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Regulation of Quality for Public Utilities under Asymmetric Information

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Abstract: This paper is concerned with the effects of unverifiability of quality on regulatory contracts under asymmetric information regarding a monopoly's costs. We show that unverifiability of quality results in a higher level of quality, lower output, higher price, and lower net consumer surplus. If the price elasticity of demand is no larger than 1, unverifiable quality increases the level of a monopoly's profits. In this case, the regulator and the monopoly have opposing preferences for verifiability of quality, which makes the regulation based on the level of quality hard to implement.

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1 Introduction

To consider regulatory policies for public utilities, the regulator has to pay attention not only to the levels of output and price but also to the level of quality. In reality, it is far more difficult to regulate public utilities based on the level of quality than to regulate them based on the levels of output and price. One reason for this is that quality is highly multidimensional and to evaluate all the aspects of quality would involve formidable costs. This difficulty, however, could be avoided, at least to some extent, by formulating an index of quality, as noted by Lynch et al. (1994). If we use this type of quality index, will it be possible to implement regulation based on the level of quality?

The purpose of this paper is to examine the implementability of regulations based on the level of quality, focusing on unverifiability of quality under asymmetric information regarding a monopoly's costs. We derive the effects of unverifiability of quality on regulatory contracts and show that regulations based on the level of quality are intrinsically difficult to implement because a monopoly benefits from unverifiability of quality. To set up a model as simply as possible, this paper focuses on service congestion as a representative aspect of service quality. Although quality is essentially multidimensional, congestion-type quality represents an important aspect of the services supplied by public utilities. For example, important quality measures in the supply of electricity are the frequency and the length of breakdowns, which mostly result from congestion. In telecommunications, ease of connection to lines and the transfer speed of data are important quality measures that are also closely related to network congestion.

The fundamental relationship we obtain in this paper is that unverifiability of quality raises the level of quality but lowers the level of output, and consequently, raises the level of price. If quality is verifiable, the regulator can reduce the informational rents a monopoly enjoys by lowering both the levels of both output and quality. If quality is unverifiable, however, the regulator's control variable is restricted to output only. In this case, the regulator cannot reduce a monopoly's informational rents by lowering the level of quality. Thus, the regulator has to make the level of output even lower when quality is unverifiable than when verifiable to cut down the monopoly's rents. As a result, unverifiable quality results in a higher level of quality and a lower level of output. This means that the price level is higher when quality is unverifiable than when verifiable.

Since the regulator can not extract a monopoly's informational rents by controlling the level of quality when quality is unverifiable, unverifiable quality presents more distortions to net consumer surplus than verifiable quality. The level of net consumer surplus is thus higher when quality is verifiable than when unverifiable. The level of the monopoly's profits, however, may be higher or lower under verifiable quality depending on the price and the quality elasticities of demand. Especially if the price elasticity of demand is no larger than 1, which would be the case for most public utilities, the level of the profits is lower when quality is verifiable than when unverifiable. Unverifiability of quality therefore raises the level of the monopoly's profits. That is, if we consider the regulation for common public utilities, unverifiability of quality lowers the level of net consumer surplus but raises the level of the monopoly's profits. This means that the monopoly has no incentives to reveal the true level of quality and thus make quality verifiable. This would be the fundamental reason why regulation based on the level of quality is hard to implement.

Our analysis is an application of optimal regulation literature for a monopoly whose costs are unknown to the regulator, in which the level of quality is taken into account. The most related studies are Beitia (1995), Laffont and Tirole (1991), and Lewis and

Sappington (1991). Beitia (1995) focuses on the case of verifiable quality only, while Laffont and Tirole (1991) focuses on the case of unverifiable quality only. Therefore, both studies do not extract the effects of unverifiability of quality on regulatory contracts. In that we explicitly compare unverifiable quality with verifiable quality, our approach is close to Lewis and Sappington (1991). Their analysis, however, focuses on the derivation of a model in which both the regulator and a monopoly benefit from verifiability of quality. Using a model that is better fit for an analysis of actual public utilities, we obtain results different from theirs: the level of quality a monopoly selects is higher when quality is unverifiable than when verifiable, and unverifiability of quality makes the level of its profits higher, i.e., a monopoly benefits from unverifiability and consequently has no incentives to reveal the level of quality truthfully, if the price elasticity of demand is no larger than 1.

The structure of this chapter is as follows. The elements of our model are described in section 2. In section 3, we analyze optimal regulation under symmetric information as a benchmark. In section 4, we derive optimal regulation under asymmetric information about a monopoly's cost, with both the cases of verifiable and unverifiable quality. In section 5, we obtain our main results by comparing the case of unverifiable quality with the case of verifiable quality. In section 6, our model is extended by numerical simulation. In section 7, we closely contrast our results with the results of Beitia (1995), Laffont and Tirole (1991), and Lewis and Sappington (1991). Section 8 concludes our analysis.

2 Elements of the model

A monopoly supplies a service. The monopoly's costs to provide the service, C, consist of unit production cost times output and capacity costs. We assume that unit production cost is fixed with a but capacity costs depend on $\beta \in [\underline{\beta}, \overline{\beta}]$[1]. For the sake of analytical simplicity, the cost function is assumed to be $C = ax + \beta Q$, where x is service output and Q is production capacity. Throughout our analysis, service output, x, is assumed to be verifiable. We assume away the shutdown of a monopoly, i.e., even the monopoly with the highest capacity cost parameter supplies the service.

The level of congestion is measured by service output per capacity, $\frac{x}{Q}$. For analytical simplicity, we use the inverse of the congestion measure as a measure of quality, q, i.e., $q \equiv \frac{Q}{x}$. *Ceteris paribus,* enhanced capacity leads to improved quality. In our analysis, we regard Q as nominal in the sense that it is possible that the quantity produced exceeds production capacity, i.e., $x > Q$, in which case the service supply continues but quality deteriorates.

Again for analytical simplicity, we focus on the case where an increase in quality (i.e., a decrease in congestion) is a perfect substitute of a decrease in price and the demand function is $x = x(p - v(q))$, where p represents price[2]. This demand function

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[1] It is possible that capacity costs are fixed but unit production cost varies. In this case, however, our analysis becomes uninteresting because asymmetric information about unit production cost leaves the level of quality unchanged.

[2] For instance, a decrease in congestion speeds up data transfer in telecommunications. The accelerated data transfer lessens the time needed to send data, and consequently, has the same effect as a decrease in price. In electricity supply, decreased congestion implies reduced possibilities of blackouts. In this case, electricity users can cut down the costs for back-up systems such as back-up generators. Thus, a decrease in risk of blackouts has virtually the same effect as a decrease in price.

yields the inverse demand function that is separable in x and q, $p(x,q)=u(x)+v(q)$. We will consider the case where the inverse demand function is not separable in section 6. The inverse market demand function satisfies $p_x \equiv \frac{\partial p(x,q)}{\partial x} = u'(x) < 0$ and $p_q \equiv \frac{\partial p(x,q)}{\partial q} = v'(q) > 0$. Hereafter, subscripts denote partial derivatives. Gross consumer surplus, S^g, is $S^g(x,q) = \int_{\tilde{x}=0}^{x} \{u(\tilde{x}) + v(q)\} d\tilde{x}$, which is assumed to be strictly concave. Thus, $v''(q) < 0$. Net consumer surplus, S^n, is defined as $S^n(x,q) \equiv S^g(x,q) - p(x,q)x$.

