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Decomposition approach to the measurement of spatial segregation

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Abstract

This paper proposes a new method of measuring spatial segregation. Many of the existing measures of spatial segregation assume spatial data aggregated by spatial units. This leads to a problem called the modifiable areal unit problem, where measures are inevitably unstable and unreliable because of their sensitivity to the definition of spatial units. Another problem of the existing measures is that the relationship between the different dimensions of spatial segregation is not fully revealed and understood. This prevents us from extracting the distinct and independent dimensions of spatial segregation. To resolve the problem, this paper takes a decomposition approach to the measurement of spatial segregation. Our primary goal is to decompose the spatial segregation into mutually exclusive and independent components. We introduce three measures of spatial segregation each of which evaluates a different dimension of spatial segregation. To test the validity of the method, we apply it to three spatial datasets of different sizes. The result shows that the method is effective for evaluating the spatial segregation from three different perspectives.

1. Introduction

Spatial segregation is a fundamental concept in geographical information science. It refers to the spatial isolation of different types of spatial objects. Spatial segregation of racial and occupational groups has drawn much attention of geographers and sociologists. Spatial segregation of plant and animal species has been discussed in biology and ecology. Landuse mixture, the lack of spatial segregation, is a critical issue in urban and regional planning.

Numerous measures have been proposed in the literature to evaluate the degree of spatial segregation quantitatively. Most of them utilize spatial data aggregated by spatial units such as census tracts, zip-code areas, and school districts. Among those, one of the most commonly used measure is the dissimilarity index proposed by Duncan and Duncan (1955). Though its suitability has been debated for a long time (White, 1983), the index has been widely used in racial segregation analysis because of its ease of calculation and interpretation.

The dissimilarity index and its variants are aspatial measures in the sense that they are insensitive to the arrangement of spatial units. Spatial measures have been developed to address this limitation by explicitly considering the distribution of spatial units. Morgan's distance-based measure of segregation (1983) and White's spatial segregation indices (1983) calculate the distance between spatial units and incorporate it into the definition of their measures. Morrill (1991) and Wong (1993) introduced an additional component to the dissimilarity index, so the degree of segregation can be moderated by the arrangement of spatial units.

The above measures are all defined based on aggregated spatial data. Considering that spatial data are often provided in an aggregated form to keep the confidentiality of individuals these unit-based approaches can be useful to some extent. This, however, causes a problem called the modifiable areal unit problem. Since spatial measures are sensitive to the definition of spatial units, they are inevitably unstable and unreliable.

Reardon and O'Sullivan (2004) appropriately points out this problem and proposes new measures of spatial segregation. They define the measures directly from the location of individual points instead of using aggregated spatial data. When spatial data are available only in an aggregated form, the measures approximate the true values by assuming the uniform distribution of point in each unit. This assures that the measures converge to the true values with an increase in the resolution of spatial data and permits us to avoid the instability in the evaluation of spatial segregation.

Another significant contribution of Reardon and O'Sullivan (2004) is that it examines in detail the five dimensions advocated by Massey and Denton (1988) and re-conceptualizes into two fundamental dimensions. They claim that the distinction between the dimension of evenness and that of clustering is arbitrary and propose two dimensions called the spatial exposure and the spatial evenness as principal and distinct dimensions of spatial segregation.

Their approach, however, has several problems that remain to be resolved. First, they did not define the spatial exposure and evenness in a formal way. Though their discussion is adequate and persuasive, they did not indicate a precise definition of the two dimensions. Second, the independency between the two dimensions is not fully discussed. They argue that the dimensions are distinct, but do not give a tangible proof of the independency between exposure and evenness measures. Third, the range of the segregation measures is not completely shown. This causes a difficulty in comparing the spatial segregation between different patterns because we cannot standardize the measures to obtain relative and comparative ones.

To resolve the above problems, this paper proposes a new method of measuring spatial segregation. Following the line of Reardon and O'Sullivan (2004), we discuss the spatial segregation of point objects whose exact location is assumed to be known. Our aim is to decompose the spatial segregation into different components that are mutually exclusive and independent. Section 2 defines several measures of evaluating the spatial segregation. Section 3 calculates the measures on three spatial datasets of different sizes. Section 4 summarizes the conclusions with discussion.

2. Method

Spatial segregation has several different aspects. Suppose the four distributions of white and black points shown in Figure 1. The same number of white and black points are distributed in Figure 1a and Figure 1b, while black points are fewer than white points in Figure 1c and Figure 1d. White and black points are uniformly distributed in Figure 1a and Figure 1c, while they are spatially separated in Figure 1b and Figure 1d.

Figure 1b and Figure 1d present complete spatial segregation of white and black points. Points are highly mixed in Figure 1a. In Figure 1c, though white and black points are globally mixed, a local segregation is observed due to the inequality in the number of points. Many white points do not have black neighborhoods except those located close to black ones.

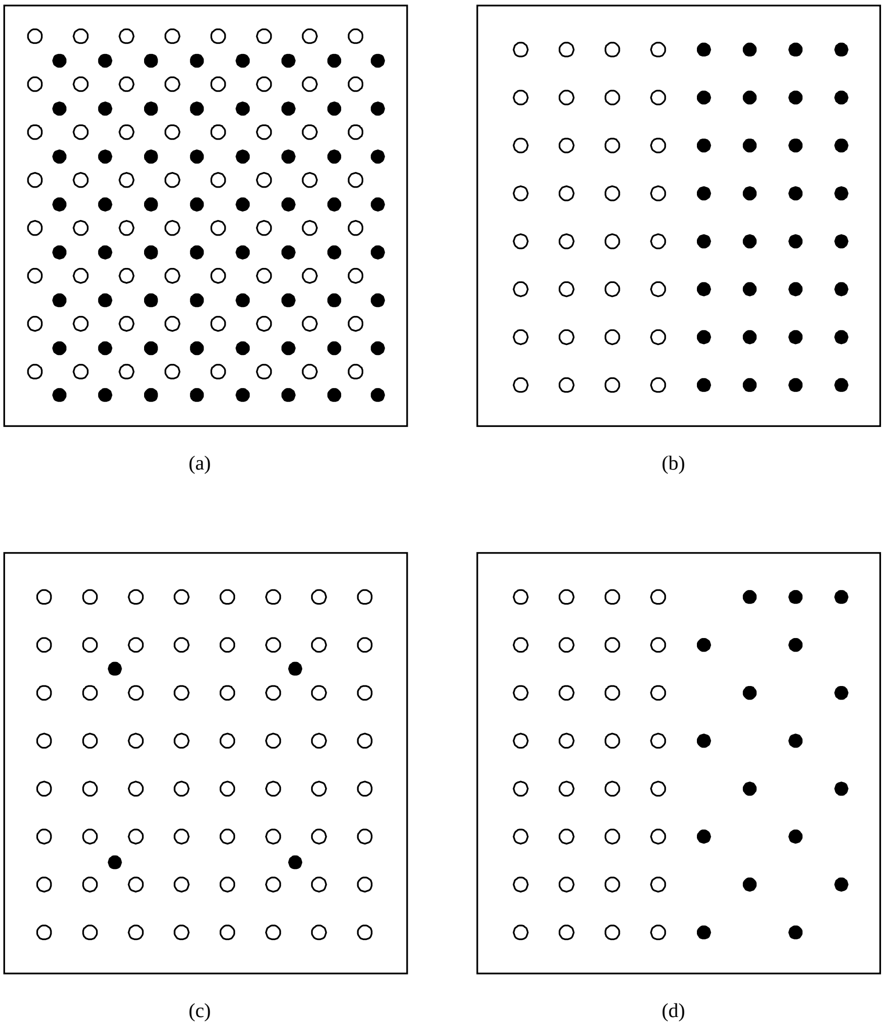

Figure 1 Four distributions of white and black points.

This implies that spatial segregation depends on at least the difference in location and that in number of points. This distinction almost corresponds to the spatial evenness and exposure advocated by Reardon and O'Sullivan (2004). This paper, however, further discusses spatial exposure from a different perspective. Figure 2 indicates the distributions of individuals of different species. If a white circle moves around its neighborhood, it is more probable to meet an individual of a different species in Figure 2b than in Figure 2a. Spatial exposure depends on the difference in not only the number but also the diversity of points.

