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Agglomeration due to the imperfect information revisited

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Abstract

The purpose of this paper is to characterize all the spatial distribution patterns of retail firms realized when consumers are uncertain about the characteristics of available varieties. Emphasis is put on the explicit treatment of taste heterogeneity, based on the spatial competition theory. By constructing a two-region model, we show that only two distribution patterns, i.e., segregation and full agglomeration, may be supported by an equilibrium. The former is an equilibrium pattern if transport costs are high, and the latter is an equilibrium pattern if transport costs are low. This closely resembles the main result of new economic geography (NEG) models.

Keywords: agglomeration; break point; characteristics cost; differentiated product; Hotelling model; imperfect information; pro-competitive effect; segregation; spatial competition; sustain point; transport cost

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1 Introduction

It has become increasingly widely recognized that economic activities tend to agglomerate within a limited space (see, among others, Combes et al. (2008) and the literature cited there). One of the most successful attempts to explain why agglomeration occurs is a series of works, bracketed together as new economic geography (NEG), originating with Krugman (1991). A key element in the explanation is circular causation brought about by demand–supply linkages. In typical models such as the standard core–periphery model (Krugman (1991)), the analytically solvable model, also known as the "footloose entrepreneurs model" (Forslid and Ottaviano (2003)), and the linear model (Ottaviano et al. (2002)), the linkages work as follows. Demand for a final product is larger in a region with greater expenditure; this enables that region to attract more production activities. This is called a "backward linkage" (Fujita et al. (1999)) or a "market-access effect" (Baldwin et al. (2003)). At the same time, the cost of living is lower in a region with more firms, or more varieties; therefore, the owners of mobile factors and, consequently, production activities, are drawn toward that region. This is a "forward linkage" or a "cost-of-living effect". Each of the two linkages can generate a snowball effect, or cumulative causation, resulting in agglomeration. The building block in this explanation is the postulate that *a shift of production activities accompanies that of income.* When production is agglomerated in a particular region, all of the owners of mobile factors migrate into that region and put their factors into the production process that takes place there. Thus, the income of such factor-owners moves to that region at the same time.[1]

Such an explanation, however effective for geographical agglomeration on a large scale, for instance in a global economy or a national economy, does not work well for agglomeration on a smaller scale, for instance that *within* a city. Now let us consider the agglomeration of commercial activities within a city, such as a retail concentration in a central business district and a shopping center represented by a shopping mall at "edge cities". It is usually the case that such an agglomeration does not entail the spatial concentration of income. This is because not many of the residents in a city engage in the commercial activities there, and many of them work for other industries, which are often located outside the city. Therefore, the location of commercial activities has at best a minor effect on the location of consumers and thus that of income.

One demand–supply linkage that explains the agglomeration of commercial activities within a city without relying on the co-movement of production and income involves *taste*

[1] A "footloose capital model" shows that if a factor (e.g., capital) is mobile but its owner is not, no agglomeration will occur because both the linkages disappear (see Martin and Rogers (1995) and Baldwin et al. (2003)).

heterogeneity and *imperfect information*.[2] More specifically, we consider the situation in which consumers have different tastes with regard to the characteristics of a product but are uncertain about the characteristics of the varieties sold in each commercial area. In this case, they *guess* the characteristics of the best variety that they will find in each area from the information on the number of commercial facilities located there: if more shops sell a product in a particular commercial area, they expect to find a more favorable variety upon going there. Therefore, they prefer visiting a commercial area with a higher concentration of commercial facilities, *ceteris paribus*. Thus, a larger number of firms or a higher level of production results in a higher level of expenditure, which attracts further production activities. This produces a backward linkage or a market-access effect and causes the circular causality.

The observation that taste heterogeneity together with imperfect information leads to agglomeration is not novel. On the contrary, it is widely known among researchers through three important pieces of research.[3]

First, Stahl (1982), analyzing consumers' search behaviors and the locational choices of sellers, derived a set of conditions for the location of all firms in one marketplace being supported by an equilibrium. However, he abstracted away price competition by assuming a given price of a differentiated product. Secondly, Wolinsky (1983) obtained another set of conditions for the geographical concentration of shops selling varieties. Although his framework is general enough to allow for price competition, he, like Stahl, did not derive an entire equilibrium configuration of economic geography: all that was shown was that agglomeration can be an equilibrium outcome.