A monopoly's profits, π, can be written as

$$\pi(x,q,T;\beta) = p(x,q)x - C(x,qx,\beta) - T \tag{1}$$

where T is a lump-sum tax (or a lump-sum subsidy if $T<0$) on the monopoly. The regulator's objective is to maximize the weighted sum of the monopoly's profits and total net consumer surplus, $S^n(x,q)+T$. The weight placed on the total net consumer surplus is α and that on profits is $1-\alpha$, where $\frac{1}{2} < \alpha \leq 1$.

3 Symmetric Information

As a benchmark, we consider the case where all information between the regulator and the monopoly is symmetric. In this case, the regulator first fixes output, x, quality, q, and a lump-sum tax, T, to maximize social welfare subject to non-negative profits (the individual rationality constraint). Second, the monopoly implements them. Social welfare, W, can be written as

$$W = \alpha\{S^n(x,q)+T\}+(1-\alpha)\pi(x,q,T;\beta). \tag{2}$$

Maximization of social welfare, W, subject to $\pi(x,q,T;\beta)\geq 0$, with respect to x, q, and T yields Lemma 1. Hereafter, we will sometimes write $p(x(\beta),q(\beta))$, $\pi(x(\beta),q(\beta),T(\beta);\beta)$, and $q(\beta)x(\beta)$ as $p(\beta)$, $\pi(\beta)$, and $Q(\beta)$, respectively, where there is no possibility of confusion.

Lemma 1

Optimal regulation under symmetric information, which is denoted by a superscript, $*$, satisfies

i) $$p^*(\beta) = a+\beta q^*(\beta), \tag{3}$$

ii) $$v'(q^*(\beta)) = \beta, \tag{4}$$

iii) $$\pi^*(\beta) = 0. \tag{5}$$

The proof is stated in the Appendix. (3) shows that service price equals its marginal service cost, $C_x(=a+\beta q)$. (4) shows that consumers' marginal willingness to pay for quality per service, $v'(q)$, equals the marginal cost of quality per service, $\frac{C_q}{x}(=\beta)$. (5) demonstrates that the monopoly's profits are zero under optimal regulation. These results are a natural extension of the standard two-part tariff theory regarding price and quantity[3].

[3] See, for example, Wilson (1993).

4 Asymmetric Information

Let us introduce asymmetric information regarding a monopoly's cost parameters. The regulator cannot observe its cost parameter, β, but it knows the probability distribution of the cost parameter. The density function of β is $f(\beta)$ and its distribution function is $F(\beta)$. We assume that the monotone-hazard-rate property holds, i.e., $\dfrac{d\left(\dfrac{F(\beta)}{f(\beta)}\right)}{d\beta} > 0$.

4-1 Verifiable Quality

First, we consider the case where production capacity, Q, is verifiable and consequently quality, q, is verifiable. In this case, the regulator designs a regulatory contract based on output, x, quality, q, and a lump-sum tax, T.

The timing of the regulation is as follows. First, the regulator and the monopoly share common knowledge about the probability distribution of β. Second, nature determines the monopoly's true β, known only to the monopoly. Third, the regulator designs a menu of regulatory contracts, $(x(\beta), q(\beta), T(\beta))$, based on $\beta \in [\underline{\beta}, \overline{\beta}]$. Fourth, the monopoly reports that its cost parameter is β' and chooses the contract designed for β', $(x(\beta'), q(\beta'), T(\beta'))$. From the Revelation Principle, we can focus on the truth-telling case, i.e., the monopoly whose true cost parameter is β announces that its cost parameter is β and chooses $(x(\beta), q(\beta), T(\beta))$. Fifth, the regulator implements the regulatory contract selected by the monopoly. We assume that both the regulator and the monopoly are fully committed to the regulation once they have signed.

Following the usual procedure in the optimal regulation literature (e.g., Besanko and Sappington (1987)), we formulate the maximization problem (P1) as follows.

(P1) $Max_{\{x(\cdot),q(\cdot),T(\cdot)\}}$

$$W = \int_{\beta=\underline{\beta}}^{\overline{\beta}} \left[\alpha\left\{S^n(x(\beta),q(\beta)) + T(\beta)\right\} + (1-\alpha)\pi(x(\beta),q(\beta),T(\beta);\beta)\right] dF(\beta)$$

s.t. (Individual rationality) $\pi(x(\beta),q(\beta),T(\beta);\beta) \geq 0$ for all β

(Incentive compatibility) $\pi(x(\beta),q(\beta),T(\beta);\beta) \geq \pi(x(\beta'),q(\beta'),T(\beta');\beta)$

for all β, $\beta'(\neq \beta)$

Solving the maximization problem (P1) yields the following proposition.

Proposition 1

Optimal regulation under asymmetric information and verifiable quality, which is denoted by a superscript, 1, satisfies

i) $p^1(\beta) = a + (\beta + k)q^1(\beta) \geq a + \beta q^1(\beta)$, (6)

ii) $v'(q^1(\beta)) = \beta + k \geq \beta$, (7)

iii) $\pi^1(\beta) = \int_{\tilde{\beta}=\beta}^{\overline{\beta}} Q^1(\tilde{\beta}) d\tilde{\beta}$, (8)

where $k \equiv \dfrac{(2\alpha-1)F(\beta)}{\alpha f(\beta)} \geq 0$.

The proof is stated in the Appendix. Unless a monopoly is the most efficient, i.e.,

$\beta = \underline{\beta}$, a monopoly's price and consumers' marginal willingness to pay for quality (per service), respectively, will exceed its marginal service cost and the marginal cost of quality (per service). These results stem from the fact that the regulator tries to extract informational rents the monopoly gains from asymmetric information about its cost parameters.

The monopoly's profits are positive, except in the most inefficient case, i.e., $\beta = \overline{\beta}$. The size of its profits depends on production capacity for the following reason. Suppose that a monopoly whose cost parameter is $\beta - \varsigma$ ($\varsigma > 0$) claims its cost parameter to be β. In this case, the monopoly's approximate gain from exaggerating its costs is:

$$\beta Q(\beta) - (\beta - \varsigma) Q(\beta) = \varsigma Q(\beta).$$

To induce truthful cost reporting, the regulator must give the monopoly the rents by $\varsigma Q(\beta)$ and compensate the monopoly for losses from truth-telling. Therefore, the monopoly's profits are determined by the scale of production capacity.

4-2 Unverifiable Quality

We proceed to the case where production capacity, Q, is unverifiable for a certain reason (e.g., high costs) and consequently, quality, q, is unverifiable. In this case, the regulator cannot implement a regulatory contract based on quality[4]. The regulator

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[4] Suppose that the regulator offers a regulatory contract based on quality and that a monopoly does not achieve the level of quality the regulator designates. If quality is unverifiable, the court cannot judge whether or not the monopoly has breached the contract. The regulator is therefore unable to punish the monopoly on the basis that the monopoly does not attain the level of quality specified in the contract. The monopoly can therefore freely choose

therefore offers a regulatory contract based on output, x, and a lump-sum tax, T. The monopoly chooses quality, q, so as to maximize its profits, given the regulatory contract it has selected. The maximizing problem in this case, (P2), is as follows.