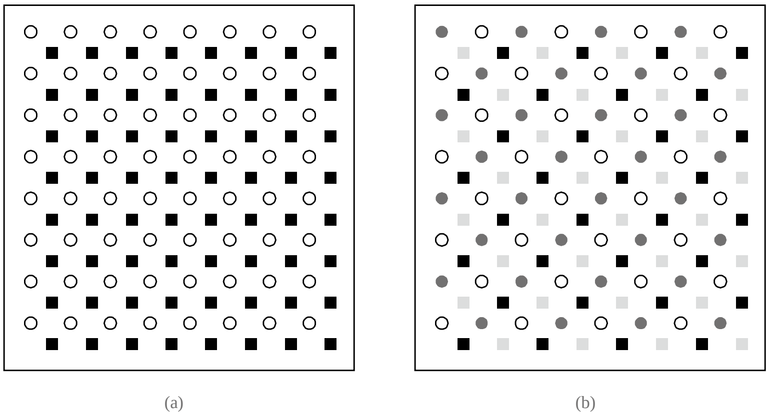

Figure 2 Distributions of individuals. Different symbols indicate different species.

The above observation suggests three factors of spatial segregation: locational unevenness, compositional unevenness, and qualitative homogeneity. The locational unevenness causes a difference in location of points, while the compositional unevenness increases the difference in the number of points, and consequently, both increase spatial segregation. The qualitative homogeneity decreases the variation of points, which also increases spatial segregation. We discuss these three factors in a more formal and quantitative way in the following.

2.1 Evaluation of spatial segregation

Suppose K types of points distributed in region R. We denote jth point of type i and its location as P_{ij} and $\mathbf{z}_{ij}$, respectively. Let $\Lambda_i=\{P_{i1}, P_{i2}, ..., P_{iN_i}\}$ be the set of type i points. The total number of points is N (=N_1+ N_2+... + N_K).

Let $\phi(\mathbf{x}, \mathbf{z}_{ij})$ be a proximity function that indicates the spatial proximity between location $\mathbf{x}$ and point P_{ij}. One definition of $\phi(\mathbf{x}, \mathbf{z}_{ij})$ is a distance-decay function such as

$$\phi\left(\mathbf{x},\mathbf{z}_{ij}\right)=\exp\left(-\alpha\left|\mathbf{x}-\mathbf{z}_{ij}\right|\right) . \tag{1}$$

We may also employ the Voronoi diagram, where points of the same type serve as generators. The proximity function $\phi(\mathbf{x}, \mathbf{z}_{ij})$ is

$$\phi\left(\mathbf{x},\mathbf{z}_{ij}\right)=\begin{cases}1 & \text{if } \mathbf{x}\in V_{ij}\\ 0 & \text{otherwise}\end{cases} , \tag{2}$$

where V_{ij} is the Voronoi region of P_{ij}.

Using the proximity function, we define the density functions of points. Though they are spatially-weighted, we simply call them *density functions* in the following. The density of P_{ij} at location **x** is defined as

$$\zeta_{ij}(\mathbf{x}) = \frac{\phi(\mathbf{x}, \mathbf{z}_{ij})}{\int_{\mathbf{y}\in R} \phi(\mathbf{y}, \mathbf{z}_{ij}) d\mathbf{y}} . \tag{3}$$

The density function of type *i* points is given by

$$\begin{aligned} D_i(\mathbf{x}) &= \sum_j \zeta_{ij}(\mathbf{x}) \\ &= \sum_j \frac{\phi(\mathbf{x}, \mathbf{z}_{ij})}{\int_{\mathbf{y}\in R} \phi(\mathbf{y}, \mathbf{z}_{ij}) d\mathbf{y}} . \end{aligned} \tag{4}$$

We can confirm

$$\int_{\mathbf{x}\in R} D_i(\mathbf{x}) d\mathbf{x} = N_i , \tag{5}$$

where N_i is the number of type *i* points.

We then define segregation measures. The *local segregation* at **x** is measured by

$$t(\mathbf{x}) = \sum_i \left\{ \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \right\}^2 . \tag{6}$$

Integration of t(**x**) over R yields the *overall segregation*:

$$S = \frac{\int_{\mathbf{x}\in R} D(\mathbf{x}) t(\mathbf{x}) d\mathbf{x}}{\int_{\mathbf{x}\in R} D(\mathbf{x}) d\mathbf{x}} , \tag{7}$$

where

$$D(\mathbf{x}) = \sum_i D_i(\mathbf{x}) . \tag{8}$$

The minimum of t(**x**) is $1/K$ as shown in Appendix A1. Its maximum is 1, and it occurs only when a single type of points exist. The range of S is thus $1/K < S \leq 1$.

The overall segregation S is a composite of segregations caused by the three factors mentioned earlier. We will decompose it into three components in the following.

The first component is the *locational segregation*. We can estimate this aspect of segregation by eliminating the locational unevenness in a given point distribution, calculating the degree of segregation for it, and comparing it with that for the original point distribution. The elimination is completed by the *locational equalization*, which converts the density distribution of every type of points into the uniform distribution with keeping the total density distribution of points. The density function of type *i* points becomes

$$D_i'(\mathbf{x}) = \frac{N_i}{N} D(\mathbf{x}). \tag{9}$$

As a result, spatial segregation reduces from S to

$$S' = \sum_i \left(\frac{N_i}{N} \right)^2 . \tag{10}$$

Figure 3a illustrates the density distributions of five types of points on a one-dimensional space. The locational equalization transforms the distributions into those shown in Figure 3b.

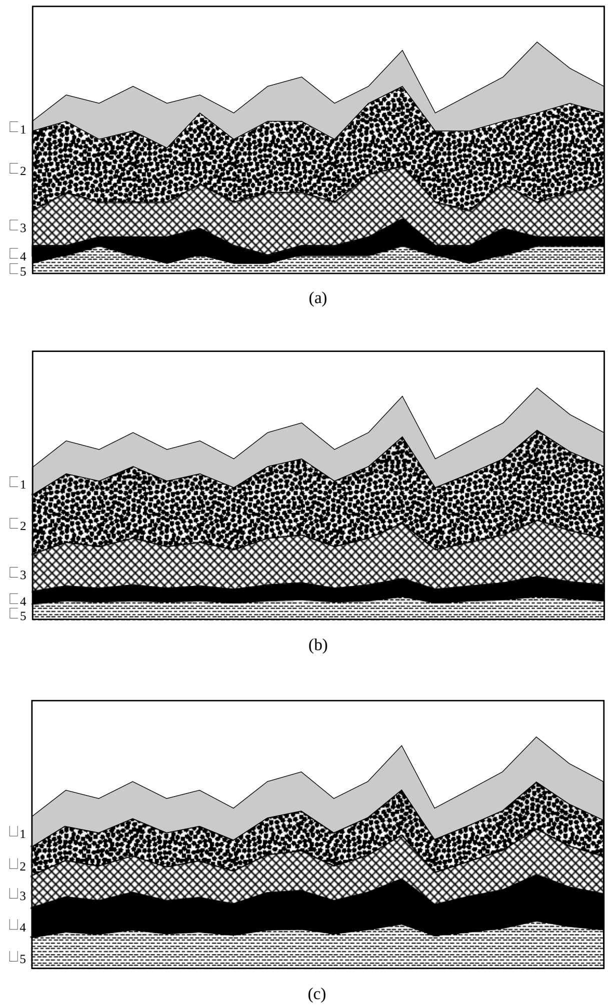

Figure 3 Elimination of locational and compositional unevenness in point distributions. (a) Density distributions of five types of points on a one-dimensional space. Different patterns indicate different types of points. (b) Density distributions obtained after the locational equalization. (c) Density distributions obtained after the compositional equalization.

