{x^{ijk}}^{ijk}(x^{ijk}, I^{ijk})x^{ijk}, \quad (10)$$

where $c_{x^{ijk}}^{ijk}(x^{ijk}, I^{ijk})x^{ijk}$ shows the congestion externality that an additional trip gives to all the trips in route ijk .

The total profits of the suppliers of transport services in route ijk , π^{ijk} , can be written as

$$\pi^{ijk} = (p^{ijk} - t^{ijk})x^{ijk} - c^{ijk}(x^{ijk}, I^{ijk})x^{ijk}, \quad (11)$$

where $p^{ijk} - t^{ijk}$ is the net price (excluding taxes) that the suppliers receive from the consumer. As a result of competition, the profits of each supplier of transport services become zero, which implies that the total profits of the suppliers of transport services are also zero:

$$\pi^{ijk} = 0. \quad (12)$$

From (11) and (12), we know that the generalized price of a trip in route ijk satisfies:

$$p^{ijk} = c^{ijk}(x^{ijk}, I^{ijk}) + t^{ijk}. \quad (13)$$

Solving (6) and (13), we have the general equilibrium demand curve in route ijk , whose arguments are the investments in transport infrastructure and the taxes, that is,

$$x^{ijk} = x^{ijk}(\dots, I^{ijk}, \dots, t^{ijk}, \dots). \quad (14)$$

Finally, since the profits of the suppliers of transport services are zero, the total social welfare, SW , can be defined as the sum of the consumers' utilities and the tax revenue of the government,

exclusive of investment costs, in all routes. From (9) and (13), we know that the tax revenue of the government in route ijk can be considered as the producers' surplus in route ijk :

$$t^{ijk} x^{ijk} = p^{ijk} x^{ijk} - C^{ijk} . \quad (15)$$

Thus, the total social welfare, SW , is

$$\begin{aligned} SW &= \sum_i U^i + \sum_i \sum_j \sum_k t^{ijk} x^{ijk} - \sum_i \sum_j \sum_k I^{ijk} \\ &= \sum_i U^i + \sum_i \sum_j \sum_k (p^{ijk} x^{ijk} - C^{ijk}) - \sum_i \sum_j \sum_k I^{ijk} . \end{aligned} \quad (16)$$

As is usual in the literature of cost-benefit analysis, we focus on the total benefits, exclusive of investment costs. The total benefits, TB , are defined as:

$$TB = \sum_i U^i + \sum_i \sum_j \sum_k t^{ijk} x^{ijk} = \sum_i U^i + \sum_i \sum_j \sum_k (p^{ijk} x^{ijk} - C^{ijk}) . \quad (17)$$

3 First-best Results

In this section, we obtain the results in the first-best case, in which the generalized prices of the trips, p^{ijk} , equal their social marginal costs, MC^{ijk} in all routes. From (10) and (13), we know that the first-best situation is attained when $t^{ijk} = c_{x^{ijk}}^{ijk}(x^{ijk}, I^{ijk})x^{ijk}$, i.e., the tax, t^{ijk} , is set equal to the congestion externality, $c_{x^{ijk}}^{ijk}(x^{ijk}, I^{ijk})x^{ijk}$ in all routes.

Suppose that an investment in transport infrastructures increases from I^{ijkWO} to I^{ijkW} in route ijk . The change in the total benefits, TB , in this case consist of:

i) a change in the consumers' surplus in route ijk :

$$\int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} \quad (18)$$

and;

ii) a change in the tax revenue in route ijk :

$$\int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} . \quad (19)$$

In sum, these two terms are reduced to:

$$\int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk} . \quad (20)$$

All mathematical developments for the analyses of Sections 3 and 4 are relegated to Appendix 1. For an intuitive understanding, we focus on the case of the simple two-node model in Figure 1. Consider an investment in transport infrastructures in route 121, I^{121} , increases from I^{121WO} to I^{121W} . We only need to analyse the consumer at node 1, who demands the trips in routes 121 and 122, because inbound trips and outbound trips are treated separately in our model, and an investment in transport infrastructures, I^{121} , will affect trips that originate at node 1 only. Even if we consider a setting in which an increase in investment in transport infrastructure, I^{121} is assumed to affect not only the trips in routes 121 and 122 but also the trips in routes 211 and 212, our argument remains unchanged. In that case, we also take into account the effects on the consumer at node 2.

Let us explain the results in detail using diagrams. Equation (18) shows a change in the consumers' surplus in the route where the investment in transport infrastructures increases, measured by the general equilibrium demand curve.⁴ In our simple example in Figure 1, (18) corresponds to a

change in the consumers' surplus in route 121 and is reduced to $\int_{I^{121W}}^{I^{121WO}} x^{121} \left(\frac{dp^{121}}{dI^{121}} \right) dI^{121}$, or

⁴ The general equilibrium demand curve is often referred to as Bailey's demand curve, because Bailey (1954) first formulated this notion. See also Boadway and Bruce (1984) for further explanation of the general equilibrium demand curve.

$\int_{p^{121W}}^{p^{121WO}} x^{121} dp^{121}$ where p^{121WO} and p^{121W} denotes the generalized prices in route 121 corresponding to

I^{121WO} and I^{121W} . Remember that the ordinary Marshallian demand curve is drawn on the basis that incomes and prices in the other markets are held constant. In our model in which the utility function is assumed to be quasi-linear, the Marshallian demand curve does not depend on income, but on prices in the other markets. The investment in transport infrastructure in route 121, not only affects the generalized price and demand of the trips in route 121, but also affects those of the trips in route 122, depending on the relationship between the trips in routes 121 and the trips in routes 122, which may be substitutes or complements. Taking into account these effects, the investment in route 121 does not decrease its generalized price along its Marshallian demand curve with the generalized price of the trips in route 122 fixed, but shifts the Marshallian demand curve. Look at Figure 2-1. With regard to the trips in route 121, an equilibrium without a project, point B, and an equilibrium with a project, point D, lie on different Marshallian demand curves. The general equilibrium demand curve for the trips in route 121 is a locus of attained equilibria, which passes through points B and D, and incorporates the accompanying change in the trips in route 122. In $x^{121} - p^{121}$ plane in Figure 2-1, the general equilibrium demand curve in route 121 is shown as $x^{121}(p^{121}, p^{122}(p^{121}, I^{122}, t^{122}))$, given I^{122} and t^{122} . A change in the consumers' surplus in route 121 needs to be measured along the general equilibrium demand curve and, consequently, is *Area ABDC*. When estimating transport demand in practice, it is often difficult to derive the Marshallian demand curve with incomes and the generalized prices in the other trips held constant, because a change in the price of a trip increases or decreases congestion in other trips, and subsequently, changes the generalized prices of other trips. In the actual forecast of transport demand, accompanying changes in other trips due to substitutability and complementarity relationships between trips are usually taken into account, at least implicitly. Thus,

the present estimation of transport demand can be regarded as a forecast of the general equilibrium demand curve, rather than of the Marshallian demand curve.

Equation (19) shows a change in the tax revenue in the route where the investments in transport infrastructures increase. In our example in Figure 1, (19) corresponds to $\int_{I^{121wo}}^{I^{121w}} t^{121} \left(\frac{dx^{121}}{dI^{121}} \right) dI^{121}$. Since

$$t^{ijk} x^{ijk} = p^{ijk} x^{ijk} - C^{ijk} \quad (21)$$

from (15), we can consider (19) as a change in the producers' surplus in the invested route. A change in the producers' surplus in route 121 can be expressed as *Area CDF* - *Area ABE* in Figure 2-2, which is rearranged as

$$\begin{aligned} & \text{Area } CDF - \text{Area } ABE \\ &= \text{Area } CDF + \text{Area } EBDC - (\text{Area } ABE + \text{Area } EBDC) \\ &= \text{Area } EBDF - \text{Area } ABDC. \end{aligned} \quad (22)$$

It should be remembered that a change in the consumers' surplus in route 121 is *Area ABDC*. By adding up the changes in the consumers' and producers' surpluses, the change in the total benefits

in route 121 is summed up as *Area EBDF*, which is expressed as $\int_{I^{121w}}^{I^{121wo}} c_{I^{121}}^{121} x^{121} dI^{121}$ in our example and corresponds to (20).