(P2) $Max_{\{x(\cdot),T(\cdot)\}}$

$$W = \int_{\beta=\underline{\beta}}^{\overline{\beta}} \left[\alpha\{S^n(x(\beta), q(x(\beta),\beta)) + T(\beta)\} + (1-\alpha)\pi(x(\beta), q(x(\beta),\beta), T(\beta);\beta) \right] dF(\beta)$$

s.t. (Individual rationality) $\pi(x(\beta), q(\beta), T(\beta);\beta) \geq 0$ for all β

(Incentive compatibility) $\pi(x(\beta), q(\beta), T(\beta);\beta) \geq \pi(x(\beta'), q(\beta'), T(\beta');\beta)$

for all β, $\beta'(\neq \beta)$

$$q(x,\beta) = \arg\max_q \left[p(x,q)x - C(x,qx,\beta) \right]$$

We obtain the following proposition by solving the maximization problem, (P2).

Proposition 2

Optimal regulation under asymmetric information and unverifiable quality, which is denoted by a superscript, 2 , satisfies

i) $p^2(\beta) = a + (\beta + k)q^2(\beta) \geq a + \beta q^2(\beta)$, (9)

ii) $v'(q^2(\beta)) = \beta$, (10)

iii) $\pi^2(\beta) = \int_{\tilde{\beta}=\beta}^{\overline{\beta}} Q^2(\tilde{\beta}) d\tilde{\beta}$, (11)

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the level of quality. Thus, unverifiability of quality makes a regulatory contract based on quality unimplementable.

where $q^2(\beta) \equiv q^2(x^2(\beta), \beta)$ and k is as defined in Proposition 1.

The proof is stated in the Appendix. (9), as well as (6), states that price is higher than the marginal service cost. This distortion arises because of the rent extraction by the regulator. (10) demonstrates that consumers' marginal willingness to pay for quality (per service), $v'(q)$, equals the marginal cost of quality (per service.) That is, regarding quality, no distortion exists. This result stems from the fact that the regulator cannot extract informational rents by controlling the level of quality, due to the unverifiability of quality. (11) shows the same result as (8): that is, the monopoly's profits are positive, except for the most inefficient case, i.e., $\beta = \overline{\beta}$.

5 A Comparison

Since we have characterized the solutions under symmetric information, asymmetric information and verifiable quality, and asymmetric information and unverifiable quality, we now compare these solutions with one another. Using Lemma 1, Proposition 1, and Proposition 2, we obtain the following Proposition 3.

Proposition 3

Given $\beta \in (\underline{\beta}, \overline{\beta})$,

i) $x^*(\beta) > x^1(\beta) > x^2(\beta)$, (12)

ii) $q^*(\beta) = q^2(\beta) > q^1(\beta)$, (13)

iii) $p^2(\beta) > p^*(\beta) > p^1(\beta)$ if $\varepsilon < 1$ and $p^2(\beta) > p^1(\beta) > p^*(\beta)$ if $\varepsilon > 1$, (14)

iv) $Q^*(\beta) > Q^2(\beta) > Q^1(\beta)$ if $\eta_p < \dfrac{p(1+\eta_q)}{p-a}$ and $Q^*(\beta) > Q^1(\beta) > Q^2(\beta)$ if

$$\eta_p > \frac{p(1+\eta_q)}{p-a}, \tag{15}$$

v) $S^{n*}(\beta) > S^{n1}(\beta) > S^{n2}(\beta)$, (16)

vi) $\pi^2(\beta) > \pi^1(\beta) > \pi^*(\beta)$ if $\eta_p < \dfrac{p(1+\eta_q)}{p-a}$ and $\pi^1(\beta) > \pi^2(\beta) > \pi^*(\beta)$ if

$$\eta_p > \frac{p(1+\eta_q)}{p-a}, \tag{17}$$

where $\varepsilon \equiv -\dfrac{d\{v'(q)\}}{dq}\dfrac{q}{v'(q)}$, $\eta_p \equiv -\dfrac{\partial x}{\partial p}\dfrac{p}{x}$, and $\eta_q = \dfrac{\partial x}{\partial q}\dfrac{q}{x}$.

The proof is stated in the Appendix. First, we focus on the effects of asymmetric information on regulatory contracts by comparing the case of symmetric information with that of asymmetric information and verifiable quality. The result we have obtained is a natural extension of Baron and Myerson (1982). The regulator lowers the levels of both output and quality to reduce a monopoly's informational rents under asymmetric information. This leads to lower levels of production capacity and net consumer surplus under asymmetric information. The monopoly gains positive profits under asymmetric information, while gaining zero profits under symmetric information.

The different result from Baron and Myerson (1982) is the effect of asymmetric information on price. In their study, the price level is higher under asymmetric information, because a decrease in output caused by asymmetric information raises the price. In our model, however, another effect exists: a decrease in quality lowers price. If this effect is large, it is probable that the price level is lower under asymmetric information. The net effect on price depends on the elasticity of consumers' marginal willingness to pay for quality, ε. Suppose that $\varepsilon > 1$. In this case, consumers' marginal willingness to pay for quality increases substantially if quality decreases. This means that the deterioration of quality lowers price only a little. As a result, the increase in price that results from a decrease in output prevails against the decrease in price caused by the change in quality. Thus, Baron and Myersons' result holds: the price level is higher under asymmetric information. On the contrary, if $\varepsilon < 1$, consumers' marginal willingness to pay for quality increases only slightly given the lower level of quality. This implies that the monopoly must lower price substantially if it reduces quality. As a result, the decrease in quality makes the price low enough to outweigh the increase in price caused by a decrease in output, and consequently the price level is lower under asymmetric information than under symmetric information.

Second, we derive the effects of unverifiability of quality on regulatory contracts by comparing the asymmetric-verifiable case with the asymmetric-unverifiable case. In the case of asymmetric information and verifiable quality, the regulator lowers the levels of both output and quality to reduce a monopoly's informational rents. When quality is unverifiable, however, the regulator cannot decrease the level of quality to reduce monopoly rents. Thus, the level of quality is higher in the asymmetric-unverifiable case than in the asymmetric-verifiable case. In our model where the inverse demand function is separable in output, x, and quality, q, the delivered level of quality under asymmetric information and unverifiable quality is the same as under symmetric information. Given that the monopoly will select a higher level of quality under unverifiable quality, the regulator has to make the level of output much lower when quality is unverifiable than when verifiable to reduce the monopoly rents. Since unverifiable quality raises the quality level and lowers the output level, the price level becomes higher under unverifiable quality. The price level in the asymmetric-unverifiable case is also higher than in the symmetric case, because the quality level in the two cases is the same but the output level is lower in the former.