Finally, Konishi (2005), whose interest was closest to ours, explained the concentration of retail stores by constructing a model with a two-dimensional geographical space, in which a given number of stores choose their locations from a set of potential shopping centers and compete with each other over prices, and consumers decide which shopping center to visit. The main difference between his work and ours is in the manipulation of taste heterogeneity, which underpins the model. Konishi assumes that the utility that a consumer receives from consumption of a given product is a random variable, which he interprets to mean that "consumers do not know their exact preferences over commodities". In this paper, in contrast, the utility itself is a deterministic variable and formulated within the framework of a spatial competition theory á la Hotelling (1929) and Salop (1979). The characteristics of

[2]Another is the demand–supply linkage for *intermediate products*, not for final products (see Krugman and Venables (1995) and Venables (1996), among others). Although agglomeration is generated by a similar intertwinement of the backward linkage (the market-access effect) and the forward linkage (the cost-of-living effect), these linkages do not necessitate the location choice of factor-owners.

[3]For the other related works, see the literature cited in Konishi (2005).

the products are the random variables: consumers do not know what kinds of products they can find upon arriving at a shopping center. We believe that the latter approach is more appropriate for studying the topic, on two grounds. First, we believe that being uncertain about available characteristics is a much more common situation in our everyday life than being uncertain about our own taste. Secondly, the spatial competition theory is one of the most powerful tools for analyzing firms' behaviors under consumers' taste heterogeneity, and there is a rich accumulation of research in that field. Our approach can take advantage of this heritage to give an explicit foundation for consumers' behaviors, whereas Konishi's approach puts taste heterogeneity in a black box.

The purpose of this paper is to characterize *all* the spatial distribution patterns of firms that are realized when consumers are uncertain about the characteristics of available varieties. Emphasis is put on the explicit treatment of taste heterogeneity based on the spatial competition theory. For this purpose, we construct a model with two regions and a differentiated product. On the one hand, there are consumers who cannot move across regions. Without knowing the characteristics of the varieties sold in respective regions, but by forming an expectation, they decide whether to go shopping or not, and, if the answer is yes, which region to visit. It is necessary for them to pay transport costs to go to a shop in a foreign region, whereas it does not cost anything to go to a counterpart in their home region. On the other hand, there are firms (sellers or shops) who are free to choose their location, or, more precisely, they freely enter and exit from the industry in each region, depending on the amount of demand they are to receive. They compete with each other over prices. We explore an emerging economic geography by formulating the consumers' and firms' behaviors in a three-stage game.

Our simple model yields two equilibrium patterns of economic geography: *segregation* and *full agglomeration*. In segregation, all the consumers visit a shop in their home region. Firms are dispersed over the two regions according to the distribution of consumers. This pattern corresponds to "dispersion" or the symmetric pattern in the standard NEG models. In full agglomeration, on the other hand, all the consumers in the economy visit a shop in the same region, i.e., a core, where firms are concentrated. This pattern corresponds to "agglomeration" in the NEG models. We derive the conditions for each pattern being supported by an equilibrium and show that the conditions depend on several parameters, in particular, transport costs. If the transport cost is lower than a critical value, i.e., a *break point*, the segregation breaks; if it is lower than another critical value, i.e., a *sustain point*, the agglomeration becomes sustainable. This closely resembles the main result of NEG models. In other words, the main result of NEG models can alternatively be derived from a demand–supply linkage in a partial equilibrium setting, i.e., a linkage which does not entail the co-movement of production and income.

The rest of the paper consists of four sections. The next section describes the model. In section 3, we derive the conditions for an equilibrium by analyzing the firms' decisions on prices to charge, the consumers' decisions on the region to visit, and the number of firms. Section 4 shows that only segregation and full agglomeration can be supported as equilibrium distribution patterns. We obtain the conditions for each of these being an equilibrium pattern and discuss the overall effect of a change in transport costs. Section 5 gives the conclusions.

2 Model

In this section, after describing the basic setting, we explain the structure of the game.