In the first-best case, it is unnecessary to consider the accompanying changes in all the other routes. To see why, let us derive the consumers' surplus and the tax revenue in route 122.

A change in the consumers' surplus in route 122 is derived from the understanding that the general equilibrium demand curve in route 122 is

$$p^{122} = MC^{122} = c^{122}(x^{122}, I^{122}) + c_{x^{122}}^{122}(x^{122}, I^{122})x^{122} \quad (23)$$

in Figure 2-3. The reason is that equilibria are always on the line for (23) in $x^{122} - p^{122}$ plane, although an increase in the investment in transport infrastructures in route 121 shifts the Marshallian demand curve for the trips in route 122. Thus, a change in the consumers' surplus relating to the trip

x^{122} is derived as an area along (23), i.e., *Area HJLK*, in Figure 2-3.

We move to a change in the tax revenue in route 122. Since we know that a change in the tax revenue equals a change in the producers' surplus in route 122, $p^{122}x^{122} - C^{122}$, from (15), it is expressed as the change in consumers' total payments, $p^{122}x^{122}$, less the change in the total cost, C^{122} . In Figure 2-3, the change in consumers' total payments, $p^{122}x^{122}$, is $-(Area HJLK + Area LJRQ)$, and the change in total cost, C^{122} , is the area below the marginal cost curve for the trips in route 122, $-Area LJRQ$. Accordingly, the change in the producers' surplus is

$$-(Area HJLK + Area LJRQ) - (-Area LJRQ) = -Area HJLK . \quad (24)$$

In total, since the change in the consumers' surplus is *Area HJLK* and the change in the producers' surplus is $-Area HJLK$, the change in the total benefits in route 122 is zero. In short, in the first-best case, in which the generalized prices of trips equal their marginal costs in all routes, we need only calculate changes in the consumers' and producers' surpluses in the invested route. This is because the sum of changes in the consumers' surplus and the tax revenue is zero in all the other routes. The above result demonstrates that the first-best results of Mohring (1976), Kanemoto and Mera (1985), Jara-Diaz (1986), and Kanemoto (1996) are applicable to any complex transport networks with congestion.

4 Second-best Results

Let us extend our analysis to incorporate the second-best case, in which the generalized prices of the trips, p^{ijk} , differ from their marginal costs, MC^{ijk} , at least in one route. In the second-best case, when the investment in transport infrastructures increases from I^{ijkWO} to I^{ijkW} in route ijk , a change in the total benefits, TB , can be expressed in three forms.

The first form of a change in the total benefits, TB , in the second-best case can be written as the

sum of:

i) a change in the consumers' surplus in all routes:

$$\sum_{i'} \sum_{j'} \sum_{k'} \int_{p^{i'j'k'w}}^{p^{i'j'k'wo}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} ; \quad (25)$$

and

ii) a change in the tax revenue in all routes:

$$\sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkwo}}^{I^{ijkw}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} , \quad (26)$$

where $1 \leq i' \leq N$, $1 \leq j' \leq N$, and $1 \leq k' \leq M$.

The second form highlights the distinction between changes in the invested route and changes in all the other routes, and is given by the sum of:

iii) a change in the consumers' surplus in route ijk :

$$\int_{I^{ijkw}}^{I^{ijkwo}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} ; \quad (27)$$

iv) a change in the tax revenue in route ijk :

$$\int_{I^{ijkwo}}^{I^{ijkw}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} ; \quad (28)$$

and

v) a change in the dead weight loss in all the other routes:

$$\sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkw}}^{I^{ijkwo}} (MC^{i''j''k''} - p^{i''j''k''}) \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk} , \quad (29)$$

where $1 \leq i'' \leq N$, $1 \leq j'' \leq N$, and $1 \leq k'' \leq M$, but (i'', j'', k'') does not contain (i, j, k) .

The third form highlights the distinction between the total benefits in the first-best case and the distortions in the second-best case, and can be rearranged as the sum of:

vi) a change in the total benefits in the first-best case:

$$\int_{I^{ijkW}}^{I^{ijkWO}} c_{ijk}^{ijk} x^{ijk} dI^{ijk} ; \quad (30)$$

and

vii) a change in the dead weight loss in all routes:

$$\sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} (MC^{i'j'k'} - p^{i'j'k'}) \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} . \quad (31)$$

In the same way as in the first-best case in Figure 1, we explain a simple case, in which the investment in transport infrastructure, I^{121} , increases from I^{121WO} to I^{121W} and affects only the trips in routes 121 and 122.

In the first-best case, as we show in Section 3, a change in the total benefits is the sum of changes in the consumers' surplus and the tax revenue in route 121 and the corresponding terms in route 122. However, with regard to the trips in route 122, the sum of changes in the consumers' surplus and the tax revenues is zero; consequently, the final benefits are the sum of changes in the consumers' surplus and the tax revenue in route 121. Even in the second-best case, the above calculation method applies without modification; from the first form, shown in (25) and (26), we know that a change in the total benefits is the sum of changes in the consumers' surplus and the tax revenue in all routes.

Two different forms of a change in the total benefits in the second-best case are derived. The second form, shown in (27)-(29), focuses on whether surpluses are generated in primary or secondary markets.⁵ Firstly, we look at changes in the primary market, which corresponds to changes in the invested route 121 in our example. A change in the consumers' surplus is *Area ABDC* in Figure 3-1, which is the same in the first-best case and corresponds to (27). A change in the tax revenue equals a

⁵ This dichotomy is based on Boardman et al. (2000).

change in producers' surplus from (15). Due to the discrepancy between the generalized price of a trip and its social marginal cost, the expression of a change in producers' surplus becomes complicated compared to the first-best case, and is $(Area\ CDG'O - Area\ F'D''G'O) - (Area\ ABG'O - Area\ E'B''G'O)$ in Figure 3-2, which corresponds to (28). (Hereafter, in Figures 3-1, 3-2, 3-3 and 4, we omit to draw the Marshallian demand curves to avoid unnecessary complications.)

Secondly, in the secondary market, which corresponds to the trips in route 122 in our example,

$$p^{122} = c^{122}(x^{122}, I^{122}) + t^{122} \quad (32)$$

always holds, regardless of shifts of the Marshallian demand curve for the trips in route 122. This means that (32) is the general equilibrium demand curve for the trips in route 122. Accordingly, a change in the consumers' surplus in route 122 is $Area\ HJLK$ in Figure 3-3, which is the same as in the first-best case. Applying (15), a change in the tax revenues in route 122 equals a change in the producers' surplus, $p^{122}x^{122} - C^{122}$. A change in consumers' total payments, $p^{122}x^{122}$, is $-(Area\ HJLK + Area\ LJRQ)$ in Figure 3-3, which is also the same result as in the first-best case. It is a change in the total cost, C^{122} , in the second-best case that differs from the first-best case, reflecting the gap between the generalized price of a trip and its social marginal cost. A change in the total cost, C^{122} , in the second-best case becomes $-Area\ L'J'RQ$ in Figure 3-3. As a result, a change in the producers' surplus in route 122 is

$$-(Area\ HJLK + Area\ LJRQ) - (-Area\ L'J'RQ) = -Area\ HJLK + Area\ L'J'JL. \quad (33)$$

In sum, a change in the total benefits to be calculated in the secondary market is $Area\ L'J'JL$ in Figure 3-3 because changes in the consumers' surplus, $Area\ HJLK$, and $-Area\ HJLK$ in (33) cancel each other out. $Area\ L'J'JL$ corresponds to (29) and shows a change in the dead weight loss in the secondary market, which stems from a gap between the generalized price of a trip and its social

marginal cost.