Whether or not the level of production capacity is higher in the asymmetric-unverifiable case than in the asymmetric-verifiable case depends on both the price and the quality elasticities of demand. If the price elasticity of demand is small enough to satisfy $\eta_p < \dfrac{p(1+\eta_q)}{p-a}$, the level of production capacity is higher when quality is unverifiable than when verifiable. As stated above, the price level is higher under unverifiable quality. When the price elasticity of demand is small, the regulator cannot reduce output by too much, even if price increases. That is, x^2 is not much smaller than x^1. Thus, the effect of $q^2 > q^1$ exceeds that of $x^1 > x^2$, and consequently the level of production capacity is higher when quality is unverifiable. On the contrary, suppose that the elasticity is so large that $\eta_p > \dfrac{p(1+\eta_q)}{p-a}$. Since the price level is higher under unverifiable quality, the regulator has to reduce the level of a monopoly's output substantially when quality is unverifiable, i.e., x^2 is much smaller than x^1. In this case, the effect of $x^1 > x^2$ outweighs that of $q^2 > q^1$, and as a result, the level of production capacity is lower when quality is unverifiable than when verifiable.

Note that when the price elasticity of demand is no larger than 1, $\eta_p < \dfrac{p(1+\eta_q)}{p-a}$ always holds, because $\dfrac{p}{p-a} > 1$ and $\eta_q > 0$ implies $\dfrac{p(1+\eta_q)}{p-a} > 1$. In this case, the level of production capacity is always higher when quality is unverifiable than when verifiable. The price elasticity of the services supplied by public utilities would usually be no larger than 1[5]. Thus, unverifiability of quality would make the level of production capacity higher.

Finally, we focus on consumer surplus and monopoly profits. The level of net consumer surplus, S^n, is always higher when quality is verifiable than when unverifiable. Unverifiability of quality leaves the regulator unable to implement regulation based on quality, and as a result, the regulator has to lower a monopoly's informational rents by using output only. This limited ability on the part of the regulator magnifies distortions which result in a lower level of net consumer surplus when quality is unverifiable.

Recall that the level of production capacity determines a monopoly's profits. Suppose common public utilities provide services whose price elasticity is no larger than 1. In this case, the level of production capacity is higher in the asymmetric-unverifiable case than in the asymmetric-verifiable case. This means that unverifiability of quality makes the level of the monopoly's profits higher.

If the regulator places a high value on net consumer surplus, S^n, the regulator will want quality to be verifiable because the level of net consumer surplus is higher in the asymmetric-verifiable case than in the asymmetric-unverifiable case. Thus, the regulator has an incentive to make quality verifiable. On the contrary, for standard public utilities with the price elasticity of their services no larger than 1, the levels of profits are higher

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[5] For instance, in telecommunications, Taylor's (1994) survey suggests that the price elasticity for short-haul toll calls of 15 to 40 miles is in the range of 0.25-0.5. In electricity supply, the recent survey by Waverman (1992) tells us that the short-run price elasticity for residential electricity demand ranges from 0.12 to 0.79.

when quality is unverifiable. That is, verifiable quality lowers the profit level. It therefore follows that standard public utilities, unlike the regulator, have no incentives to reveal quality levels. This result suggests that regulation based on the level of quality is intrinsically hard to implement because the regulator and the public utilities have opposing preferences for verifiability of quality.

6 An Extension by a Simulation

We have so far analyzed the case where the inverse demand function is separable in x and q. In this case, as first shown by Spence (1975), the level of quality under profit maximization of a monopoly yields the same level of quality under social welfare maximization. Recall that the monopoly chooses the level of quality to maximize its profits if quality is unverifiable. Thus, the level of quality in the asymmetric-unverifiable case is the same as the level in the symmetric case. That is, the result of $q^* = q^2$ stems from our separable inverse demand function. We now consider the case where x and q are inseparable in the inverse demand function.

In the case of an inseparable inverse demand function, we cannot derive straightforward results by using a theoretical model; thus, we use a numerical simulation method. For the sake of analytical simplicity, we assume that β is $\overline{\beta}$ with a probability of 0.5 and $\underline{\beta}$ with a probability of 0.5[6]. The following simple functions and parameters are used in our simulation.

(Case 1) $p = -x + q^{0.5} + \gamma xq + 5$, $C = \beta xq$, $\overline{\beta} = 0.6$, $\underline{\beta} = 0.4$

[6] If we assume such a probability distribution, 1) – 5) of Proposition 3 holds for $\overline{\beta}$ and 6) holds for $\underline{\beta}$.

(Case 2) $p = -x - q^{-0.5} + \gamma xq + 5$, $C = \beta xq$, $\overline{\beta} = 0.6$, $\underline{\beta} = 0.4$

Case 1 is the case where $\varepsilon < 1$, while Case 2 is the case where $\varepsilon > 1$. In the ensuing simulation, the price elasticity of demand, η_p, will be always no larger than 1, and consequently, $\eta_p < \frac{p(1+\eta_q)}{p-a}$ will always hold. We try three different values for γ (0, 0.01, and -0.01) and two different values for α (0.51 and 1).

Spence (1975) points out that given x, a monopoly oversupplies quality compared to social welfare maximization when $p_{xq} > 0$ (i.e., $\gamma > 0$ in our setting) and undersupplies it when $p_{xq} < 0$ (i.e., $\gamma < 0$ in our setting.) The results of our simulation, which is presented in Table 1, shows that when $p_{xq} \neq 0$, this oversupply or undersupply of quality modifies our results derived in Proposition 3.

First, in the case of $p_{xq} = 0$, the results are the same as in Proposition 3, for any value of α. This is because there exists no oversupply or undersupply of quality when $p_{xq} = 0$.

Second, we consider the case of $p_{xq} > 0$, where the monopoly oversupplies quality. The effect of an oversupply of quality causes $q^2 > q^*$, which results in $Q^2 > Q^*$ for $\alpha = 0.51$ in Cases 1-2 and 2-2. When $\alpha = 1$, however, this result is not robust. In Case 2-2, $Q^* > Q^2$ holds for $\alpha = 1$, which is the same results as in Proposition 3. We can interpret these results as follows. The results we show in Proposition 3 stem from the rent extraction by the regulator. When $\alpha = 0.51$, the regulator does not need to reduce a monopoly's profits so harshly, because consumer surplus and profits are almost equally important for the regulator. Thus, the effects from the rent extraction in

Proposition 3 are so small as to be counteracted by the effect of an oversupply of quality pointed out by Spence (1975), i.e., the effect of $q^2 > q^*$ exceeds that of $x^* > x^2$ and consequently $Q^2 > Q^*$. On the contrary, when $\alpha = 1$, the profits of the monopoly are totally meaningless to the regulator. Therefore, the regulator has to significantly lower the rents that the monopoly gains. In this case, the effects from the rent extraction in Proposition 3 are so strong that they possibly outweigh the overprovision of quality, i.e., the effect of $x^* > x^2$ is possibly larger than that of $q^2 > q^*$ and as a result, $Q^* > Q^2$. This simulation result suggests that the results in Proposition 3, exclusive of $q^* = q^2$, possibly hold even in the case of $p_{xq} > 0$ if α is sufficiently close to 1.