The locational segregation is measured by the decrease from S to S':

$$\begin{aligned} S_L &= S - S' \\ &= \frac{\int_{\mathbf{x}\in R} D(\mathbf{x})\, t(\mathbf{x})\, d\mathbf{x}}{\int_{\mathbf{x}\in R} D(\mathbf{x})\, d\mathbf{x}} - \sum_i \left(\frac{N_i}{N}\right)^2 . \end{aligned} \tag{11}$$

Since 0<S' and $0 \leq S_L$ (a proof is shown in Appendix A2), the range of S_L is $0 \leq S_L < S$. It shows a large value when the locational equalization is very effective, that is, the spatial segregation greatly varies between different locations. The measure S_L becomes zero if the locational equalization does not at all reduce the spatial segregation. This occurs when all the density distributions of points are already uniform before the locational equalization.

The second component of spatial segregation is the *compositional segregation*. We define it as the segregation further reduced from *S*' by eliminating the compositional unevenness in point distributions. This elimination is performed by the *compositional equalization*, which equalizes the proportion of the total density between different types of points. The compositional equalization transforms the density function of type *i* points into

$$D_i''(\mathbf{x}) = \frac{1}{K}. \tag{12}$$

and reduces spatial segregation to

$$S'' = \frac{1}{K^2}. \tag{13}$$

Figure 3c shows the result of the compositional equalization applied to the distributions shown in Figure 3b. The compositional segregation is measured by

$$\begin{aligned} S_C &= S' - S'' \\ &= \sum_i \left(\frac{N_i}{N}\right)^2 - \frac{1}{K^2} \end{aligned}. \tag{14}$$

The range of S_C is $0 \leq S_C < S'$, which can be proved in a similar way as that of Appendix A1. It shows a large value if the number of points greatly varies between different types of points. The measure S_C becomes zero when the compositional equalization is completely ineffective, that is, all the types of points have the same number of points.

The third component of spatial segregation is the *qualitative segregation*. We define it as the segregation reduced from *S*'' by eliminating the qualitative homogeneity of points. The elimination is completed by increasing infinitely the types of points. As *K* approaches infinity, the spatial segregation infinitely reduces. Consequently, the qualitative segregation is measured by

$$\begin{aligned} S_Q &= S'' - 0 \\ &= \frac{1}{K^2} \end{aligned}. \tag{15}$$

The range of S_Q is obviously $0 < S_Q \leq 1$.

The locational segregation evaluates the spatial aspect of segregation while the compositional and qualitative segregations consider the aspatial aspects. The decomposition of spatial segregation into the three components permits us to discuss the different aspects of spatial segregation separately. Figure 4 displays the relationship between the segregation measures. As seen in the figure, the locational, compositional and qualitative segregations are mutually exclusive. They are also independent with each other in that we can change one aspect of segregation with preserving the other two dimensions. For instance, the locational and compositional segregations are independent with each other, and they are both independent of the qualitative segregation. We will show concrete examples in the next section. The qualitative segregation is also independent of the locational and compositional segregations. A proof is given in Appendix A3.

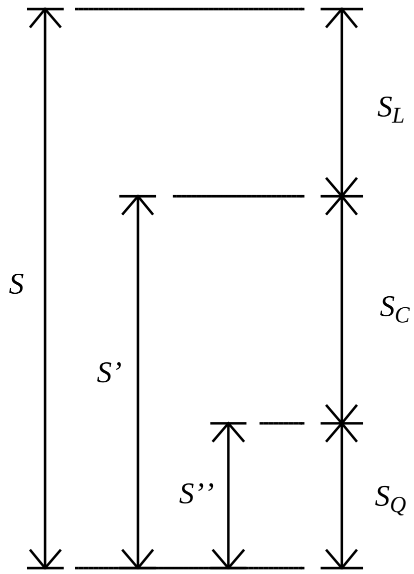

Figure 4 Relationship between the segregation measures.

The segregation measures proposed above do not meet the criterion of composition invariance defined by James and Taeuber (1985). Their definition claims that a measure should remain unchanged if the distribution of spatial objects changes but its relative spatial distribution does not change. Though some papers raise objections to this definition (Coleman *et al.*, 1982; Reardon and O'Sullivan, 2004), we can modify the proposed measures to satisfy it by using the standardized density distribution of points instead of $D_i(\mathbf{x})$:

$$\eta_i(\mathbf{x}) = \frac{D_i(\mathbf{x})}{\int_{\mathbf{y}\in R} D_i(\mathbf{y})\,d\mathbf{y}}. \tag{16}$$

Since it is independent of the absolute distribution of points, the segregation measures defined based on Equation (16) meet the criterion of composition invariance.

The segregation measures proposed above evaluate the three components of spatial segregation by using the result of eliminating the three factors of spatial segregation successively from the overall segregation. This implies that the measures cannot distinguish point distributions that share the same values of segregation measures. Suppose the checkerboard patterns (Wong, 2005; O'Sullivan and Wong, 2007) of different resolutions as shown in Figure 5. Each cell contains the same number of white or black points. If we define the density functions of points by simply dividing the number of points by the cell size, the two patterns show the same values of segregation measures, and consequently, we cannot distinguish the two patterns.

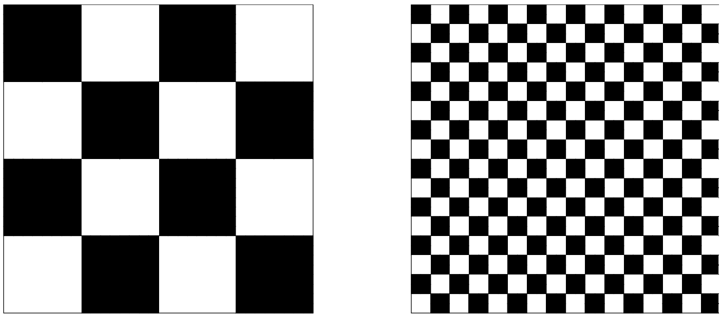

Figure 5 Checkerboard patterns of different resolutions that cannot be distinguished by the segregation measures.

One option to resolve the problem is to use a distance-decay function such as those defined by Equation (1). This enables us to take into account explicitly the spatial dimension of density distributions, and yields different values of segregation measures between the two patterns. Interpretation of the result, however, is not straightforward since the result heavily depends on the definition of distance-decay function. Its interpretation requires us to fully understand the properties of distance-decay function and the relationship between the function and result, especially when the point distributions present more complicated patterns.

The above problem arises because the locational unevenness is evaluated in the aspatial rather than the spatial dimension. The locational segregation S_L is dimensionless as seen in Equation (11). To resolve the problem, we evaluate the locational equalization in the spatial dimension using the earth moving transformation (Peleg *et al.*, 1989; Rubner *et al.*, 2000; Zhao *et al.*, 2010, Sadahiro, 2012). The earth moving transformation is originally a transportation problem that converts a pile of dirt into another form by transferring the dirt between regions with the least cost. We regard the locational equalization defined by Equation (9) as the earth moving transformation from $D_i(\mathbf{x})$ into $D_i'(\mathbf{x})$. The solution gives the volume of transfer between locations, from which we calculate two equalization measures useful for evaluating the locational equalization.

To discretize the density distributions of points, we divide region R by M subregions $\{R_1,$

R_2, ..., R_M} of the same size. The subregions should be small enough for a close approximation. Let D_{ij} be the density of type i points in R_j. The density of type i points transferred from R_j to R_k is denoted by x_{ijk} ($x_{ijk}\geq 0$). The locational equalization of $D_i(\mathbf{x})$ is formulated as the following optimization problem where the total volume of transfer is minimized.

Problem LE (Locational equalization):

$$\begin{aligned}
&\underset{x_{ijk},\, j,k\in\mathbf{N}}{\text{minimize}} \quad \sum_j \sum_k x_{ijk} \\
&\text{subject to} \\
&\qquad D_{ij} - \sum_j x_{ijk} + \sum_k x_{ikj} = \frac{N_i}{A(R)} \\
&\qquad x_{ijk} \geq 0 \quad \text{if } j \text{ and } k \text{ are adjacent} \\
&\qquad x_{ijk} = 0 \quad \text{otherwise}
\end{aligned}$$

Operator $A(R)$ gives the area of region R. Problem LE is a linear optimization problem whose computational complexity is O(n). It is thus solvable in a linear time.