The third form of a change in the total benefits focuses on its relationship to the first-best result. In route 121, a change in the consumers' surplus is *Area ABDC* in Figure 3-1 and a change in the producers' surplus is $(Area\ CDG''O - Area\ F'D''G''O) - (Area\ ABG'O - Area\ E'B''G'O)$ in Figure 3-2. Superimposing Figure 3-1 on Figure 3-2 and adding up these terms, we obtain

$$\begin{aligned}
& Area\ ABDC + (Area\ CDG''O - Area\ F'D''G''O) - (Area\ ABG'O - Area\ E'B''G'O) \\
&= (Area\ ABDC + Area\ CDG''O - Area\ ABG'O) + Area\ E'B''G'O - Area\ F'D''G''O \\
&= Area\ BDG''G' + Area\ E'B''G'O - Area\ F'D''G''O \\
&= Area\ E'B''BDG''O - Area\ F'D''G''O \\
&= Area\ E'B'D'F' + Area\ B'B''B - Area\ D'D''D,
\end{aligned} \tag{34}$$

where *Area E'B'D'F'* is a change in the total benefits in the first-best case, which corresponds to (20), and *Area B'B''B - Area D'D''D* is a change in the dead weight loss in route 121. We know that a change in the benefits to be calculated in route 122 is a change in the dead weight loss in route 122, which is *Area L'J'JL* in Figure 3-3, from the second form that focuses on the difference between the primary market and the secondary market. In the end, we can rearrange a change in the total benefits in the second-best case as the sum of a change in the total benefits in the first-best case, (30), and a change in the dead weight loss in all routes, (31). The third form is an extension of Mohring's (1976) argument on the total benefit in the second-best case.

We have focused so far on an increase in an investment in transport infrastructures in existing routes. However, our analysis also applies even when a new route is built, if we consider that the investment in a new route lowers the generalized price of the trips and generates their demand. For example, consider the building of a bridge over a river between nodes 1 and 2. Without an investment, no bridge exists and, thus, residents cannot travel between nodes 1 and 2. With an investment, residents can travel between nodes 1 and 2 by using the bridge. This situation can be expressed in the following way. Without an investment, the generalized price of the trips between nodes 1 and 2 is

prohibitively high and, consequently, the demand for the trips between nodes 1 and 2 is zero. With an investment, however, the generalized price of the trips between nodes 1 and 2 declines, and the decrease in the generalized price generates the demand for the trips between nodes 1 and 2. This interpretation implies that our analysis applies directly to newly introduced routes.

As we will show in Section 5, the first and the second forms are useful in practical benefit-estimation. In conducting benefit estimation in practice, almost all the cases are the second-best. Thus, it is difficult to estimate a hypothetical first-best change in the total benefits. However, the third form, which highlights a first-best change in the total benefits and a change in the dead weight loss in all routes, does have practical validity and is worthy to be remembered. Applying the third form, we can easily derive a change in the total benefits when the tax increases from t^{ijkWO} to t^{ijkW} in route ijk , for example, by introducing the congestion toll. A change in the total benefits, TB , in this case becomes

$$\sum_{i'} \sum_{j'} \sum_{k'} \int_{t^{ijkW}}^{t^{ijkWO}} (MC^{i'j'k'} - p^{i'j'k'}) \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) dt^{ijk}, \quad (35)$$

which is the same form as (31) and shows a change in the dead weight loss in all routes.

Again, we explain this on the basis of a simple example in Figure 1 and suppose that the tax in route 121 increases. An increase in the tax in route 121 does not shift the social marginal cost curve for the trips in route 121, unlike an increase in the investment in transport infrastructures. This implies that *Area E'B'D'F'* in Figure 3-2, which corresponds to a change in the total benefits in the first-best case and is expressed in (30), does not exist. In the end, the final benefits are the sum of a change in the dead weight loss in routes 121 and 122, which corresponds to (35).

The same result can be derived if we calculate a change in the total benefits as the sum of changes in the consumers' surplus and the tax revenue. In Figure 4 of route121, a change in the consumers' surplus is $-Area SVYT$, and a change in the tax revenues, which equals the producers'

surplus from (15), is $(Area\ SVV''O - Area\ WV'V''O) - (Area\ TYY''O - Area\ WY'Y''O)$. Adding up these terms, we obtain a change in the total benefits in route 121 of

$$\begin{aligned}
& -Area\ SVYT + (Area\ SVV''O - Area\ WV'V''O) - (Area\ TYY''O - Area\ WY'Y''O) \\
& = -Area\ SVYT + (Area\ SVV''O - Area\ TYY''O) + (Area\ WY'Y''O - Area\ WV'V''O) \\
& = -Area\ SVYT + (Area\ SVYT - Area\ VYY''V'') + Area\ V'Y'Y''V'' \\
& = Area\ V'Y'Y''V'',
\end{aligned} \tag{36}$$

which is a change in the dead weight loss in route 121. In route 122, by the same argument as an increase in an investment in transport infrastructures in the second-best case, a change in the benefits to be calculated is a change in the dead weight loss in route 122, which is the same as $Area\ L'J'JL$ in Figure 3-3. Thus, we can confirm that the final benefits are summed up as a change in the dead weight loss in all routes.

5 Examples

Based on the analyses in Sections 3 and 4, let us check the present benefit-estimation methods, which claim to take into account the “network characteristic” of transport. We focus on two typical inadequacies: Examples 1 and 2 are applications of benefit estimation in the first-best and second-best cases respectively.

Example 1: The Weighted-Average Method in the Institute for Transport Policy Studies (ITPS) (1999)

ITPS (1999) is a manual for cost-benefit analyses of railway projects in Japan. Although ITPS (1999) claims to take into account the effects for railways and the effects for other transport modes, the way it estimates benefits is inconsistent with the theory developed here. We illustrate this by using a numerical example from pp. 104–124 of ITPS (1999). The data used to calculate benefits are given in Table 1, from which we know that this railway project shortens rail travel time and increases

rail demand.

In this example, it is assumed that the time cost does not depend on the volume of demand (i.e., no congestion exists) and no tax exists. Since the tax and the congestion externality equal zero, we can apply the benefit-estimation method in the first-best case. Thus, a change in the total benefits is derived as the sum of a change in the consumers' surplus and a change in the tax revenues in the invested route, i.e., rail. In this example, given the assumption of no tax, a change in the total benefits is simply a change in the consumers' surplus relating to rail. We assume that time costs are 39.3 yen/minute, as in ITPS (1999), and that the general equilibrium demand curve is linearly approximated. Rail's generalized price without a project is

$$39.3 \times 190 + 11,000 = 18,467 \text{ (yen)}. \quad (37)$$

The corresponding price with a project is

$$39.3 \times 140 + 11,000 = 16,502 \text{ (yen)}. \quad (38)$$

Hence, the final benefits are

$$0.5 \times (18,467 - 16,502) \times (100 + 135) = 230,888 \text{ (yen)}. \quad (39)$$

In contrast, the benefit-estimation method in ITPS (1999) is as follows. First, we obtain the origin-destination (OD) generalized price, which is the demand-weighted average of generalized prices in all routes. The OD generalized price without a project is

$$\begin{aligned} & (39.3 \times 190 + 11,000) \times \frac{100}{220} + (39.3 \times 120 + 14,000) \times \frac{110}{220} + (39.3 \times 400 + 5,500) \times \frac{10}{220} \\ & = 18,717 \text{ (yen)} \end{aligned} \quad (40)$$

The corresponding price with a project is

$$\begin{aligned} & (39.3 \times 140 + 11,000) \times \frac{135}{220} + (39.3 \times 120 + 14,000) \times \frac{80}{220} + (39.3 \times 400 + 5,500) \times \frac{5}{220} \\ & = 17,414 \text{ (yen)} \end{aligned} \quad (41)$$

Second, this change in the OD generalized price is assumed to apply to all the demand in the OD.

Thus, the final benefits are

$$0.5 \times (18,717 - 17,414) \times (220 + 220) = 286,660 \text{ (yen)}.^6 \quad (42)$$

Hence, the benefit-estimation method in ITPS (1999) overestimates true benefits by about 24 per cent.