Third, we consider the case of $p_{xq} < 0$. In this case, the same explanation as the case of $p_{xq} > 0$ holds. When $\alpha = 0.51$, the underprovision of quality as a result of monopoly profit maximization in the case of unverifiable quality is significant, while its underprovision due to the rent extraction in the case of verifiable quality is minor. Thus, $q^* > q^1 > q^2$ holds. This undersupply of the unverifiable quality causes $p^* > p^1 > p^2$ in Case 1-3, $p^1 > p^* > p^2$ in Case 2-3, and $Q^* > Q^1 > Q^2$ and $\pi^1 > \pi^2 > \pi^*$ in both cases. When $\alpha = 1$, however, the regulator wants to reduce a monopoly's profits drastically. Thus, the regulator has to lower the level of quality under verifiable quality when $\alpha = 1$ compared to when $\alpha = 0.51$. This causes $q^* > q^2 > q^1$, which yields the same results as in Proposition 3 exclusive of $q^* = q^2$.

The above simulation shows that the effect of oversupply or undersupply of quality is added to the results in Proposition 3 when $p_{xq} \neq 0$. As the regulator gives greater weight to total net consumer surplus, i.e., as α is closer to 1, however, the effects in

Proposition 3 become stronger, because the regulator reduces the monopoly's profits. Thus, when α is sufficiently close to 1, the effects in Proposition 3 possibly prevails over the effect of an oversupply or undersupply of quality, even if $p_{xq} \neq 0$.

7 Relation between Our Results and Others in the Literature

In this section, we compare our results with others in the literature. First, Beitia (1995) extends the model of Baron and Myerson (1982), taking into account product quality, which is assumed to be verifiable. The result he obtains is that the levels of output and quality are lower under asymmetric information and verifiable quality than under symmetric information if quantity and quality are net complements ($\frac{\partial(p-C_x)}{\partial q} > 0$ in his definition). In our analysis, under asymmetric information and verifiable quality, quantity and quality are also net complements because of $\frac{\partial(p-C_x)}{\partial q} = v'(q) - \beta = k > 0$ for $\beta \in (\underline{\beta}, \overline{\beta}]$. Thus, our results, which state that the levels of output and quality are lower under asymmetric information and verifiable quality than under symmetric information, are consistent with his result. In his analysis, however, the price level is not considered. Our analysis of the case of asymmetric information and verifiable quality differs from his in that we derive the effect of asymmetric information on the price level focusing on the fact that a decrease in output raises prices but a decrease in quality lowers prices.

Second, Laffont and Tirole (1991) analyze the regulation of quality, assuming that quality is unverifiable and that the inverse demand function is linear in output and quality.

In their study, if quantity and quality are net complements ($\frac{\partial(p-C_x)}{\partial q} > 0$ in their definition), the level of quality is lower under asymmetric information and unverifiable quality than under symmetric information. On the contrary, if they are net substitutes ($\frac{\partial(p-C_x)}{\partial q} < 0$), the results are reversed. In our analysis under asymmetric information and unverifiable quality, quality is "independent" of quantity, i.e., is not a net complement or a net substitute of quantity, because $\frac{\partial(p-C_x)}{\partial q} = v'(q) - \beta = 0$. Applying the results of Laffont and Tirole (1991) to our model, we know that the level of quality under asymmetric information and unverifiable quality is the same as under symmetric information. Our results derived in Proposition 3 indeed indicate $q^2 = q^*$, which is consistent with their results. The main focus of their analysis, however, is on the difference between whether consumers know the level of quality before or after purchasing the product. Thus, they do not derive the effects of the unverifiability of quality itself; in this paper, these effects are derived by comparing the case of unverifiable quality with that of verifiable quality.

Third, Lewis and Sappington (1991) consider both the cases of verifiable and unverifiable quality and compare them with each other. As they themselves note, however, their focus is to derive a simple model in which the regulator and a monopoly both prefer verifiable quality to unverifiable quality. A feature of their model is that the monopoly's efforts improve the level of quality. In so doing, they assume that the marginal production costs, exclusive of disutilities of the efforts, do not vary with the level of quality, and that the regulator and the monopoly share the same perfect knowledge of production costs exclusive of disutilities of the efforts. They state that if one of these assumptions is relaxed, their main result, i.e., that both the regulator and the monopoly want quality to be verifiable, may not hold. Our analysis deals with the case where their assumptions are not satisfied. In our analysis, the marginal service costs, $C_x = a + \beta q$, depend on the level of quality and the cost parameter of production capacity that is known only by the monopoly. By relaxing the assumptions in Lewis and Sappington (1991) to construct a model better suited to public utilities, we obtain the following results different from theirs: 1) the level of quality is higher when quality is unverifiable than when verifiable, and 2) if the price elasticity of demand is small, the level of the profits of a monopoly is higher when quality is unverifiable than when verifiable and consequently the monopoly prefers quality to be unverifiable.

8 Concluding Remarks

In this paper, we show that unverifiability of quality raises the level of quality, lowers output, and consequently raises price. In the case of common public utilities with the price elasticity no larger than 1, unverifiable quality increases the level of production capacity, and as a result, increases the level of a monopoly's profits. Unverifiable quality, however, always lowers the level of net consumer surplus. Thus, the regulator wants quality to be verifiable, but the monopoly wants quality to be unverifiable. These opposing preferences for verifiability of quality would make the regulation based on the level of quality difficult to implement.

Finally, we comment on two issues.

First, in our paper, asymmetric information stems from the cost parameters of production capacity. As analyzed in Lewis and Sappington (1988a), (1988b), and

(1992), however, there exist cases in which a monopoly has superior information about the demand function. Analyses including demand unknown to the regulator deserve future research.

Second, our analysis uses a static model. If we set up a dynamic model, however, we would have to consider the regulator's commitment power, which is pointed out by Dag (1997). His analysis shows that the unverifiability of quality does not hurt the regulator if the regulator's commitment power is limited. In a dynamic framework, the reputation effect of quality, which is analyzed by Laffont and Tirole (1991), is also important. It is delegated to future research to show how these effects alter our results using a dynamic framework.

Appendix: Proofs of Lemmas and Propositions

Proof of Lemma 1

Since $\frac{1}{2} < \alpha \leq 1$, the regulator puts more weight on consumer surplus. Therefore, the regulator sets a monopoly's profits at zero. That is, we obtain (5):

$$\pi^*(\beta) = 0. \tag{A1}$$

Substituting (A1) into (1) yields

$$T = p(x,q)x - C(x,qx,\beta). \tag{A2}$$

Substituting (A1) and (A2) into (2) gives us

$$\begin{aligned} W &= \alpha\left\{S^n(x,q) + p(x,q)x - C(x,qx,\beta)\right\} \\ &= \alpha\left\{S^g(x,q) - C(x,qx,\beta)\right\}. \end{aligned} \tag{A3}$$

Maximizing (A3) with respect to x and q, we obtain the following first order conditions.

$$\frac{\partial W}{\partial x} = \alpha\{p - a - \beta q\} = 0 \tag{A4}$$

$$\frac{\partial W}{\partial q} = \alpha\{v'(q)x - \beta x\} = 0 \tag{A5}$$

Rewriting (A4) and (A5) yields (3) and (4).

Q.E.D.

Proof of Proposition 1

We solve the maximization problem (P1), following the standard procedure in the optimal regulation literature.

First, we tackle the incentive compatibility constraint. $\pi(\beta,\beta')$ denotes the profits that a monopoly with cost parameter β gains when it reports to the regulator that its cost parameter is β'.