From the solution of Problem LE, we calculate the weighted average distance of locational equalization. Let l_{jk} be the distance between R_j and R_k. The *equalization distance* is defined by

$$\begin{aligned}
L_E &= \frac{2\sum_i \sum_j \sum_k l_{jk} x_{ijk}}{\sum_i \sum_j \sum_k x_{ijk}} \\
&= \frac{2\sum_i \sum_j \sum_k l_{jk} x_{ijk}}{\sum_i \sum_j \left| D_{ij} - \frac{N_i}{A(R)} \right|} .
\end{aligned} \tag{17}$$

This distance measure permits us to evaluate the locational equalization in the spatial dimension. Another measure useful for evaluating the locational equalization is the ratio of the total volume of transfer called the *equalization ratio*:

$$\begin{aligned}
R_E &= \frac{\sum_i \sum_j \sum_k x_{ijk}}{\sum_i \sum_j D_{ij}} \\
&= \frac{\sum_i \sum_j \left| D_{ij} - \frac{N_i}{A(R)} \right|}{2\sum_i \sum_j D_{ij}} .
\end{aligned}$$

(18)

This measure indicates the discrepancy of the density distributions of points from the uniform distribution. It ranges from zero to one.

2.2 Evaluation of spatial segregation by the entropy index

Existing studies of spatial segregation often adopt the entropy index as a measure of spatial segregation (Pielou, 1977; White, 1986; Allen and Turner, 1989; Wong, 1993; Reardon and O'Sullivan, 2004). This paper, on the other hand, defines segregation measures in a different manner as shown in Equation (6). This is because the entropy index prevents us from decomposing the compositional and qualitative segregations separately as shown later. However, if they can be treated as a single component of segregation, we can use the entropy index within our framework. This subsection briefly describes the evaluation of spatial segregation by the entropy index.

We replace the local segregation measure $t(\mathbf{x})$ defined by Equation (6) with that based on the entropy index:

$$t_e(\mathbf{x}) = 1 + \sum_i \left\{ \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \log \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \right\}. \tag{19}$$

The overall segregation becomes

$$S_e = \frac{\int_{\mathbf{x}\in R} D(\mathbf{x}) t_e(\mathbf{x}) d\mathbf{x}}{\int_{\mathbf{x}\in R} D(\mathbf{x}) d\mathbf{x}} \tag{20}$$

Segregation measures *S*' and *S*'' are defined as

$$S_e' = 1 + \sum_i \frac{N_i}{N} \log \frac{N_i}{N}$$

and

$$\begin{aligned} S_e'' &= 1 + \sum_i \frac{1}{K} \log \frac{1}{K}, \\ &= 0 \end{aligned} \tag{21}$$

respectively. The locational, compositional, and qualitative segregation measures become

$$S_{Le} = S_e - S_e{}' = \frac{\int_{\mathbf{x}\in R} D(\mathbf{x}) \sum_i \left\{ \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \log \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \right\} d\mathbf{x}}{\int_{\mathbf{x}\in R} D(\mathbf{x}) d\mathbf{x}} - \sum_i \frac{N_i}{N} \log \frac{N_i}{N}, \quad (22)$$

$$S_{Ce} = S_e{}' - S_e{}'' = 1 + \sum_i \frac{N_i}{N} \log \frac{N_i}{N}, \quad (23)$$

and

$$S_{Qe} = S_e{}'' = 0, \quad (24)$$

respectively. Equation (24) indicates that the entropy index conceals the qualitative segregation. This prevents us from evaluating spatial segregation caused by the qualitative homogeneity shown in Figure 2. The ranges of the segregation measures are $0 \le S_{Le} < S_e$ and $0 \le S_{Ce} \le S_e$ (a proof of the former is given in Appendix A4). The equalization measures can be calculated in a similar way as that in the previous subsection.

Segregation measures defined from Equation (6) and those based on Equation (19) have both advantages and disadvantages. One strength of the former is that it permits us to decompose the aspatial components of spatial segregation into the compositional and qualitative segregations. The latter is advantageous in that the entropy index has already been widely used in the measurement of spatial segregation. Its implementation can be done with a slight modification of existing programs.

2.3 Comparison with Reardon and O'Sullivan's paper

This paper shares an intention and a principle with Reardon and O'Sullivan (2004). Both aim to evaluate spatial segregation from different perspectives and define segregation measures based on the location of individual points. This subsection briefly compares the two papers in terms of segregation dimensions and measures.

Reardon and O'Sullivan (2004) advocates the spatial exposure and the spatial evenness as the two dimensions of spatial segregation. The latter corresponds to the locational segregation proposed in this paper. Spatial exposure contains at least the compositional segregation, but it is not clear whether it also contains the qualitative segregation since they do not explicitly discuss the spatial exposure in terms of the multigroup segregation.

As shown in Section 2.1, the locational, compositional and qualitative segregations are independent with each other. This implies that the spatial exposure and evenness are also independent. We can change the former with keeping the latter, and vice versa. We should note, however, that this does not assure the independency between their segregation measures. It is unknown whether their spatial exposure index and spatial evenness indices are independent with each other.

To evaluate the spatial evenness, they propose the spatial information theory segregation index defined by

$$\widetilde{H} = 1 - \frac{1}{TE} \int_{p \in R} \tau_p \tilde{E}_p dp \, , \tag{25}$$

where T and E are the total number of points and the overall entropy, respectively. This corresponds to S_L in this paper in that both compare the spatial segregation with the overall segregation. While S_L considers the difference between the segregations, $\widetilde{H}$ evaluates their ratio.

The range of S_L is known as $0 \leq S_L < S$, while that of $\widetilde{H}$ is not known. The latter can take a negative value, and we cannot compare the spatial segregation between different patterns as mentioned in Section 1. This is presumably because the local entropy is calculated from the local density of points while the overall entropy is based on the total population of points. Unlike Equation (5), the summation of the local density is not always equal to the total population.

To avoid this problem, we suggest using the variables defined in the local environment consistently to calculate segregation measures. We modify the spatial information theory segregation index as

$$\widetilde{H}' = 1 - \frac{1}{T'E'} \int_{p \in R} \tau_p \tilde{E}_p dp \, , \tag{26}$$

where

$$T' = \int_{p \in R} \tilde{\tau}_p dp \tag{27}$$

and

$$E' = -\sum_{m=1}^{M} \frac{\int_{p \in R} \tilde{\tau}_{pm} dp}{T'} \log_M \frac{\int_{p \in R} \tilde{\tau}_{pm} dp}{T'} \, . \tag{28}$$

The modified index would range from 0 to 1.

3. Applications

To test the validity of the method proposed above, this section applies it to an analysis of three datasets of different sizes.

3.1 Small synthetic dataset

This subsection aims to investigate the properties of the segregation measures. Figure 6 and Figure 7 show a small synthetic dataset of the density distributions of points on a one-dimensional space and their segregation measures.

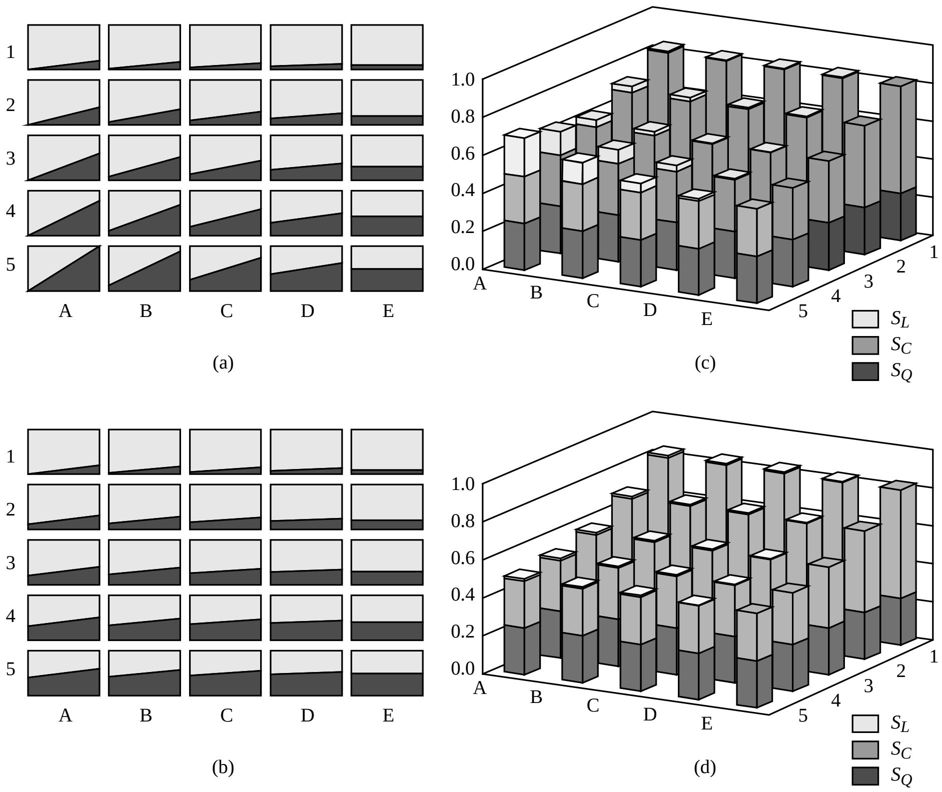

Figure 6 Density distributions of points (a, b) and their segregation measures (c, d). Different colors indicate different types of points in Figure 6a and b.