In general, the benefit-estimation method in ITPS (1999) is unreliable. However, if all routes—rail, air, and car in this example—are assumed to be perfect substitutes, i.e., the utility function is assumed to be like (2), the ITPS (1999) method happens to give the right value. This is because the generalized prices of the trips between nodes i and j are equal in all routes actually used in such a case and, consequently, the generalized price in each route is equal to the demand-weighted average of the generalized prices in all routes. However, it is not necessary to calculate the demand-weighted average of the generalized prices in such a case when the generalized prices of the trips are equal in all routes. Thus, even in this case, we can conclude that the ITPS (1999) method lacks practical validity.

It is sometimes suggested that the ITPS (1999) method, which uses the OD generalized price, is consistent with the estimation of transport demand using the logit model.⁷ However, this is incorrect. If a discrete choice model such as the logit model is used to estimate transport demand, the general method of estimating welfare changes in discrete choice models developed by Williams (1977) and Small and Rosen (1981) must be used to calculate welfare changes. As Anderson et al. (1992) show, the logit model can be interpreted as a special case of the ordinary utility-maximization problem of a representative consumer. Thus, the methods derived in Sections 3 and 4 are still valid with logit-based estimation of transport demand. That is, we can accurately calculate a change in the total

⁶ Although the total benefit is given as 326,260 (yen) in ITPS (1999), this figure is the result of a simple error in calculation.

⁷ See, for example, Ueda et al. (2002).

benefits by using the methods in Sections 3 and 4, whether or not the logit model is used to estimate transport demand⁸. The ITPS (1999) OD-based method is theoretically inconsistent with both the method developed by Williams (1977) and Small and Rosen (1981) and the method derived in this paper; therefore, it cannot correctly compute welfare changes even when the estimation of transport demand is logit-based.

Example 2: “Network effects” in Button (1993)

Button (1993, pp. 182–184) proposes a benefit-estimation method for transportation projects that takes into account “network effects.” He claims that a change in the total benefits from an investment in a route in the road network is the sum of a change in the consumers’ surplus in all routes. However, this argument is inadequate, because it ignores a change in tax revenue. In the first-best case, this omission obviously could be very significant; the true benefits are the sum of changes in consumers’ surplus and the tax revenue in the invested route, not the sum of changes in consumers’ surplus in all routes. In the second-best case, the omission of the tax revenues would be justified if the tax was regarded as zero. However, as Button himself suggests (e.g., Table 4.10 in p. 81), the revenues from road tax (e.g., fuel tax) are too large to ignore. If we assume that a change in the tax revenue is zero when it is actually quite large, the final benefits could be quite different from those calculated by Button’s method.

Let us check how the omission of the producers’ surplus affects the results, using an illustrative example, in which rail and car are available. To simplify the calculation, the time cost is assumed to be 40 (yen/minute). Consider a transport project that upgrades rail tracks to shorten rail travel time. Consequently, the project increases rail demand and reduces car demand. A decrease in car demand

⁸ See Kidokoro (2003) for a detailed analysis.

and the subsequent relief in car congestion reduce revenues from fuel tax. The fuel tax rate is assumed to be 100 (%), i.e., gross fuel prices are twice as high as net fuel prices. Rail receives a subsidy from the government. Upgraded rail tracks require new rail cars, which raises rail's average (monetary) cost from 600 (yen) to 700 (yen) but the rise in the fare is only 50 (yen). The gap between the increase in the rail's average cost and the increase in the fare, 50 (yen), is financed by an increase in the government's subsidy, which is regarded as a negative tax. This example can be analysed by applying the benefit-estimation method in the second-best case from Section 4. In this example, we cannot estimate the change in the total benefits in the first-best case and, accordingly, it is impossible to calculate the final benefits by the third method, shown in (30) and (31). Thus, we need to obtain the final benefits by applying the first method, shown in (25) and (26), or by applying the second method, shown in (27) to (29).

First, applying the results of (25) and (26), we calculate a change in the total benefits as the sum of changes in the consumers' surplus and the tax revenue relating to both rail and car. (As in Example 1, the general equilibrium demand curve is assumed to be linearly approximated.)

A change in the consumers' surplus relating to rail is

$$0.5 \times (2,500 - 1,950) \times (300 + 400) = 192,500 \text{ (yen)}. \quad (43)$$

A change in the tax revenue relating to rail, which is a change in the government's subsidy here, is

$$(550 - 700) \times 400 - (500 - 600) \times 300 = -30,000 \text{ (yen)}. \quad (44)$$

A change in the consumers' surplus relating to car is

$$0.5 \times (3,200 - 2,700) \times (200 + 150) = 87,500 \text{ (yen)}. \quad (45)$$

A change in the tax revenue relating to car, which is a change in the revenue from fuel tax here, is

$$150 \times 150 - 200 \times 200 = -17,500 \text{ (yen)}. \quad (46)$$

Adding up (43) to (46) gives a change in the total social welfare of 232,500 (yen).

Second, by applying the results of (27) to (29), we calculate a change in the total benefits as the

sum of changes in the consumers' surplus and the tax revenues relating to rail, and a change in the dead weight loss relating to car. If we re-use Figure 3-3 and regard route 122 as by car, calculating a change in the dead weight loss relating to car travel is reduced to calculating *Area L'J'JL*, which is *Area L'J'RQ* less *Area LJRQ*. Since

$$(\text{Area } L'J'RQ) = (70 \times 40 + 200) \times 200 - (60 \times 40 + 150) \times 150 = 217,500 \text{ (yen)}^9 \text{ and} \quad (47)$$

$$(\text{Area } LJRQ) = 0.5 \times (3,200 + 2,700) \times (200 - 150) = 147,500 \text{ (yen)}, \quad (48)$$

a change in the dead weight loss relating to the car is $217,500 - 147,500 = 70,000$ (yen), which equals the sum of (45) and (46). Adding this value to (43) and (44), which are changes in the consumers' surplus and the tax revenue relating to rail, we again obtain the correct change in the total benefits of 232,500 (yen).

By applying Button's method, the benefit derived is 280,000 (yen), which is the sum of (43) and (44), and represents an overestimate of 47,500 (yen). This overestimate results from disregarding (44) and (46), which represent a decrease in government revenues because of an increase in the subsidy relating to rail, and a decrease in the tax revenues relating to car, respectively. This example illustrates the possibility of incorrectly calculating benefits when the producers' surplus is not properly included. Thus, we must pay attention, not only to the consumers' surplus, but also to the producers' surplus when calculating a change in the total benefits, unless the change in the producers' surplus is negligible.

6 Concluding Remarks

We have argued the benefit-estimation for transport projects both in the first-best case and in the

⁹ Recall that *Area L'J'RQ* represents a change in the total cost. Thus, when calculating *Area L'J'RQ*, which is the area below the social marginal cost curve of car, we do not need to know the social marginal cost of the car itself, which is often difficult to know in practice.

second best case, explicitly modeling congestion-prone transport networks. In a practical benefit-estimation for transport projects, the second-best situations are common, although they have not been analysed systematically in the literature. Our main results are summed up as follows. In the second-best situation, a change in the total benefits from an investment in transport infrastructures can be calculated in three ways. The first method shows that a change in total benefits is the sum of changes in consumers' and producers' surpluses in all routes. In all the non-invested routes, the sum of changes in consumers' and producers' surpluses result in a change in the dead weight loss, caused by the discrepancy between the generalized prices of trips and their social marginal costs. Focusing on this feature, the second method shows that a change in the total benefits is the sum of changes in the consumers' and producers' surpluses in the invested route and a change in the dead weight loss in all the other routes. The third method shows that a change in the total benefits in the second-best case equals the sum of a change in the total benefits in the first-best case and a change in the dead weight loss in all routes, highlighting the relationship between the total benefits in the first-best case and those in the second-best case. Although the first and the second methods are easy to implement practically, the implications of the third method is very important. Applying the third method, we know that a change in the total benefits is a change in the dead weight loss in all routes only, if the social marginal cost curve does not shift and consequently, a change in the total benefits in the first best case is zero. For example, when the congestion tax is introduced, a change in the total benefits can be calculated as a change in the dead weight loss in all routes.