For truth-telling, the first order condition is

$$\pi_{\beta'}(\beta,\beta')\Big|_{\beta'=\beta}=0, \tag{A6}$$

and the second order condition is

$$\pi_{\beta'\beta'}(\beta,\beta')\Big|_{\beta'=\beta}\le 0. \tag{A7}$$

From (A6), we obtain

$$\dot{\pi}(\beta)=-Q(\beta)<0, \tag{A8}$$

where $\bullet$ denotes a partial derivative with respect to β. From (A7), we have

$$\dot{Q}(\beta)\le 0. \tag{A9}$$

For the moment, we neglect (A9).

Next, we transform the individual rationality constraint. From (A8), we can replace the individual rationality constraint by

$$\pi(\overline{\beta})=0. \tag{A10}$$

We set up a Hamiltonian as follows:

$$H=\left[\alpha\left\{S^{g}(x(\beta),q(\beta))-C(x(\beta),x(\beta)q(\beta),\beta)\right\}+(1-2\alpha)\pi(\beta)\right]f(\beta)-\mu(\beta)x(\beta)q(\beta) \tag{A11}$$

where $\pi(\beta)$ is the state variable, $\mu(\beta)$ is the costate variable associated with $\pi(\beta)$, and $x(\beta)$ and $q(\beta)$ are the control variables. By the maximum principle,

$$\dot{\mu}(\beta)=-\frac{\partial H}{\partial\pi(\beta)}=-(1-2\alpha)f(\beta). \tag{A12}$$

Since the boundary $\beta=\underline{\beta}$ is unconstrained, the transversality condition at $\beta=\underline{\beta}$ is

$$\mu(\underline{\beta})=0. \tag{A13}$$

Integrating (A12) yields

$$\mu(\beta)=(2\alpha-1)F(\beta). \tag{A14}$$

Maximizing the Hamiltonian, (A11), with respect to $x(\beta)$ and $q(\beta)$, we obtain

$$\frac{\partial H}{\partial x(\beta)} = \alpha(p - a - \beta q(\beta))f(\beta) - \mu(\beta)q(\beta) = 0, \tag{A15}$$

$$\frac{\partial H}{\partial q(\beta)} = \alpha(v'(q(\beta))x(\beta) - \beta x(\beta))f(\beta) - \mu(\beta)x(\beta) = 0. \tag{A16}$$

Defining k as $k \equiv \dfrac{(2\alpha - 1)F(\beta)}{\alpha f(\beta)} \geq 0$, we obtain (6) and (7) from (A14) - (A16). Now we check the second order condition of the incentive compatibility constraint, (A9). Totally differentiating and rewriting (6) and (7) gives us

$$\dot{x}(\beta) = \frac{(1 + k_\beta)q}{u'(x)} < 0, \tag{A17}$$

$$\dot{q}(\beta) = \frac{1 + k_\beta}{v''(q)} < 0, \tag{A18}$$

where $k_\beta > 0$ from the monotone-hazard-rate property. Since $\dot{Q}(\beta) = x(\beta)\dot{q}(\beta) + \dot{x}(\beta)q(\beta)$, we obtain $\dot{Q}(\beta) < 0$ from (A17) and (A18). Therefore, (A9) is always satisfied.

Lastly, rewriting (A8) and (A10) yields (8).

Q.E.D.

Proof of Proposition 2

In the same way as the proof of Proposition 1, the true cost parameter of a monopoly is β and the cost parameter that the monopoly reports is β'. The monopoly chooses q to maximize its profits, given $x(\beta')$ and $T(\beta')$ the regulator designates based on the reported β'. The monopoly faces the following profit-maximizing problem:

$$\max_q \ \pi(\beta, \beta') = p(x(\beta'), q)x(\beta') - ax(\beta') - \beta x(\beta')q.$$

Solving this problem, we obtain (10):

$$v'(q) = \beta . \tag{A19}$$

Rewriting (A19) yields

$$q = q(\beta) . \tag{A20}$$

We consider the incentive compatibility constraint. The first order condition for truth-telling is (A6), and the second order condition is (A7). Thus, we obtain

$$\dot{\pi}(\beta) = -Q(\beta) < 0 \tag{A21}$$

as the first order condition and

$$\dot{x}(\beta) \leq 0 \tag{A22}$$

as the second order condition. For the moment, we neglect (A22).

Next, we transform the individual rationality constraint. From (A21), we can replace the individual rationality constraint by

$$\pi(\overline{\beta}) = 0 . \tag{A23}$$

We set up a Hamiltonian as follows.

$$H = \left[\alpha\left\{S^g(x(\beta), q(\beta)) - C(x(\beta), x(\beta)q(\beta), \beta)\right\} + (1-2\alpha)\pi(\beta)\right]f(\beta) - \mu(\beta)x(\beta)q(\beta) \tag{A24}$$

where $\pi(\beta)$ is the state variable, $\mu(\beta)$ is the costate variable associated with $\pi(\beta)$, and $x(\beta)$ is the control variable. By the maximum principle,

$$\dot{\mu}(\beta) = -\frac{\partial H}{\partial \pi(\beta)} = -(1-2\alpha)f(\beta) . \tag{A25}$$

Since the boundary $\beta = \underline{\beta}$ is unconstrained, the transversality condition at $\beta = \underline{\beta}$ is

$$\mu(\underline{\beta}) = 0 . \tag{A26}$$

Integrating (A25) gives us

$$\mu(\beta) = (2\alpha - 1)F(\beta). \tag{A27}$$

Maximizing the Hamiltonian, (A24), with respect to $x(\beta)$, we obtain

$$\frac{\partial H}{\partial x(\beta)} = \alpha(p - a - \beta q(\beta))f(\beta) - \mu(\beta)q(\beta) = 0. \tag{A28}$$

Rewriting (A27) and (A28) yields (9). Now we check the second order condition of the incentive compatibility constraint, (A22). By totally differentiating and rewriting (9) and (10), we have

$$\dot{x}(\beta) = \frac{k + (1 + k_\beta)v''(q)q}{u'(x)v''(q)}. \tag{A29}$$

Because of $u'(x) < 0$ and $v''(q) < 0$,

$$k + (1 + k_\beta)v''(q)q \leq 0 \tag{A30}$$

must hold to satisfy (A22). Otherwise, "bunching" may occur. Since "bunching" causes us unessential complications, (A30) is assumed to be hold in our analysis. (See Guesnerie and Laffont (1984) for the derivation of the solution in the case where bunching exists.)

Lastly, rewriting (A21) and (A23) yields (11).

Q.E.D.