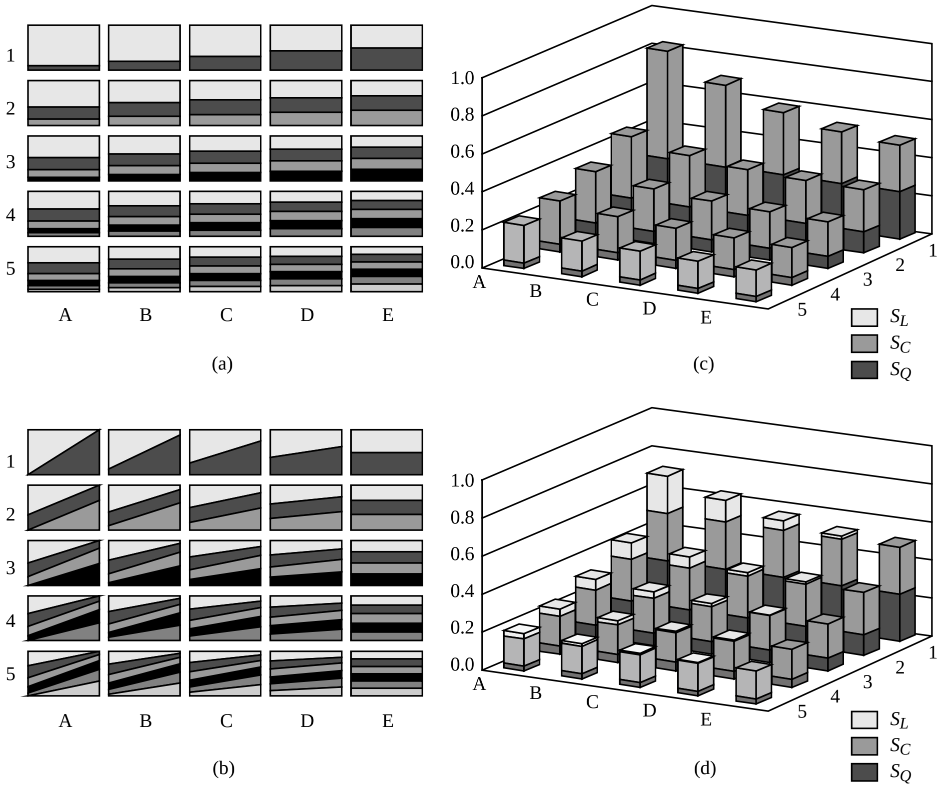

Figure 7 Density distributions of points (a, b) and their segregation measures (c, d). Different colors indicate different types of points in Figure 7a and b.

We first discuss the relationship between the locational and compositional segregations in Figure 6. To focus only on the two segregations, we consider the distributions of two types of points. In every row from column A to E, the proportion of two types of points remains unchanged while the gradient of the boundary between two types of points changes. The gradient also changes from row 1 to 5 in Figure 6a while it remains unchanged in Figure 6b. Since the number of types of points does not change, S_Q is constant in Figure 6c and Figure 6d. In column E, the locational segregation S_L is equal to zero in all the cases due to the uniform density distributions. The compositional segregation S_C increases from row 5 to 1 with the unevenness in the composition of points. This is consistent with our earlier observation in Figure 1c, where a difference in the number of points causes a local segregation. The locational unevenness decreases from column A to E, which results in a decrease in S_L.

The overall segregation $S(=S_L+S_C+S_Q)$ first decreases and then increases from row 5 to 1 in columns from A to D in Figure 6c. This is because the locational and compositional segregations can change simultaneously. While S_L decreases monotonically from row 5 to 1, S_C increases continuously. This confirms us that we cannot distinguish the locational and compositional segregations by only using S. The distinction requires at least two measures.

In Figure 6c, the locational segregation S_L changes with keeping S_C and S_Q unchanged from column A to E. In Figure 6d, on the other hand, S_C changes with keeping S_L and S_Q from row 5 to 1. This clearly indicates that the locational and compositional segregations are independent with each other.

We then consider the relationship between the compositional and qualitative segregations. To exclude the effect of locational segregation, we consider the uniform density distributions in Figure 7a. The number of point types K ranges from two to six. Figure 7c shows that the qualitative segregation S_Q increases with a decrease of K from row 5 to 1, so does the overall segregation. This supports our earlier discussion in Figure 2, that is, the locational segregation increases with the qualitative homogeneity. We also confirm our observation in Figure 1c by looking at an increase in S_C and S from column E to A, where the compositional unevenness monotonically increases.

We finally discuss the relationship between the locational and qualitative segregations. To exclude the effect of compositional segregation, we consider the density distributions of points shown in Figure 7b, in each of which all the distributions have the same total density. Similar to Figure 7c, Figure 7d shows that a decrease of K increases S_Q from row 5 to 1. The locational segregation S_L increases while S_C and S_Q remain unchanged from column E to A. This again supports the independency of locational segregation.

3.2 Larger synthetic datasets

We then apply the method to larger synthetic datasets to test the validity of equalization measures. We consider five checkerboard patterns of side 32, each of which consists of 2*2, 4*4, 8*8, 16*16, and 32*32 cells as shown in Figure 8. Every cell contains the same number of either white or black points.

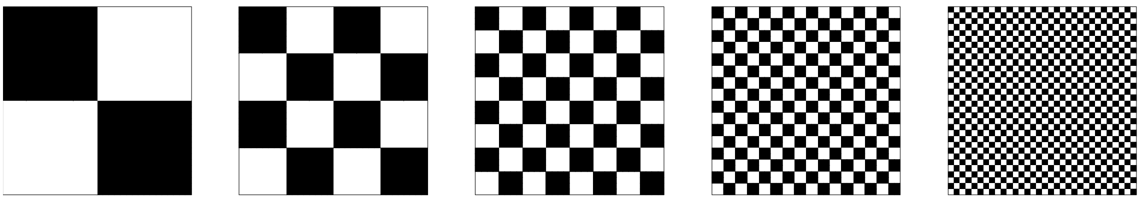

Figure 8 Checkerboard patterns of white and black points. Every cell contains the same number of either white or black points.

The result is shown in Table 1. While the segregation measures fail to distinguish the checkerboard patterns of different resolutions, the equalization distance L_E clearly detects the difference between them. In checkerboard patterns, the locational equalization is completed by transferring a half of points in each cell to one of its adjacent cell. Consequently, the equalization distance is theoretically equal to the distance between the centroids of adjacent cells, though it is violated by the boundary effect. This permits L_E to distinguish the checkerboard patterns of different resolutions.

Table 1 Segregation and equalization measures of checkerboard patterns

Resolution	S_L	S_C	S_Q	L_E	R_E
2*2	0.00	0.00	0.25	10.688	0.50
4*4	0.00	0.00	0.25	4.031	0.50
8*8	0.00	0.00	0.25	1.773	0.50
16*16	0.00	0.00	0.25	1.063	0.50
32*32	0.00	0.00	0.25	1.000	0.50

We then consider the distributions of white and black points of the same number in a rectangular region of 16*32 cells. Figure 9 shows the five patterns of the density distribution of black points. Patterns P1, P2, and P3 have a single peak, while P4 and P5 have two and four peaks, respectively.