In concluding the paper, we comment on two issues. Firstly, in our model, the dead weight loss arises because the generalized prices of trips differ from their social marginal costs due to the gap between the tax, t^{ijk} , and the congestion externality, $c_{x^{ijk}}^{ijk}(x^{ijk}, I^{ijk})x^{ijk}$. Our analysis also applies without modification when the dead weight loss stems from other distortions. As an example, consider the case in which suppliers of transport services behave less competitively. Even in this case,

our analysis is directly applicable, if the tax, t^{ijk} , is redefined to include the price-cost margin that results from imperfect competition. Our analysis also applies to the case where the generalized prices differs from the social marginal costs due to environmental externality, if environmental costs are additionally included in the total cost function of trips.

Secondly, we have assumed away the price-cost divergences in non-transport sectors. Although it would be difficult to include them in practical benefit estimation of transport projects, our analysis is applicable in principle, even if we explicitly consider the second-best situation in non-transport sectors. In that case, all we have to do is to add changes in consumers' and producers' surpluses in non-transport sectors. Since the same logic as changes in the non-invested routes applies to changes in non-transport sectors, the sum of changes in consumers' and producers' surpluses in a non-transport sectors equals a change in the dead weight loss in the non-transport sectors. Thus, we can add a change in the dead weight loss in all non-transport sectors, instead of the sum of changes in consumers' and producers' surpluses.

Appendix 1: Derivation of Results in Sections 3 and 4

First, we consider the case in which investment in transport infrastructure, I^{ijk} , increases from I^{ijkWO} to I^{ijkW} . We begin by analysing the second-best case, in which the generalized prices of trips are not necessarily equal to their social marginal costs. We then show the first-best result, in which the generalized prices of trips equal their social marginal costs, as a special case of the second-best results.

Substituting (1) and (3) into (17) and using i' , j' , and k' , where $1 \leq i' \leq N$, $1 \leq j' \leq N$, and $1 \leq k' \leq M$, instead of i , j , and k , we obtain

$$TB = \sum_{i'} y^{i'} - \sum_{i'} \sum_{j'} \sum_{k'} p^{i'j'k'} x^{i'j'k'} + \sum_{i'} u^{i'} + \sum_{i'} \sum_{j'} \sum_{k'} t^{i'j'k'} x^{i'j'k'}. \quad (A1)$$

Totally differentiating the total benefits, (A1), with respect to I^{ijk} yields

$$\begin{aligned} \frac{dTB}{dI^{ijk}} &= - \sum_{i'} \sum_{j'} \sum_{k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) x^{i'j'k'} - \sum_{i'} \sum_{j'} \sum_{k'} p^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) \\ &\quad + \sum_{i'} \sum_{j'} \sum_{k'} u_{x^{i'j'k'}}^{i'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) + \sum_{i'} \sum_{j'} \sum_{k'} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) \\ &= - \sum_{i'} \sum_{j'} \sum_{k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) x^{i'j'k'} + \sum_{i'} \sum_{j'} \sum_{k'} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) \end{aligned} \quad (A2)$$

where we have used the result of the utility maximization; i.e., $p^{ijk} = u_{x^{ijk}}^i$ in (5).

Integrating (A2), from I^{ijkWO} to I^{ijkW} , the change in the total benefits is

$$\begin{aligned} \Delta TB &= - \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \\ &= \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \end{aligned} \quad (A3)$$

where $\sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk}$ is the change in the consumers' surplus in all routes, and

$\sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk}$ is the change in the tax revenue in all routes.

Distinguishing changes in the primary market, which is the invested-in route, from those in the secondary market, which incorporates all other routes, using (10) and (13), we rearrange (A3) to obtain the second form as follows:

$$\begin{aligned}
\Delta TB &= \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkW}}^{I^{ijkWO}} x^{i''j''k''} \left(\frac{dp^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk} \\
&\quad + \int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i''j''k''} \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk} \tag{A4} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkW}}^{I^{ijkWO}} \left(c_{x^{i''j''k''}}^{i''j''k''} x^{i''j''k''} - t^{i''j''k''} \right) \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkW}}^{I^{ijkWO}} \left(MC^{i''j''k''} - p^{i''j''k''} \right) \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk}
\end{aligned}$$

where $1 \leq i'' \leq N$, $1 \leq j'' \leq N$, and $1 \leq k'' \leq M$, but (i'', j'', k'') does not contain (i, j, k) . In (A4),

$\int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk}$ is the change in the consumers' surplus in route ijk , $\int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk}$ is the

change in the tax revenue in route ijk , and $\sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkW}}^{I^{ijkWO}} \left(MC^{i''j''k''} - p^{i''j''k''} \right) \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) dI^{ijk}$ is the change

in the deadweight loss in all other routes.

Highlighting the distinction between total benefits in the first-best case and distortions in the second-best case, using (10) and (13) to rearrange (A3), we obtain the third form as follows:

$$\begin{aligned}
\Delta TB &= \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} x^{i'j'k'} \left(\frac{dp^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkWO}}^{I^{ijkW}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left[c_{I^{ijk}}^{ijk} + c_{x^{ijk}}^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) \right] dI^{ijk} + \sum_{i''} \sum_{j''} \sum_{k''} \int_{I^{ijkW}}^{I^{ijkWO}} x^{i''j''k''} \left[c_{x^{i''j''k''}}^{i''j''k''} \left(\frac{dx^{i''j''k''}}{dI^{ijk}} \right) \right] dI^{ijk} - \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} \left(c_{x^{i'j'k'}}^{i'j'k'} x^{i'j'k'} - t^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk} \\
&= \int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} \left(MC^{i'j'k'} - p^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk}.
\end{aligned} \tag{A5}$$

In (A5), $\int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk}$ is the change in the total benefits in the first-best case, which is derived

subsequently, and $\sum_{i'} \sum_{j'} \sum_{k'} \int_{I^{ijkW}}^{I^{ijkWO}} \left(MC^{i'j'k'} - p^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dI^{ijk}} \right) dI^{ijk}$ is the change in the deadweight loss relating to all routes.

In the first-best case, in which the generalized prices of the trips equal their social marginal costs in all routes, from (A4) and (A5), we obtain

$$\Delta SW = \int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk} + \int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk} = \int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk}. \tag{A6}$$

Equation (A6) shows that a change in total benefits in the first-best case consists of a change in the

consumers' surplus in route ijk , $\int_{I^{ijkW}}^{I^{ijkWO}} x^{ijk} \left(\frac{dp^{ijk}}{dI^{ijk}} \right) dI^{ijk}$, and a change in the tax revenue in route ijk ,

$$\int_{I^{ijkWO}}^{I^{ijkW}} t^{ijk} \left(\frac{dx^{ijk}}{dI^{ijk}} \right) dI^{ijk}, \text{ and that these two terms are reduced to } \int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk}. \text{ Thus, } \int_{I^{ijkW}}^{I^{ijkWO}} c_{I^{ijk}}^{ijk} x^{ijk} dI^{ijk}$$

corresponds to the change in total benefits in the first-best case.

Second, let us focus on the case in which the tax in route ijk , t^{ijk} , increases from t^{ijkWO} to t^{ijkW} .

Totally differentiating total benefits, (A1) with respect to t^{ijk} yields

$$\begin{aligned}
\frac{dTB}{dt^{ijk}} &= -\sum_{i'} \sum_{j'} \sum_{k'} \left(\frac{dp^{i'j'k'}}{dt^{ijk}} \right) x^{i'j'k'} - \sum_{i'} \sum_{j'} \sum_{k'} p^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) \\
&\quad + \sum_{i'} \sum_{j'} \sum_{k'} u_{x^{i'j'k'}}^{i'} \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) + \sum_{i'} \sum_{j'} \sum_{k'} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) + x^{ijk} \\
&= -\sum_{i'} \sum_{j'} \sum_{k'} c_{x^{i'j'k'}}^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) x^{i'j'k'} - x^{ijk} + \sum_{i'} \sum_{j'} \sum_{k'} t^{i'j'k'} \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) + x^{ijk} \\
&= -\sum_{i'} \sum_{j'} \sum_{k'} \left(c_{x^{i'j'k'}}^{i'j'k'} x^{i'j'k'} - t^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right)
\end{aligned} \tag{A7}$$

where we have used the result of the utility maximization; i.e., $p^{ijk} = u_{x^{ijk}}^i$ in (5).