2.1 Basic setting

The economy consists of two regions, 1 and 2. In each region, firms produce a horizontally differentiated product, which is formulated as in Salop (1979). Varieties are "located" in a one-dimensional characteristics space, which is a circle of length 1. Each firm produces one variety, and each variety is produced by one firm. The number of firms is therefore equal to the number of varieties in each region. Furthermore, the one-to-one correspondence between a firm and a variety implies that each firm can be identified by the location of its variety in the characteristics space. Thus, we say that a firm is "located" at a particular point on the circle when it produces a variety at that point.

Our attention is limited to the case where the firms, and thus the varieties, are located equidistantly on the circle in each region. They line up at distances $1/n_i$ from each other in region i, where $n_i \in \mathbf{N}_0$, the set of non-negative integers, is the number of firms in region i ($i = 1, 2$). Furthermore, without loss of generality, we measure the distances on the circle in the clockwise direction. The location of the first firm is at a distance of $\bar{x}_i$ from the origin of the circle in region i (see Figure 1). Firms are numbered in the order of their locations in the clockwise direction, and the varieties are referred to accordingly. Firm k is therefore located at $x_i^k \equiv \bar{x}_i + (k-1)/n_i$ in region i for $k \in N_i \equiv \{1, 2, \cdots, n_i\}$.

Figure 1: Characteristics space

To produce a variety, each firm pays a constant marginal cost, which is normalized to 0, and a fixed cost f. The profit of firm k in region i is therefore equal to

$$\Pi_i^k = \pi_i^k - f \qquad \text{with} \qquad \pi_i^k \equiv p_i^k d_i^k, \tag{1}$$

where π_i^k is its operating profit, p_i^k is the price of its variety, and d_i^k is the demand for it (or the mass (number) of consumers who buy it).

Moreover, there are $\bar{L}$ consumers in the economy. They do not move across regions. The share of consumers in region i is denoted by λ_i, with $\lambda_1 + \lambda_2 = 1$ $(i = 1, 2)$. We assume, without loss of generality, that region 1 is no smaller than region 2: $\lambda_1 \geq 1/2$.

Each consumer prefers a certain characteristic.[4] In parallel with the terminology used in the theory of spatial competition, we say that in the characteristics space, a consumer is "located" at the point of her most-preferred characteristic. To keep the analysis simple, we concentrate on the case where the most-preferred characteristics are distributed uniformly over the circle. Furthermore, the farther away the consumed variety is located from the consumer's most-preferred characteristic, the lower the level of utility she receives. More specifically, the decrease in utility is a quadratic function of the distance between the consumed variety and the consumer: the utility she gets by consuming variety k in region i is given by

$$v_i^k = r - \left[p_i^k + c\left(z - x_i^k\right)^2\right], \tag{2}$$

where z is the location of the consumer, p_i^k is the price of the variety, and r is the reservation price of the product: consumers are willing to pay up to this amount for its consumption. Here, $c > 0$ is a constant, which we call a characteristics-cost coefficient, and we refer to $c\left(z - x_i^k\right)^2$ and $p_i^k + c\left(z - x_i^k\right)^2$ as the "characteristics cost" and "subjective price" of the variety, respectively.[5] The reason why we assume a quadratic characteristics cost, in contrast to the original setting by Salop (1979), who assumes a linear cost, will be explained later.

Finally, in order to get a variety, consumers need to visit the firm, or the "shop", that produces the variety. To visit a shop in a foreign region, they must pay a transport cost, t. To visit a shop in their home region, on the other hand, they do not have to pay any transport costs. The overall utility of a consumer who lives in region j and buys variety k in region i is thus represented by

$$V_{ij}^k = v_i^k - \tau_{ij}, \qquad (k \in N_i; i = 1, 2; j = 1, 2), \tag{3}$$

where $\tau_{ij} = 0$ if $i = j$ and $\tau_{ij} = t$ when $i \neq j$.

[4]In our model, each consumer consumes at most one variety. In the NEG models, on the other hand, consumers have a "love for variety", so each of them consumes *all* the varieties, however small the amount of each variety is. This difference is not fundamental. It is known that the aggregate demand of heterogeneous consumers each of whom consumes only one variety coincides with its counterpart in the NEG models with the love for variety (see Anderson et al. (1992) and Combes et al. (2008)).