Proof of Proposition 3

Now $\beta \in (\underline{\beta}, \overline{\beta})$ is given, and consequently, k is given. First, we focus on q. From (4) and (7), we obtain

$$v'(q^1) = \beta + k > \beta = v'(q^*). \tag{A31}$$

Because of $v''(q) < 0$, we have

$$q^* > q^1. \tag{A32}$$

From (4) and (10), we obtain

$$v'(q^2) = \beta = v'(q^*), \tag{A33}$$

which shows

$$q^2 = q^*. \tag{A34}$$

From (A32) and (A34), we obtain (13):

$$q^* = q^2 > q^1. \tag{A35}$$

Second, we concentrate on x. From $u^{-1'} < 0$, $v''(q) < 0$, (3), (4), (6), (7) and (A35),

$$\begin{aligned} x^* - x^1 &= u^{-1}(a + \beta q^* - v(q^*)) - u^{-1}(a + (\beta + k)q^1 - v(q^1)) \\ &= u^{-1}(a + v'(q^*)q^* - v(q^*)) - u^{-1}(a + v'(q^1)q^1 - v(q^1)) \\ &= \int_{q^1}^{q^*} u_q^{-1} dq \\ &= \int_{q^1}^{q^*} u^{-1'} v''(q) q dq > 0 \end{aligned} \tag{A36}$$

holds with respect to x^* and x^1. In the same way, regarding x^1 and x^2, from $u^{-1'} < 0$, $v''(q) < 0$, (6), (9), and (A35), we obtain

$$\begin{aligned} x^1 - x^2 &= u^{-1}(a + (\beta + k)q^1 - v(q^1)) - u^{-1}(a + (\beta + k)q^2 - v(q^2)) \\ &= \int_{q^2}^{q^1} u_q^{-1} dq \\ &= \int_{q^2}^{q^1} u^{-1'} \{(\beta + k) - v'(q)\} dq > 0. \end{aligned} \tag{A37}$$

(A36) and (A37) yields (12):

$$x^* > x^1 > x^2. \tag{A38}$$

Third, we focus on p. From (3), (9), and (A35), we obtain

$$p^2 = a + (\beta + k)q^2 = a + (\beta + k)q^* > a + \beta q^* = p^*. \tag{A39}$$

From (6), (9), and (A35), we also have

$$p^2 = a + (\beta + k)q^2 > a + (\beta + k)q^1 = p^1. \tag{A40}$$

Moreover, rewriting (3), (4), (6), and (7) gives us

$$\begin{aligned} p^* - p^1 &= a + \beta q^* - a - (\beta + k)q^1 \\ &= v'(q^*)q^* - v'(q^1)q^1 \\ &= \int_{q^1}^{q^*} \{v'(q)(1-\varepsilon)\} dq, \end{aligned} \tag{A41}$$

which shows

$$p^* > p^1 \text{ if } \varepsilon < 1 \text{ and } p^1 > p^* \text{ if } \varepsilon > 1. \tag{A42}$$

From (A39), (A40), and (A42), we obtain (14):

$$p^2 > p^* > p^1 \text{ if } \varepsilon < 1 \text{ and } p^2 > p^1 > p^* \text{ if } \varepsilon > 1. \tag{A43}$$

Fourth, we focus on Q. From (A35) and (A38), we have

$$Q^* > Q^1 \text{ and } Q^* > Q^2. \tag{A44}$$

With respect to Q^1 and Q^2, in the same way as we derive (A37), we obtain

$$\begin{aligned} Q^1 - Q^2 &= q^1 u^{-1}(a + (\beta + k)q^1 - v(q^1)) - q^2 u^{-1}(a + (\beta + k)q^2 - v(q^2)) \\ &= \int_{q^2}^{q^1} \{q u^{-1}(a + (\beta + k)q - v(q))\}' dq \\ &= \int_{q^2}^{q^1} x \left\{ \frac{p(1+\eta_q)}{p-a} - \eta_p \right\} dq. \end{aligned} \tag{A45}$$

Since $q^2 > q^1$ from (A35), we have

$$Q^2 > Q^1 \text{ if } \eta_p < \frac{p(1+\eta_q)}{p-a} \text{ and } Q^1 > Q^2 \text{ if } \eta_p > \frac{p(1+\eta_q)}{p-a}. \tag{A46}$$

From (A44) and (A46), we obtain (15):

$$Q^* > Q^2 > Q^1 \text{ if } \eta_p < \frac{p(1+\eta_q)}{p-a} \text{ and } Q^* > Q^1 > Q^2 \text{ if } \eta_p > \frac{p(1+\eta_q)}{p-a}. \tag{A47}$$

Fifth, we concentrate on net consumer surplus. First, note that

$$S^n(p,q) \equiv S^g(x(p,q),q) - px(p,q) = \int_p^\infty x(\tilde{p},q) d\tilde{p}. \tag{A48}$$

With respect to S^{n*} and S^{n1}, from $v''(q) < 0$, $S^n_p = -x$, $S^n_q = v'(q)x$, (3), (4), (6), (7), and (A35), we obtain

$$\begin{aligned} S^{n*}(p^*,q^*)-S^{n1}(p^1,q^1) &= S^{n*}(a+v'(q^*)q^*,q^*)-S^{n1}(a+v'(q^1)q^1,q^1) \\ &= \int_{q^1}^{q^*}\left[\frac{dS^n}{dq}\right]dq \\ &= \int_{q^1}^{q^*}\left[S_p^n\frac{d\{a+v'(q)q\}}{dq}+S_q^n\right]dq \\ &= \int_{q^1}^{q^*}(-xv''(q)q)dq>0. \end{aligned} \tag{A49}$$

In the same way, we compare S^{n1} with S^{n2}. From $v''(q)<0$, (6), (9), and (A35), we obtain

$$\begin{aligned} S^{n1}(p^1,q^1)-S^{n2}(p^2,q^2) &= S^{n1}(a+(\beta+k)q^1,q^1)-S^{n2}(a+(\beta+k)q^2,q^2) \\ &= \int_{q^2}^{q^1}\left[\frac{dS^n}{dq}\right]dq \\ &= \int_{q^2}^{q^1}\left[S_p^n(\beta+k)+S_q^n\right]dq \\ &= \int_{q^2}^{q^1}x\{v'(q)-(\beta+k)\}dq>0. \end{aligned} \tag{A50}$$

From (A49) and (A50), we have (16):

$$S^{n*}>S^{n1}>S^{n2}. \tag{A51}$$

Lastly, we focus on π. From (A46), if $\eta_p<\dfrac{p(1+\eta_q)}{p-a}$, $Q^2>Q^1$ holds, in which case

$$\int_{\underline{\beta}}^{\overline{\beta}}Q^2(\tilde{\beta})d\tilde{\beta}>\int_{\underline{\beta}}^{\overline{\beta}}Q^1(\tilde{\beta})d\tilde{\beta}>0. \tag{A52}$$

From (A52) and (5), we obtain

$$\pi^2>\pi^1>\pi^*. \tag{A53}$$

In the same way, if $\eta_p>\dfrac{p(1+\eta_q)}{p-a}$, $Q^1>Q^2$ which implies

$$\int_{\underline{\beta}}^{\overline{\beta}}Q^1(\tilde{\beta})d\tilde{\beta}>\int_{\underline{\beta}}^{\overline{\beta}}Q^2(\tilde{\beta})d\tilde{\beta}>0. \tag{A54}$$

From (A54) and (5), we have

$$\pi^1 > \pi^2 > \pi^*. \tag{A55}$$

From (A53) and (A55), we obtain (17):

$$\pi^2(\beta) > \pi^1(\beta) > \pi^*(\beta) \text{ if } \eta_p < \frac{p(1+\eta_q)}{p-a} \text{ and } \pi^1(\beta) > \pi^2(\beta) > \pi^*(\beta) \text{ if } \eta_p > \frac{p(1+\eta_q)}{p-a} \tag{A56}$$

Q.E.D.