Integrating (A7) from t^{ijkWO} to t^{ijkW} , from (10) and (13), we obtain the change in the total benefits as follows:

$$\begin{aligned}
\Delta TB &= -\sum_{i'} \sum_{j'} \sum_{k'} \int_{t^{ijkWO}}^{t^{ijkW}} \left(c_{x^{i'j'k'}}^{i'j'k'} x^{i'j'k'} - t^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) dt^{ijk} \\
&= \sum_{i'} \sum_{j'} \sum_{k'} \int_{t^{ijkW}}^{t^{ijkWO}} \left(c_{x^{i'j'k'}}^{i'j'k'} x^{i'j'k'} - t^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) dt^{ijk} \\
&= \sum_{i'} \sum_{j'} \sum_{k'} \int_{t^{ijkW}}^{t^{ijkWO}} \left(MC^{i'j'k'} - p^{i'j'k'} \right) \left(\frac{dx^{i'j'k'}}{dt^{ijk}} \right) dt^{ijk},
\end{aligned} \tag{A8}$$

which shows the change in the deadweight loss in all routes.

Figures and Tables

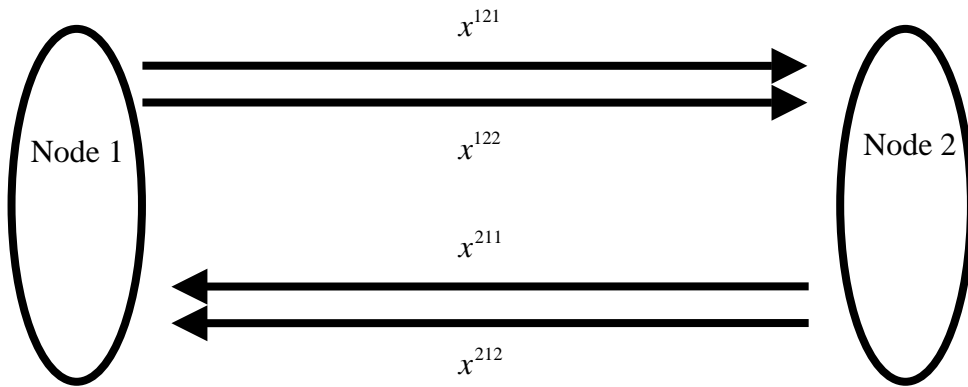


Figure 1: N -node Network with M routes ($N = 2, M = 2$)