[5]In the spatial competition literature, the subjective price is usually called a "delivered price". However, this paper deals with two spaces, namely a characteristics space and a geographical space. To avoid confusion between them, we use the term subjective price whenever the characteristics space is concerned.

2.2 Structure of the game

We consider the following three-stage game.

In the first stage, firms enter the industry if they will obtain a non-negative profit by doing so, and exit from it if they have been incurring a loss. At the equilibrium, therefore, each firm earns a non-negative profit, and if one additional firm enters the market, it will suffer a loss. In other words, the pari of equilibrium number of firms, (n_1^*, n_2^*), must satisfy the following equations:[6]

$$\Pi_i(n_1^*, n_2^*) \geq 0 \quad \text{if} \quad n_i^* > 0 \qquad (i = 1, 2) \tag{4}$$

and

$$\Pi_1(n_1^* + 1, n_2^*) < 0 \quad \text{and} \quad \Pi_2(n_1^*, n_2^* + 1) < 0. \tag{5}$$

Here, the profit of a firm in region i, defined by (1), is expressed as a function of the numbers of firms in the two regions. (Because it turns out that firms in the same region earn an equal profit, the superscript k is dropped here.) Note that, for the second requirement, we consider only the entry of a single firm: we do not take into account the possibility that more than one firm sets up at the same time.[7]

In the second stage, firms and consumers move simultaneously.

On the one hand, firms decide the price of a variety so as to maximize their profit, given the prices charged by their competitors. To keep the analysis tractable, we focus on the symmetric Nash equilibrium, in which all the firms in the same region charge the same price. It then suffices to examine the problem of a representative firm in each region when all the other firms in that region charge an equal price. We can write the operating profit of the representative firm in region i as $\pi_i(p_i, \bar{p}_i)$, where p_i and $\bar{p}_i$ are the prices charged by that firm and the competitors in that region, respectively. The equilibrium price, p_i^*, is the solution to $\max_{p_i} \pi_i(p_i, p_i^*)$.

On the other hand, consumers decide whether to go shopping or to stay at home, and which region to visit if they decide to shop.[8] In this decision-making process, consumers know the number of varieties provided in each region, which is observable at the end of the first stage, as well as the fact that they line up equidistantly. However, they do not know the locations of the provided varieties. More precisely, for them, the *absolute* position of the first firm, $\bar{x}_i$, is a random variable. Because the location of the available variety nearest to

[6] The assumption that firms do enter the industry if $\Pi_i(n_1, n_2) = 0$ is not important. We would get similar results if we assumed otherwise.

[7] This is consistent with the concept of the Nash equilibrium.

[8] In this paper, we assume that consumers can visit a shop at most once. This considerably simplifies the analysis, eliminating the possibility that they engage in search. An important agenda for future research is to relax this assumption so as to take into account consumers' search behaviors.

their own location also becomes a random variable, the consumers' decision-making is based on their *expectation* of it. In relation to this point, two simplifying assumptions are imposed. First, we suppose that the location of the first firm is designated randomly: the probability that it is at a given point on the circle is uniformly distributed. Secondly, consumers are assumed to be risk neutral: they choose which region to visit by comparing the levels of utility computed from *the expected location of the nearest variety.*

Let $A = \{0, 1, 2\}$ be a set of consumers' choices on a shopping trip: 1 and 2 represent visiting a shop in regions 1 and 2, respectively, and 0 means refraining from going shopping. We denote by U_{aj} the *expected* utility of a consumer in region j when she takes $a \in A$. Then (3) implies that visiting a shop in region i yields

$$U_{ij} = u_i - \tau_{ij} \quad \text{where} \quad u_i \equiv E\left[\max_{k \in N_i}\left[v_i^k\right]\right] \qquad (i = 1, 2; j = 1, 2). \tag{6}$$

On the other hand, refraining from going shopping yields $U_{0j} = 0$. The problem of consumers in region j is then to find $a \in A$ that maximizes U_{aj}. We denote the set of the solutions by A_j^*: $A_j^* \equiv \left\{a : a = \arg\max_{a \in A}\left[U_{aj}\right]\right\}$. Furthermore, let us denote the mass of region j consumers who choose option $a \in A$ as a share of the region j population by $s_{aj} \in [0, 1]$ ($s_{0j} + s_{1j} + s_{2j} = 1$). We must then have $\sum_{a \in A_j^*} s_{aj} = 1$ ($j = 1, 2$). The share of consumers who take option $a \in A$ in the *entire economy* is given by $s_a \equiv \lambda_1 s_{a1} + \lambda_2 s_{a2}$ ($a \in A$).