The result is shown in Table 2. We again confirm the effectiveness of the equalization distance L_E. It increases from P1 to P3 as the peak shifts from the center to the boundary of the region. It decreases from P1 to P4 and P5 as the distribution becomes smooth. The equalization ratio R_E can also distinguish P4 and P5 from P1, P2, and P3. This is because the total volume of transfer necessary for flattening peaks decreases as the density distribution becomes smooth.

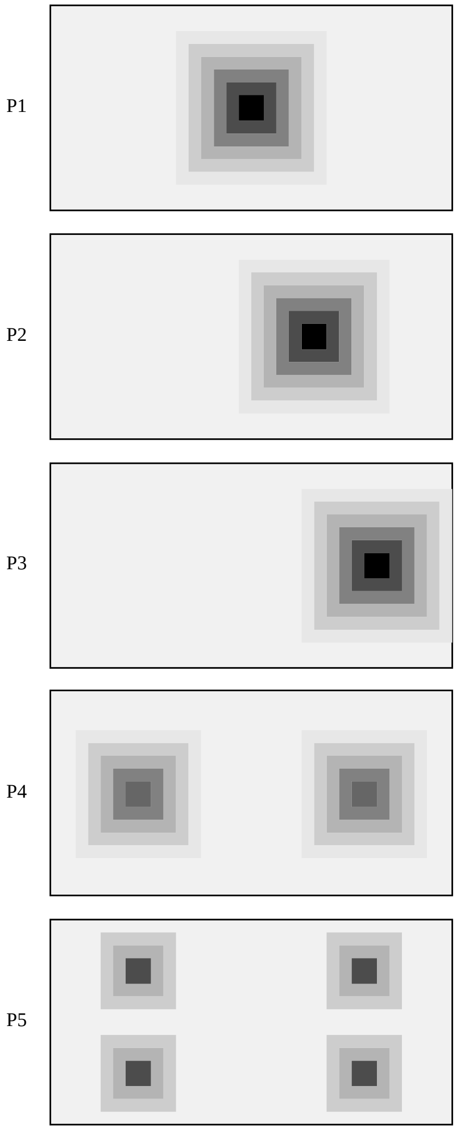

Figure 9 Distributions of white and black points of the same number. Grey shades indicate the density distribution of black points.

Table 2 Segregation and equalization measures of density distributions shown in Figure 9.

Pattern	S_L	S_C	S_Q	L_E	R_E
P1	0.00	0.00	0.25	10.422	0.72
P2	0.00	0.00	0.25	11.880	0.72
P3	0.00	0.00	0.25	16.350	0.72
P4	0.00	0.00	0.25	6.933	0.65
P5	0.00	0.00	0.25	5.090	0.61

3.3 Real dataset

We finally use the method to analyze a real dataset to test its practical feasibility. We examine the urbanization process from 1974 to 1994 in Chiba Prefecture, Japan. Figure 10 shows the population distribution of Chiba in 1994. Since Chiba is adjacent to the east of Tokyo Metropolis, its population density is higher along railway lines in the western area that is densely inhabited by people working in Tokyo.

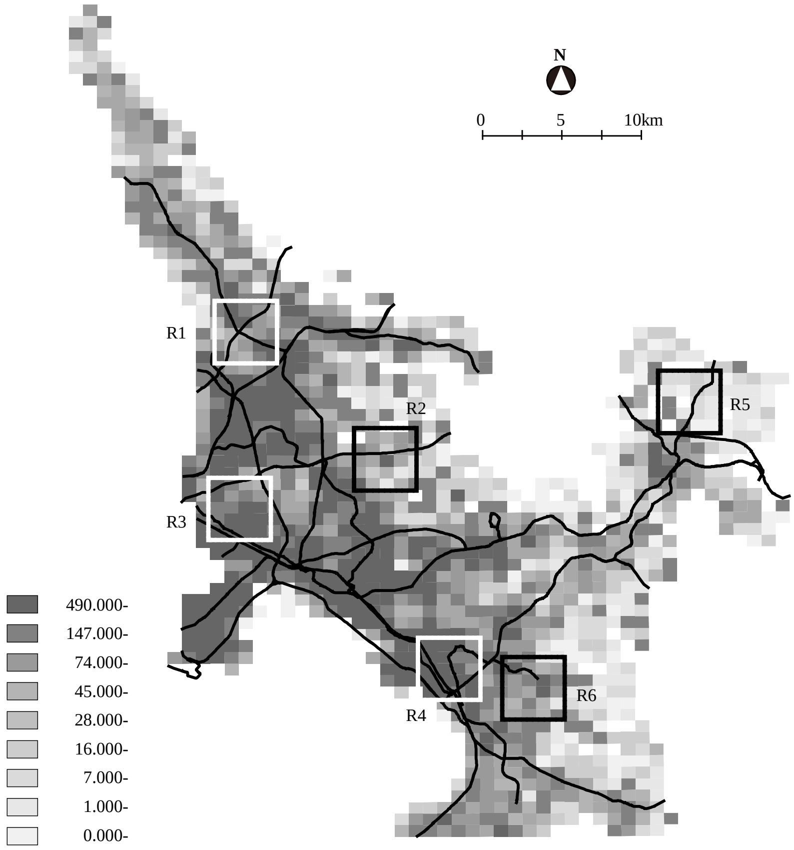

Figure 10 Population distribution of Chiba Prefecture, Japan in 1994. White and black broken lines indicate subregions in which landuse mixture is analyzed in detail.

We obtained the landuse data of 10m resolution from Geospatial Information Authority of Japan, and aggregated them into the raster format data of 500m resolution. We calculated the segregation measures of landuse pattern in each cell and summed them up over the entire region.

Urbanization is usually accompanied with a progress of landuse mixture. The transition proceeds from the dominance of natural landuse to the mixture of urban landuse such as residential, commercial, and industrial areas. Consequently, a decrease in the segregation measures is often observed with the progress of urbanization.

Figure 11 shows the distribution of the overall segregation *S*. Note that darker shades indicate smaller values of *S* where the landuse is highly mixed. Comparing Figure 11b with Figure 10, we notice that the landuse mixture and the population density are highly correlated. This confirms us of the close relationship between the landuse mixture and urbanization. The urban area expanded especially in the western area of Chiba during this period because of the rapid concentration of population in the metropolitan area of Tokyo.

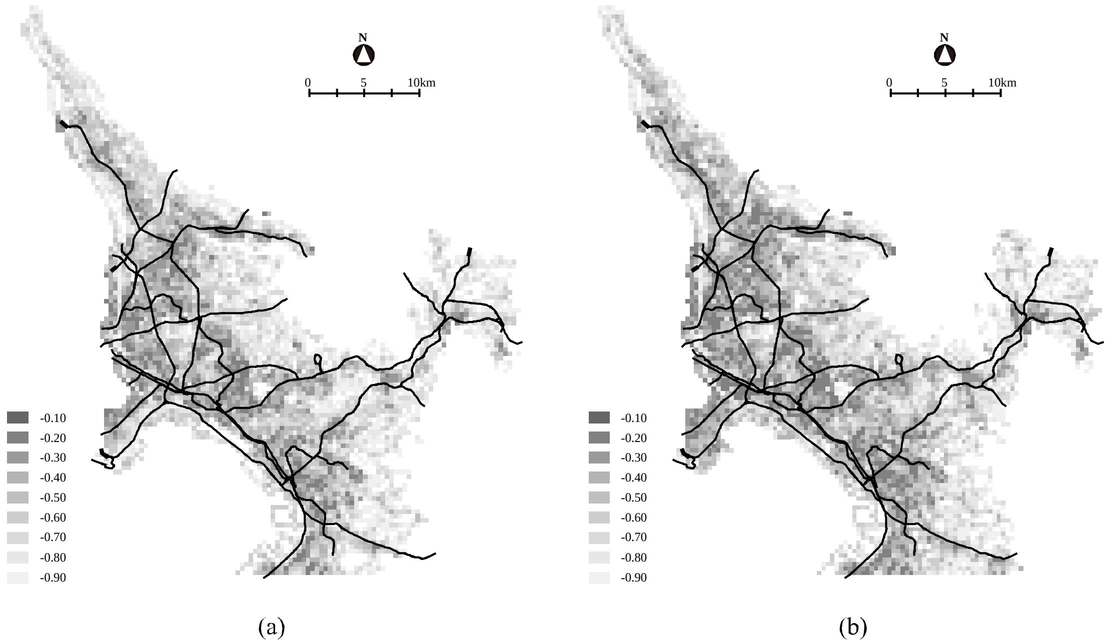

Figure 11 The overall segregation *S* of landuse pattern in (a) 1974 and (b) 1994.