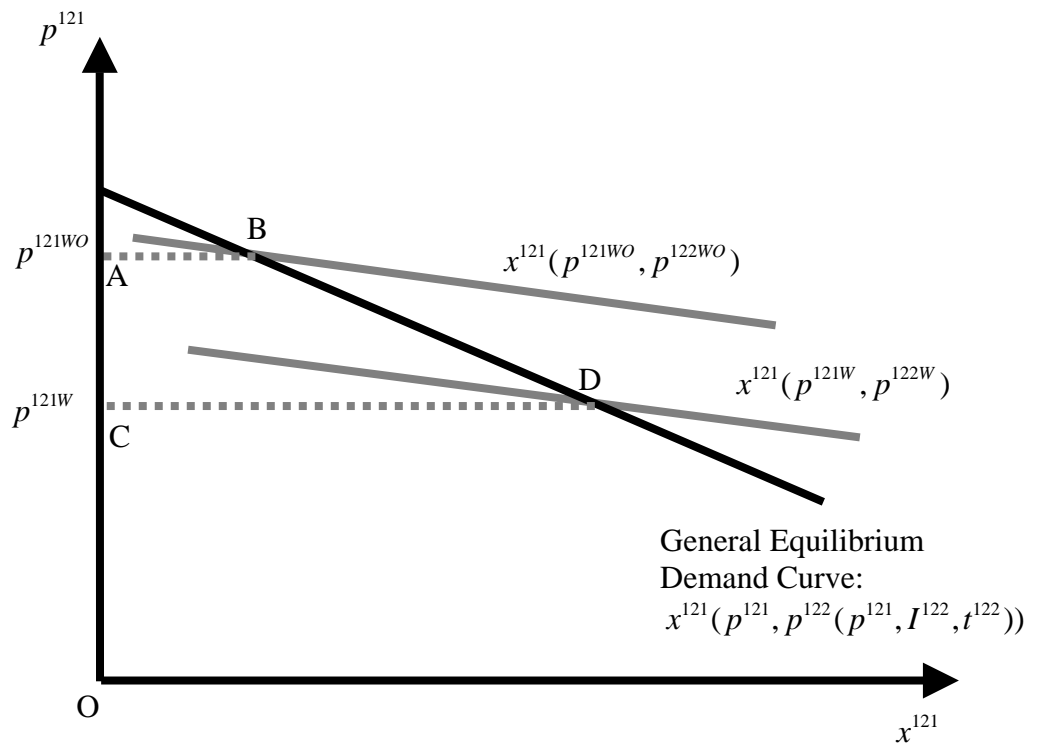


Figure 2-1: Consumers' surplus in route 121 in the first-best case

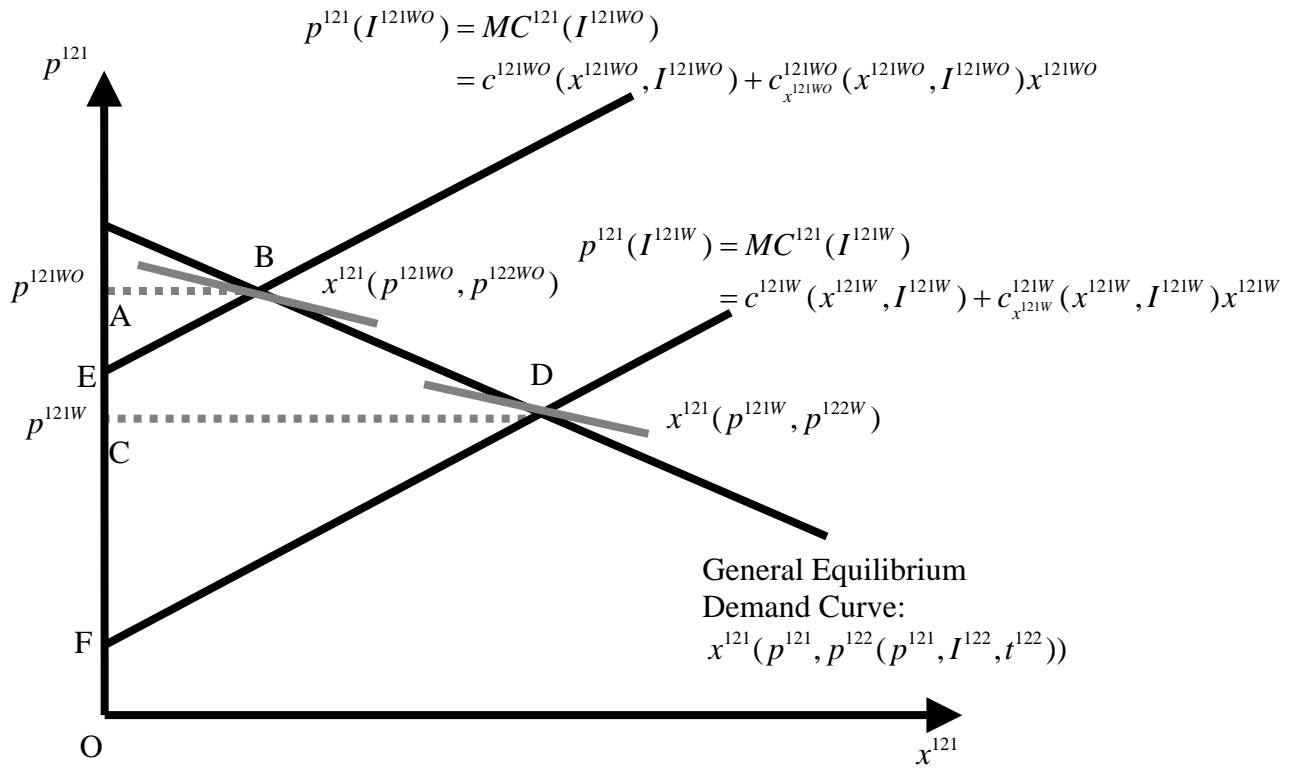


Figure 2-2: Producers' surplus in route 121 in the first-best case

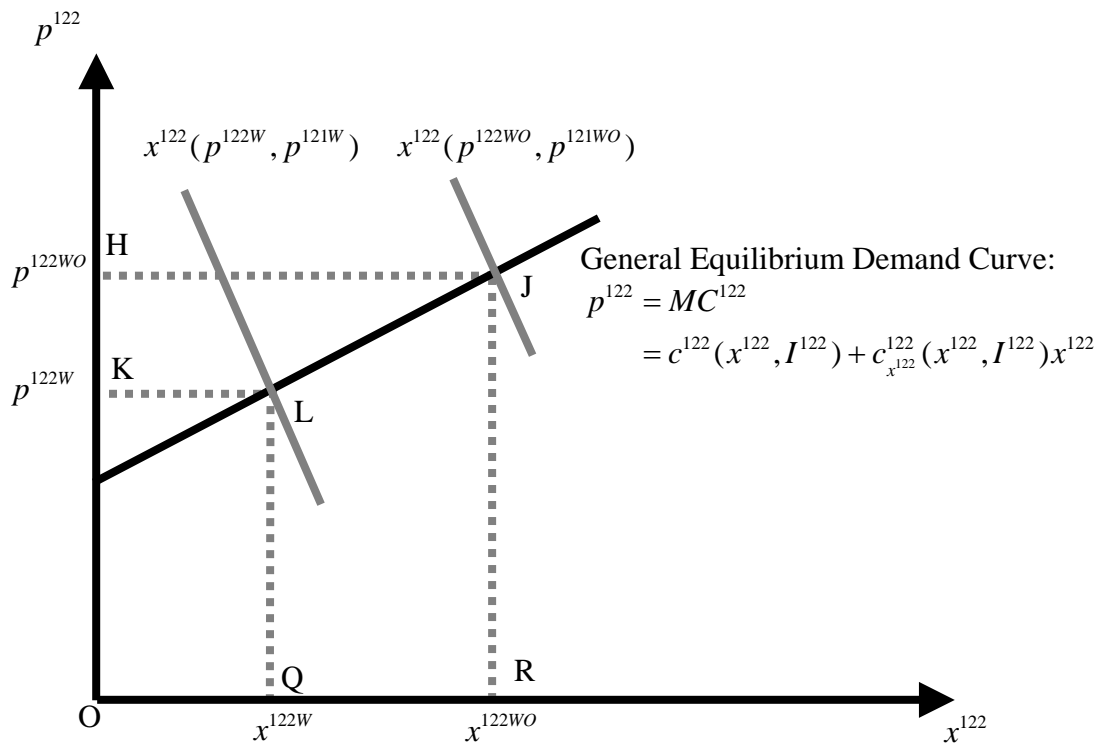


Figure 2-3: Consumers' and producers' surpluses in route 122 in the first-best case