In the third and last stage, consumers realize $\bar{x}_i$ upon arriving at region i, and decide whether to buy a variety or not and, if any, which variety to buy. For the sake of simplicity, we focus on the case where they buy at most 1 unit of the product. In this stage, furthermore, the transport cost paid in the second stage is already sunk. Therefore, a consumer who is located at z and visits region i

$$\begin{cases} \text{buys a variety at } x_i^*(z) = \arg\max_{x_i^k | k \in N_i}\left[v_i^k\right] & \text{if } \max_{x_i^k | k \in N_i}\left[v_i^k\right] \geq 0 \\ \text{buys no variety} & \text{otherwise.} \end{cases} \tag{7}$$

To derive the demand that each firm receives in the third stage, let us consider the set of the locations of consumers who buy from firm k in region i, $M_i^k \equiv \{z : x_i^*(z) = x_i^k\}$. Since the characteristics cost is quadratic, the set becomes a segment, which we call the "market area" of firm k or variety k. Since everything is symmetric, we can express the size of the market area of a firm as a function of its own price and of that of its competitors, i.e., as $m_i(p_i, \bar{p}_i)$. Finally, since the circle has a unit length, the demand for each firm is given by

$$d_i(p_i, \bar{p}_i) = s_i \bar{L} m_i(p_i, \bar{p}_i). \tag{8}$$

Note that no mention is made of the firms' choices with regard to the variety to produce. It would be possible to incorporate this into our model by expanding the price game in the second stage to a location–price game (here, "location" obviously refers to that in the characteristics space). However, it has already been established that in a location–price game with a circular characteristics space, *there is an equilibrium with a given number of firms being placed evenly spaced apart along the circle when the characteristics cost is quadratic* (see Economides (1988)). We therefore *postulate* this equilibrium configuration. Furthermore, it is known that equal spacing is not necessarily supported by an equilibrium if the characteristics cost is not quadratic. If it is linear, for instance, there is the possibility of a firm deviating from the equal spacing configuration to obtain a higher profit. This is why we assume quadratic characteristics costs in this paper.

3 Equilibrium conditions

As the solution concept, we use a subgame perfect Nash equilibrium. The equilibrium is obtained by the standard method of backward induction. Because the third-stage subgame has already been solved (see (7)), we begin with the second-stage subgame. First, the firms' decisions on the prices to charge are discussed. Secondly, we analyze the consumers' decisions on the shopping trip. Lastly, we turn to the firms' decisions on entry and exit in the first stage.

3.1 Price

To begin with, let us examine the firms' pricing strategies determined in the second stage. The setup of this part is the same as that in Salop (1979), except that he assumes a linear characteristics cost, not a quadratic cost.

As has been mentioned, we focus on a symmetric equilibrium and examine the behavior of a representative firm, say firm k in region i, who charges p_i, and all the other firms in that region charge the same price, $\bar{p}_i$. From (1) and (8), the firm's operating profit is expressed as

$$\pi_i\left(p_i, \bar{p}_i\right) = s_i \bar{L} p_i m_i(p_i, \bar{p}_i). \tag{9}$$

We begin the analysis with the case of $n_i \geq 2$.