We then examine the progress of landuse mixture in more detail in six subregions shown in Figure 10. Subregions R1 and R2 experienced a rapid expansion of urban areas in 1980's primarily caused by the new town construction. Subregions R3 and R4 both contain urban areas of a long history, where landuse pattern had been stable during the period. Subregions R5 and R6 present a mixture of urban, suburban, and rural areas. The former contains an old historical town while the latter is a new town located at the boundary of urban and rural areas.

Figure 12 shows the segregation measures from 1974 to 1994. The overall segregation *S* decreased in all the regions, which suggests the progress of urbanization all over Chiba. Comparing Figure 12b and Figure 12c, we notice that the decrease of *S* was caused by the rapid reduction of S_C. Subregions R1, R2 and R6 show a drastic decrease of S_C during this period. The locational segregation S_L, on the other hand, slightly increased in many regions. The compositional unevenness of landuse decreased while the locational unevenness increased with the progress of urbanization.

This implies that urban areas expanded scatteredly rather than uniformly in Chiba. We confirm this in Figure 13 where the equalization distance L_E increased in R1 and R2.

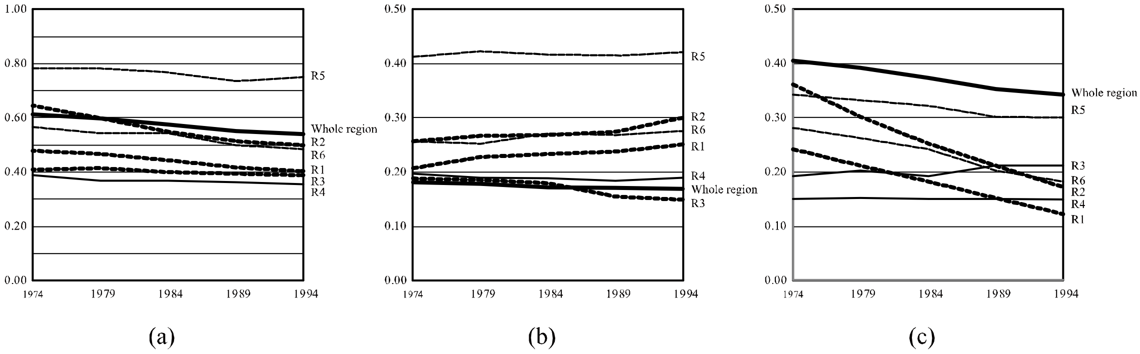

Figure 12 The segregation measures of landuse pattern from 1974 to 1994. (a) The overall segregation S, (b) the locational segregation S_L, (c) the compositional segregation S_C.

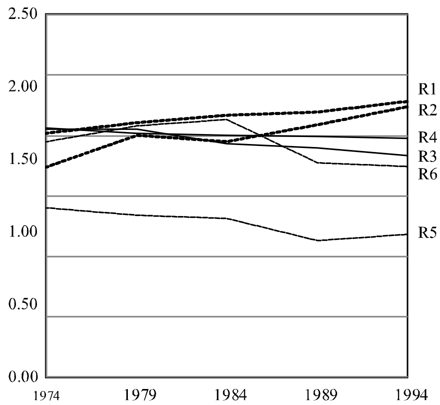

Figure 13 The equalization distance L_E from 1974 to 1994.

4. Concluding discussion

This paper has developed a new method of evaluating the spatial segregation. Using the locational and compositional equalizations, the method decomposes the spatial segregation into three exclusive and independent components. The locational segregation evaluates the spatial aspect of segregation caused by the locational unevenness. The compositional and qualitative segregations reflect the compositional unevenness and the qualitative homogeneity, both of which are aspatial factors of spatial segregation. The paper proposes three segregation and two equalization measures to evaluate the three components of spatial segregation quantitatively. To test the validity of the method, the paper applied it to an analysis of three spatial datasets of different sizes. The result indicated that the method is effective for evaluating the spatial segregation from three different perspectives.

Our method has several advantages over existing method. First, the method defines the three components of spatial segregation in a formal and quantitative way. It clearly reveals the exclusive and independent relationship between the components. Second, the paper proposes three segregation measures whose range is analytically obtained. This permits us to compare the spatial segregations between different patterns. Third, following the line of Reardon and O'Sullivan (2004), the paper defines the segregation measures based on the location of individual points. This enables us to apply the method to both point data and their aggregated data, the latter of which permits us to approximate the segregation measures calculated from the former.

We finally discuss some limitations and extensions of the paper for future research.

First, the paper considers three components of spatial segregation. Existing papers, on the other hand, have proposed further dimensions. Massey and Denton (1988), for instance, advocates five dimensions called evenness, exposure, clustering, centralization, and concentration. Though they may not be independent with each other, they are still effective and convenient for understanding the spatial structure of segregation from various perspectives. For a better usage of existing dimensions, their properties need to be clarified in more detail. To this end, we should analyze and describe the relationship between our components and existing dimensions.

Second, we should consider the spatial segregation in the spatiotemporal dimension. Numerous moving objects exist in the real world such as human beings, animal species, vehicles and planes. Their spatial distributions change continuously, and so does their spatial segregation. Fortunately, a rapid progress of data acquisition tools in GIS enables us to capture the trajectories of moving objects. An analytical method of spatiotemporal segregation needs to be developed.

Third, this paper evaluates the similarity of different types of points equally between any pair of different types. In the real world, however, this treatment is sometimes inappropriate. For instance, the difference between European and Asian peoples is larger than that between Asian and Pacific peoples. When we discuss their spatial segregation, segregation of European and Asian peoples may be more critical than that of Asian and Pacific peoples. To treat such cases, we should take into account explicitly the attributes of spatial objects and their differences in the evaluation of spatial segregation.

Fourth, the behavioral approach seems useful in discussing the spatial segregation of moving objects. The concept of exposure (Massey and Denton, 1988; Reardon and O'Sullivan, 2004) implicitly considers the movement of spatial objects that is represented as the possibility of contact between objects. Assuming a more realistic behavior of objects, we can define segregation measures that are practically more useful to describe the spatial segregation. Extension in this direction sounds promising.

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Appendix A1

This appendix derives the minimum value of $t(\mathbf{x})$ defined by Equation (6) using the Langrangian multiplier method. Let us define $d_i(\mathbf{x})$ as

$$d_i(\mathbf{x}) = \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})}. \tag{29}$$

We minimize

$$f = \sum_i d_i^2(\mathbf{x}) + \lambda\left(1 - \sum_i d_i(\mathbf{x})\right) \tag{30}$$

with constrain

$$\sum_i d_i(\mathbf{x}) = 1, \tag{31}$$

where λ is the Langrangian multiplier. Partial differentiation of f by $d_i(\mathbf{x})$ and that by λ are

$$\frac{\partial f}{\partial d_i(\mathbf{x})} = 2d_i(\mathbf{x}) - \lambda \tag{32}$$

and

$$\frac{\partial f}{\partial \lambda} = 1 - \sum_i d_i(\mathbf{x}), \tag{33}$$

respectively. Since

$$\frac{\partial f}{\partial d_i(\mathbf{x})} = 0, \tag{34}$$

we obtain

$$d_i(\mathbf{x}) = \frac{\lambda}{2}. \tag{35}$$

Substitution of Equation (35) into Equation (33) yields

$$1-\sum_i d_i(\mathbf{x}) = 1-\frac{\lambda K}{2} = 0\,.$$

(36)

Solving this equation, we have

$$\lambda = \frac{2}{K}\,.$$

(37)

Substituting the above equation into Equation (36), we obtain

$$d_i(\mathbf{x}) = \frac{1}{K}\,.$$

(38)

This gives the minimum of $t(\mathbf{x})$, that is,

$$\begin{aligned} t(\mathbf{x}) &= \sum_i \left\{d_i(\mathbf{x})\right\}^2 \\ &= \frac{1}{K} \end{aligned}\,.$$

(39)

Appendix A2

This appendix derives the minimum value of S_L. To this end, we divide region R into M subregions denoted by $\{R_1, R_2, ..., R_M\}$. Let D_{ij} be the density of type i points in R_j, which corresponds to $\zeta_{ij}(\mathbf{x})$ in Equation (3). The proportion of type i points in R_j is given by

$$p_{ij} = \frac{D_{ij}}{\sum_i D_{ij}}\,.$$

(40)

The total density of points in R_j is denoted by ρ_i. The total density of all the points is

$$T = \sum_j A\left(R_j\right)\rho_j\,,$$

(41)

where $A(R_j)$ is an operator that gives the area of R_j.