Case 1-1： $p=-x+q^{0.5}+5$

α	0.51			1		
x	x^* 5.4167	x^1 5.4113	x^2 5.4112	x^* 5.4167	x^1 5.3126	x^2 5.2778
q	q^* 0.6944	q^2 0.6944	q^1 0.6766	q^* 0.6944	q^2 0.6944	q^1 0.3906
Q	Q^* 3.7614	Q^2 3.7575	Q^1 3.6613	Q^* 3.76136	Q^2 3.6649	Q^1 2.0751
p	p^2 0.4221	p^* 0.4167	p^1 0.4112	p^2 0.5556	p^* 0.4167	p^1 0.3124
S^n	S^{n*} 14.6702	S^{n1} 14.6412	S^{n2} 14.6407	S^{n*} 14.6702	S^{n1} 14.1116	S^{n2} 13.9274
π	π^2 0.7516	π^1 0.7323	π^* 0	π^2 0.733	π^1 0.415	π^* 0
η_p	η^2_p 0.0780	η^*_p 0.0769	η^1_p 0.0760	η^2_p 0.1053	η^*_p 0.0769	η^1_p 0.0588

Case 2-1： $p=-x-q^{-0.5}+5$

α	0.51			1		
x	x^* 3.406	x^1 3.3992	x^2 3.3992	x^* 3.406	x^1 3.2455	x^2 3.229
q	q^* 0.8855	q^2 0.8855	q^1 0.8779	q^* 0.8855	q^2 0.8855	q^1 0.7309
Q	Q^* 3.0160	Q^2 3.0100	Q^1 2.9842	Q^* 3.0160	Q^2 2.8593	Q^1 2.3721
p	p^2 0.5382	p^1 0.5335	p^* 0.5313	p^2 0.7083	p^1 0.5848	p^* 0.5313
S^n	S^{n*} 5.8005	S^{n1} 5.7773	S^{n2} 5.7772	S^{n*} 5.8005	S^{n1} 5.2666	S^{n2} 5.2133
π	π^2 0.6020	π^1 0.5968	π^* 0	π^2 0.5719	π^1 0.4744	π^* 0
η_p	η^2_p 0.1583	η^1_p 0.1569	η^*_p 0.1560	η^2_p 0.2194	η^1_p 0.1802	η^*_p 0.1560

Table 1： When $\gamma=0$

Case 1-2： $p = -x + q^{0.5} + 0.01xq + 5$

α	0.51			1		
x	x^* 5.4573	x^1 5.4506	x^2 5.4458	x^* 5.4573	x^1 5.3345	x^2 5.2537
q	q^2 0.8400	q^* 0.7622	q^1 0.7414	q^2 0.8341	q^* 0.7622	q^1 0.4180
Q	Q^2 4.5745	Q^* 4.1596	Q^1 4.0411	Q^2 4.3821	Q^* 4.1596	Q^1 2.2298
p	p^2 0.5165	p^* 0.4573	p^1 0.4509	p^2 0.7035	p^* 0.4573	p^1 0.3304
S^n	S^{n*} 14.7777	S^{n1} 14.7444	S^{n2} 14.7038	S^{n*} 14.7777	S^{n1} 14.1690	S^{n2} 13.6853
π	π^2 0.9149	π^1 0.8082	π^* 0	π^2 0.8764	π^1 0.4460	π^* 0
η_p	η_p^2 0.0940	η_p^* 0.0832	η_p^1 0.0821	η_p^2 0.1328	η_p^* 0.0832	η_p^1 0.0617

Case 2-2： $p = -x - q^{-0.5} + 0.01xq + 5$

α	0.51			1		
x	x^* 3.4369	x^1 3.4297	x^2 3.4288	x^* 3.4369	x^1 3.2698	x^2 3.2439
q	q^2 0.9210	q^* 0.9029	q^1 0.8948	q^2 0.9190	q^* 0.9029	q^1 0.7412
Q	Q^2 3.1579	Q^* 3.1032	Q^1 3.0689	Q^* 3.1032	Q^2 2.9811	Q^1 2.4236
p	p^2 0.5608	p^1 0.5439	p^* 0.5417	p^2 0.7427	p^1 0.5929	p^* 0.5417
S^n	S^{n*} 5.8525	S^{n1} 5.8287	S^{n2} 5.3062	S^{n*} 5.8525	S^{n1} 5.8242	S^{n2} 5.2131
π	π^2 0.6316	π^1 0.6138	π^* 0	π^2 0.5962	π^1 0.4847	π^* 0
η_p	η_p^2 0.1620	η_p^1 0.1572	η_p^* 0.1562	η_p^2 0.2268	η_p^1 0.1800	η_p^* 0.1562

Table 1： When $\gamma = 0.01$

Case 1-3： $p=-x+q^{0.5}-0.01xq+5$

α	0.51			1		
x	x^* 5.3817	x^1 5.3772	x^2 5.3760	x^* 5.3817	x^1 5.2928	x^2 5.2821
q	q^* 0.6361	q^1 0.6206	q^2 0.5849	q^* 0.6361	q^2 0.5866	q^1 0.3660
Q	Q^* 3.423299	Q^1 3.3371	Q^2 3.1444	Q^* 3.423299	Q^2 3.0985	Q^1 1.9372
p	p^* 0.3817	p^1 0.3772	p^2 0.3574	p^2 0.4528	p^* 0.3817	p^1 0.2928
S^n	S^{n*} 14.5733	S^{n1} 14.5469	S^{n2} 14.5351	S^{n*} 14.5733	S^{n1} 14.0580	S^{n2} 14.0322
π	π^1 0.6674	π^2 0.6289	π^* 0	π^2 0.6197	π^1 0.3874	π^* 0
η_p	η_p^* 0.0714	η_p^1 0.0706	η_p^2 0.0669	η_p^2 0.0862	η_p^* 0.0714	η_p^1 0.0555

Case 2-3： $p=-x-q^{-0.5}-0.01xq+5$

α	0.51			1		
x	x^* 3.3765	x^1 3.3701	x^2 3.3699	x^* 3.3765	x^1 3.2222	x^2 3.2124
q	q^* 0.8693	q^1 0.8621	q^2 0.8539	q^* 0.8693	q^2 0.8553	q^1 0.7213
Q	Q^* 2.9352	Q^1 2.9054	Q^2 2.8776	Q^* 2.9352	Q^2 2.7476	Q^1 2.3242
p	p^1 0.5239	p^* 0.5216	p^2 0.5191	p^2 0.6789	p^1 0.5771	p^* 0.5216
S^n	S^{n*} 5.7500	S^{n1} 5.7277	S^{n2} 5.7265	S^{n*} 5.7500	S^{n1} 5.2288	S^{n2} 5.2037
π	π^1 0.5811	π^2 0.5755	π^* 0	π^2 0.5495	π^1 0.4648	π^* 0
η_p	η_p^1 0.1568	η_p^* 0.1558	η_p^2 0.1554	η_p^2 0.2131	η_p^1 0.1804	η_p^* 0.1558

Table 1： When $\gamma=-0.01$

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