To derive the size of the market area, let us define $z_i(p_i, \bar{p}_i)$ as the location of a marginal consumer who is located in the interval (x_k, x_{k+1}) and is indifferent with respect to buying from firm k and buying from firm $k+1$: $z_i(p_i, \bar{p}_i) \equiv n_i\left(\bar{p}_i - p_i\right)/(2c) + (x_{k+1} + x_k)/2$. For variety k, such a marginal consumer pays a subjective price equal to $q_i\left(p_i, \bar{p}_i\right) \equiv p_i + c\left[z_i\left(p_i, \bar{p}_i\right) - x_k\right]^2$. Depending on the relative size of this subjective price compared to the reservation price, we can identify three types of market area,

$$m_i(p_i, \bar{p}_i) = \begin{cases} m^M(p_i) \equiv 2\left(\dfrac{r - p_i}{c}\right)^{\frac{1}{2}} & \text{if} \quad q_i(p_i, \bar{p}_i) > r, \\ m^B(\bar{p}_i) \equiv 2\Big[z_i\big(\rho_i(\bar{p}_i), \bar{p}_i\big) - x_k\Big] & \text{if} \quad q_i(p_i, \bar{p}_i) = r, \\ m^C(p_i, \bar{p}_i) \equiv 2\big[z_i(p_i, \bar{p}_i) - x_k\big] & \text{if} \quad q_i(p_i, \bar{p}_i) < r, \end{cases} \tag{10}$$

as long as $z_i(p_i, \bar{p}_i) \in (x_k, x_{k+1})$. Here,

$$\rho_i(\bar{p}_i) \equiv r - c\left[\frac{1}{n_i} - \left(\frac{r - \bar{p}_i}{c}\right)^{\frac{1}{2}}\right]^2$$

gives, as a function of the price charged by the competitors, firm k's price for which the associated subjective price equals the reservation price for the marginal consumer. In other words, $\rho_i(\bar{p}_i)$ is equal to the value of p_i that solves $q_i(p_i, \bar{p}_i) = r$.

The first case in (10) describes the situation in which the market area of firm k does not touch those of the neighboring firms (see the first panel in Figure 2). We refer to this type of market area as a *monopoly type*, or an M type. It follows from (9) that the operating profit of firm k is equal to $\pi_i^M(p_i) \equiv 2s_i\bar{L}p_i\left[(r - p_i)/c\right]^{\frac{1}{2}}$. Furthermore, the last case describes the situation in which the competition between firm k and its neighbors is so intense that it will lose some of its market area when the competitors slightly lower their prices (see the last panel in Figure 2). For this type, namely a *competitive type*, or a C type, the operating profit of firm k becomes $\pi_i^C(p_i, \bar{p}_i) \equiv s_i\bar{L}p_i\Big[n_i^2(\bar{p}_i - p_i) + c\Big]\Big/(cn_i)$. Finally, the second case is a borderline case between the above two cases: if firm k slightly enhances its price, its market area becomes M type, whereas if it slightly reduces its price, its market area becomes C type (see the second panel in Figure 2). When the market area of firm k is this *borderline type*, or B type, its profit is given by $\pi_i^B(\bar{p}_i) \equiv s_i\bar{L}\rho_i(\bar{p}_i)\Big[n_i^2\{\bar{p}_i - \rho_i(\bar{p}_i)\} + c\Big]\Big/(cn_i)$.

Figure 2: Three types of market area at equilibrium

A necessary condition for firm k's profit maximization is given by

$$-\frac{\partial \ln m_i(p_i, \bar{p}_i)}{\partial \ln p_i} = 1, \tag{11}$$

where the left-hand side is the price elasticity of the size of the market area. Furthermore, note that at any symmetric equilibrium, the market area of each firm must be one of the above three types.

First, suppose that the equilibrium involves a market area of type M. Solving (11) for $m^M(p_i)$ yields $p_i = p^M \equiv 2r/3$. Since the market area is isolated, the optimal price does not depend on the price charged by the other firms. Therefore, all the firms charge p^M at the equilibrium. The size of each market area and the operating profit of each firm are given by $m^M(p^M) = 2r^{\frac{1}{2}}(3c)^{-\frac{1}{2}}$ and $\pi_i^M(p^M) = 4s_i\bar{L}r^{\frac{3}{2}}c^{-\frac{1}{2}}/3$, respectively. At the equilibrium, $q_i(p_i, \bar{p}_i) > r$ is reduced to

$$r < r_i^M \equiv \frac{3c}{4n_i^2}. \tag{12}$$

Secondly, suppose that a C-type market area is realized at the equilibrium. Computing the fixed point $p_i = \bar{p}_i$ for the profit maximization prescribed by (11), we derive the equilibrium price for type C: $p_i^C \equiv c/n_i^2$. The size of each market area and the operating profit of each firm are given by $1/n_i$ and $\pi_i^C(p_i^C, p_i^C) = cs_i\bar{L}/n_i^3$, respectively. We can rewrite $q_i(p_i, \bar{p}_i) < r$ as

$$r > r_i^C \equiv \frac{5c}{4n_i^2}. \tag{13}$$

Note that $r_i^M < r_i^C$.