The locational segregation S_L is defined as

$$S_L = \frac{1}{T}\sum_j A(R_j)\rho_j \sum_i p_{ij}^2 - \sum_i \left(\frac{N_i}{N}\right)^2$$
$$= \frac{1}{T}\sum_i \left\{ \sum_j A(R_j)\rho_j p_{ij}^2 - \left(\frac{N_i}{N}\right)^2 T \right\}.$$

(42)

We use the Langrangian multiplier method, where we minimize

$$f_i = \sum_j A(R_j)\rho_j p_{ij}^2 - \left(\frac{N_i}{N}\right)^2 T + \lambda\left(\frac{N_i}{N}T - \sum_j p_{ij}A(R_j)\rho_j\right).$$

(43)

The constraints are

$$\forall i, \sum_j p_{ij}A(R_j)\rho_j = \frac{N_i}{N}T$$

(44)

Partial differentiation of f_i by p_{ij} and that by λ are

$$\frac{\partial f_i}{\partial p_{ij}} = 2A(R_j)\rho_j p_{ij} - \lambda A(R_j)\rho_j .$$

(45)

and

$$\frac{\partial f_i}{\partial \lambda} = \frac{N_i}{N}T - \sum_j p_{ij}A(R_j)\rho_j ,$$

(46)

respectively. Solving

$$\frac{\partial f_i}{\partial p_{ij}} = 0,$$

(47)

we obtain

$$p_{ij} = \frac{\lambda}{2}.$$

(48)

This equation indicates that p_{ij} does not depend on i. We thus obtain

$$p_{ij} = \frac{N_i}{N}.$$

(49)

and the minimum of S_L:

$$S_L = \frac{1}{T}\sum_i \left\{ \sum_j A\left(R_j\right)\rho_j \left(\frac{N_i}{N}\right)^2 - \left(\frac{N_i}{N}\right)^2 T \right\}.$$
$$= 0$$

(50)

Appendix A3

This appendix proves that the qualitative segregation is independent of the locational and compositional segregations. To this end, we add *m* types of points to the point set considered in Section 2. We keep the total number of points but can modify the number of each type of points. Let N_i' be the number of type *i* points after the addition of new points.

We first consider the independency of the qualitative segregation from the locational segregation. The locational segregation remains unchanged when the following equation holds for any **x**:

$$\sum_i \left\{ \frac{D_i(\mathbf{x})}{\sum_k D_k(\mathbf{x})} \right\}^2 = \sum_i \left\{ \frac{D_i'(\mathbf{x})}{\sum_k D_k'(\mathbf{x})} \right\}^2 .$$

(51)

This condition is equivalent to

$$\sum_i p_i^2 = \sum_i p'^2_i ,$$

(52)

under the constraint:

$$\sum_i p_i = \sum_i p_i' = 1.$$

(53)

We assume *m*=1 and $p_i = p_i$' for *i*=1, ..., *K*-2. Equations (52) and (53) become

$$p_{K-1}^2 + p_K^2 = p'^2_{K-1} + p'^2_K + p'^2_{K+1},$$

(54)

and

$$p_{K-1} + p_K - p'_K - p'_{K+1} = p'_{K-1}.$$

(55)

We can rewrite p_{K-1} as

$$p_{K-1} = p_K + a$$

(56)

without losing generality. Substituting Equations (55) and (56) into Equation (54), we solve it in terms of p_K:

$$p_K = \frac{-\left(p'_K + p'_{K+1}\right) + 2\sqrt{p'_K\, p'_{K+1} + ap'_K + ap'_{K+1}}}{2}$$

(57)

The condition of the existence of a positive p_K is

$$4a\left(p'_K + p'_{K+1}\right) > \left(p'_K - p'_{K+1}\right)^2 .$$

(58)

It is always possible to satisfy the above inequality by choosing close values of p'_K and p'_{K+1}. Consequently, we can change the locational segregation with keeping the qualitative segregation.

We then consider the independency of the qualitative segregation from the compositional segregation. The compositional segregation is unchanged when

$$\sum_i \left(\frac{N_i}{N}\right)^2 - \frac{1}{K^2} = \sum_i \left(\frac{N_i'}{N}\right)^2 - \frac{1}{\left(K+m\right)^2} .$$

(59)

Solving Equation (59) in terms of m, we obtain

$$m = \sqrt{\frac{K^2}{K^2 \sum_i \left(\frac{N_i'}{N}\right)^2 - K^2 \sum_i \left(\frac{N_i}{N}\right)^2 + 1}} - K .$$

(60)

Since m is positive, the following inequality needs to hold:

$$\sum_i \left(\frac{N_i}{N}\right)^2 - \frac{1}{K^2} < \sum_i \left(\frac{N_i'}{N}\right)^2 < \sum_i \left(\frac{N_i}{N}\right)^2 .$$

(61)

It is always possible to satisfy the above inequality by slightly reducing every N_i and add a new type of points. Consequently, we can change the qualitative segregation with keeping the compositional segregation.

Appendix A4

This appendix derives the minimum value of the locational segregation S_{Le}. We use the same setting as that of Appendix A2. The locational segregation S_{Le} is written as

$$
\begin{aligned}
S_{Le} &= \frac{1}{T}\sum_{j} A\left(R_j\right)\rho_j \sum_{i} p_{ij}\log p_{ij} - \sum_{i} p_{ij}\log p_{ij} \\
&= \frac{1}{T}\sum_{i}\left\{\sum_{j} A\left(R_j\right)\rho_j p_{ij}\log p_{ij} - \frac{N_i}{N}T\log\frac{N_i}{N}\right\}
\end{aligned}
.
$$

(62)

We use the Langrangian multiplier method to calculate the minimum value of S_{Le}. We minimize

$$
f_i = \sum_{j} A\left(R_j\right)\rho_j p_{ij}\log p_{ij} - \frac{N_i}{N}T\log\frac{N_i}{N} + \lambda\left(\frac{N_i}{N}T - \sum_{j} p_{ij}A\left(R_j\right)\rho_j\right).
$$

(63)

with constraints

$$
\forall i, \sum_{j} p_{ij}A\left(R_j\right)\rho_j = \frac{N_i}{N}T .
$$

(64)

Partial differentiation of f_i by p_{ij} and that by λ are

$$
\frac{\partial f_i}{\partial p_{ij}} = \rho_j A\left(R_j\right)\left(\log p_{ij} - 1 - \lambda\right).
$$

(65)

and

$$
\frac{\partial f_i}{\partial \lambda} = \frac{N_i}{N}A\left(R\right) - \sum_{j} p_{ij}A\left(R_j\right),
$$

respectively. Solving

$$
\frac{\partial f_i}{\partial p_{ij}} = 0 ,
$$

(66)

we obtain

$$
p_{ij} = K^{1+\lambda} .
$$

(67)

This implies that p_{ij} is independent of i. From this we derive

$$
p_{ij} = \frac{N_i}{N} .
$$

(68)

Substitution of Equation (68) into (62) yields

$$\begin{aligned} S_{Le} &= \frac{1}{T}\sum_i \left\{ \sum_j A\left(R_j\right)\rho_j \frac{N_i}{N}\log\frac{N_i}{N} - \frac{N_i}{N} T \log\frac{N_i}{N} \right\}. \\ &= 0 \end{aligned} \tag{69}$$