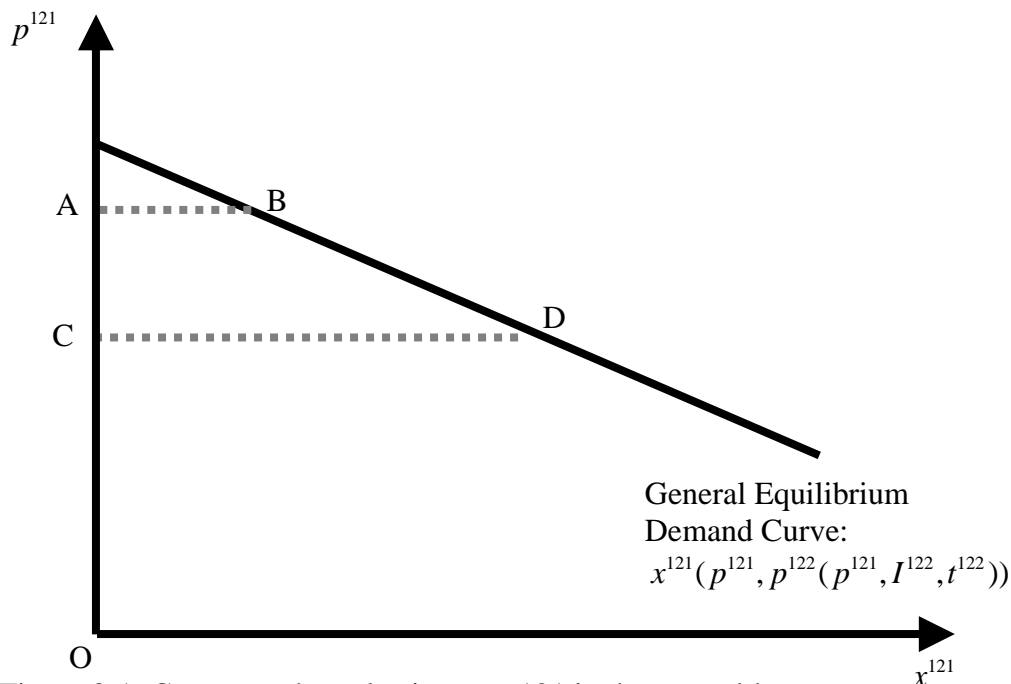


Figure 3-1: Consumers' surplus in route 121 in the second-best case

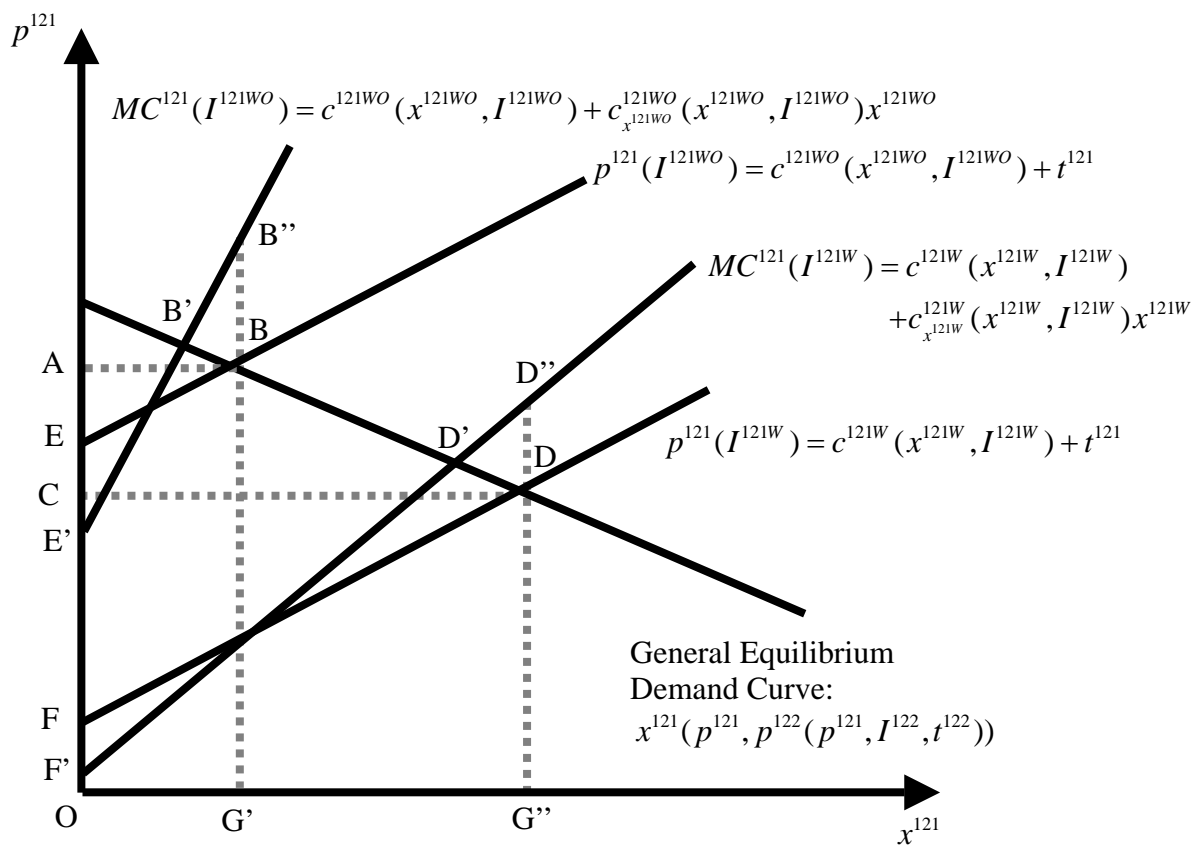


Figure 3-2: Producers' surplus in route 121 in the second-best case

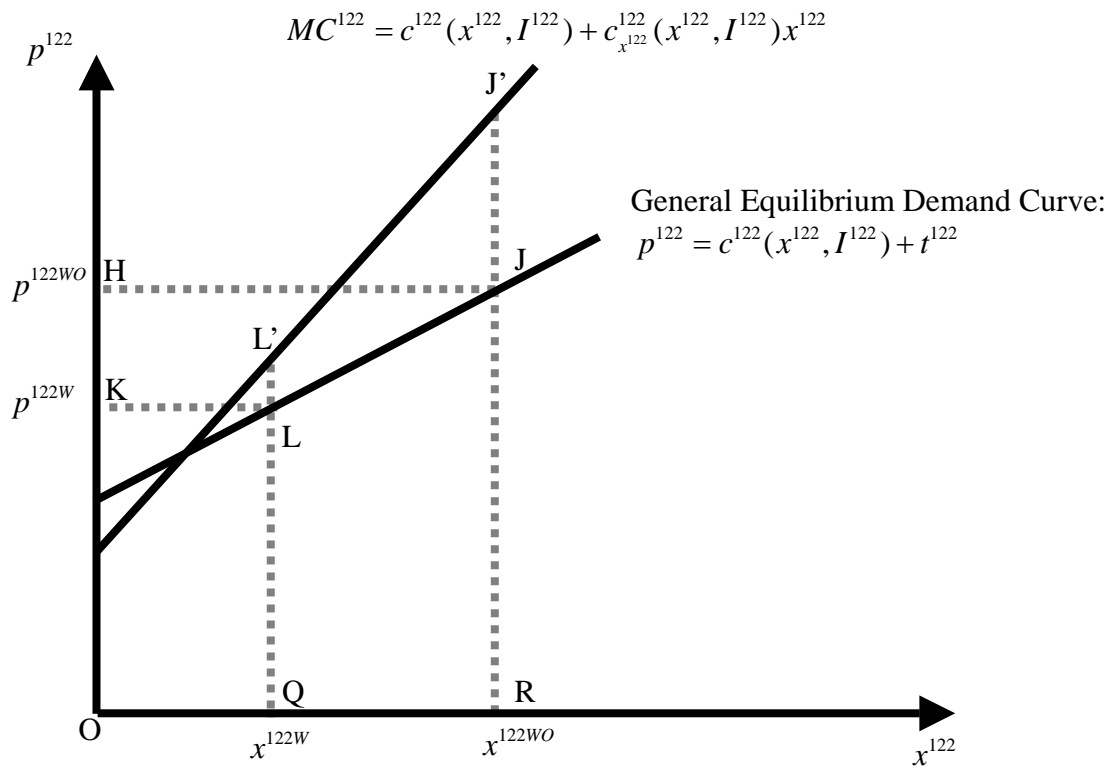


Figure 3-3: Consumers' and producers' surpluses in route 122 in the second-best case

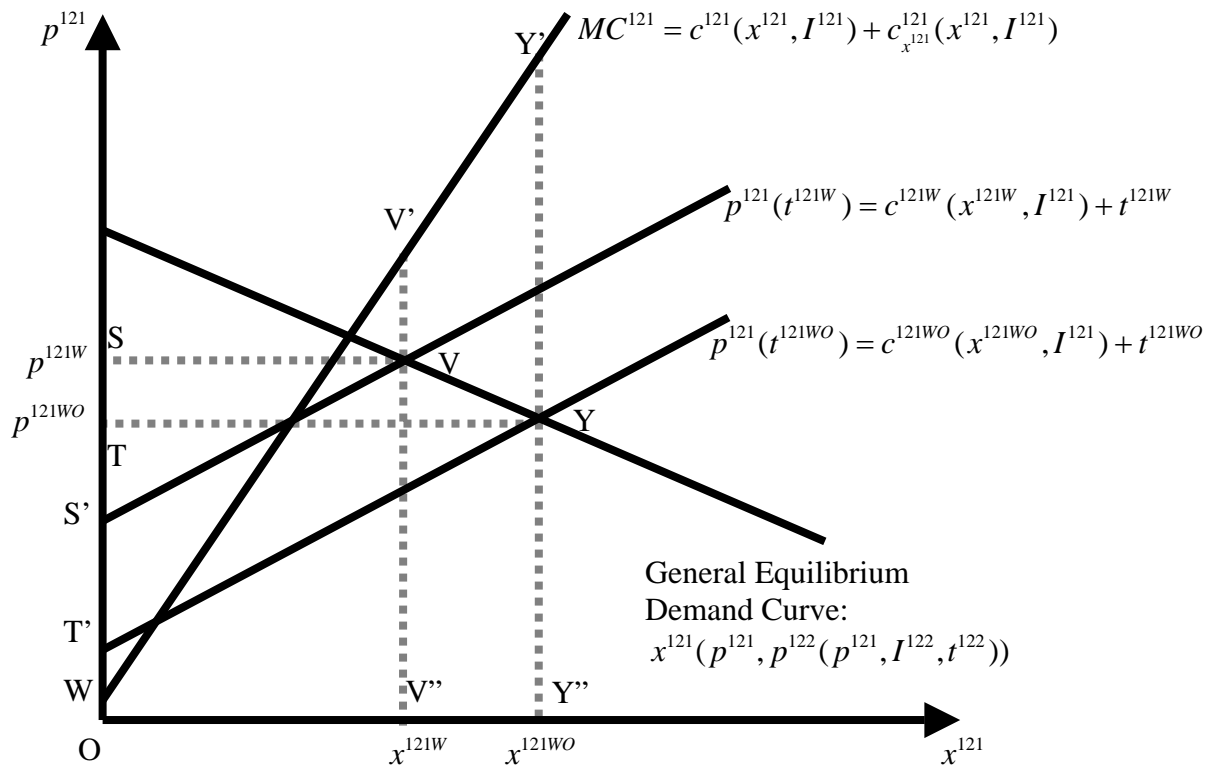


Figure 4: The effects of an increase in the tax t^{121} in route 121

Table 1: An example from ITPS (1999)

Without a Project

Mode	Travelling time (minutes)	Monetary price (yen)	Demand per day
Rail	190	11,000	100
Air	120	14,000	110
Car	400	5,500	10

With a Project

Mode	Travelling time (minutes)	Monetary price (yen)	Demand per day
Rail	140	11,000	135
Air	120	14,000	80
Car	400	5,500	5

Table 2: An example in which the producers' surplus plays an important role

Without a Project

Mode	Travelling time (minutes)	Monetary price (yen)	Net fuel price (yen)	Fuel tax (yen)	Rail's average cost (yen)	Generalized price (yen)	Demand per day
Rail	50	500	/	/	600	2,500	300
Car	70	400	200	200	/	3,200	200

With a Project

Mode	Travelling time (minutes)	Monetary price (yen)	Net fuel price (yen)	Fuel tax (yen)	Rail's average cost (yen)	Generalized price (yen)	Demand per day
Rail	35	550	/	/	700	1,950	400
Car	60	300	150	150	/	2,700	150

Note: Monetary price of car travel \equiv Net fuel price + Fuel tax

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