Thirdly, and finally, suppose that a B-type market area is realized at the equilibrium. At a symmetric equilibrium with $p_i = \bar{p}_i$, the price is given by $p_i^B \equiv r - c/(4n_i^2)$. The size of each market area and the operating profit of each firm are $1/n_i$ and $\pi_i^B(p_i^B) = s_i\bar{L}(4n_i^2r - c)/(4n_i^3)$, respectively.

We can now establish the following result.

Lemma 1. *Suppose that $n_i \geq 2$. There exists a unique symmetric Nash equilibrium such that*

i) every firm charges p^M if $r < r_i^M$,

ii) every firm charges p_i^C if $r > r_i^C$, and

iii) every firm charges p_i^B if $r_i^M \leq r \leq r_i^C$.

The proof is tedious and is relegated to the Appendix.

This is a good place to introduce one simplifying assumption. We confine our analysis to the case where the reservation price is sufficiently high that $r > r_i^C$ for any $n_i \geq 2$: *the market area of each firm becomes C type at equilibrium whenever there are no fewer than two firms in a region.* A sufficient condition is given by the following inequality:

Assumption 1. (Competitive market) $r > 5c/16$.

Let us turn our attention from a regional economy with $n_i \geq 2$ to that with $n_i = 1$. Suppose that the only firm charges p^M. If it is sufficiently high for only part of the circle to be served by the firm, p^M maximizes its profit. This occurs when the associated subjective price at the point farthest from the location of the firm exceeds the reservation price, that is, when $p^M + c/4 > r$ or $r < 3c/4 = r_i^M\big|_{n_i=1}$. If p^M is low enough for the firm to deliver its product to the entire market, it is not a profit-maximizing price; with a price higher than this, the firm can still serve the entire market while raising its revenue. In this case, the firm chooses a price such that the associated subjective price at the point farthest from its location exactly reaches the reservation price. Such a price turns out to be $p_i^B\big|_{n_i=1} = r - c/4$. Assumption 1 guarantees that this is positive.

To sum up, the equilibrium market area of each firm is C type if $n_i \geq 2$, M type if $n_i = 1$ and $r < 3c/4$, and, finally, B type if $n_i = 1$ and $r \geq 3c/4$.

Lemma 2. *Given the number of firms in region i, the price, the size of the market area, and the profit of a firm in that region at the equilibrium are obtained as follows:*

$$p_i^* = \begin{Bmatrix} p_i^C \\ p^M \\ p^B\big|_{n_i=1} \end{Bmatrix}, \quad m_i^* = \begin{Bmatrix} 1/n_i \\ m^M\left(p^M\right) \\ 1/n_i \end{Bmatrix}, \quad \pi_i^* = \begin{Bmatrix} \pi_i^{*C} & \equiv & \pi_i^C\left(p_i^C, p_i^C\right) \\ \pi_i^{*M} & \equiv & \pi_i^M\left(p^M\right) \\ \pi_i^{*B} & \equiv & \pi_i^B\left(p_i^B\right) \end{Bmatrix}$$

$$\textit{if} \quad \begin{Bmatrix} n_i \geq 2 \\ n_i = 1 \ \textit{and} \ \ r < 3c/4 \\ n_i = 1 \ \textit{and} \ \ r \geq 3c/4 \end{Bmatrix}.$$

3.2 Shopping trip

In this subsection, we turn to the consumers' decisions on the shopping trip in the second stage. At this stage, the number of firms in each region is known and given to consumers. From this piece of information, they can correctly infer the equilibrium prices that will be realized at the end of the second stage. Therefore, the consumers' decision-making in the second stage involves the equilibrium outcome of the firms' decision-making in that stage, which is summarized in Lemma 2, despite the fact that they move simultaneously.

Consider a consumer in region j who visits a shop in region i ($i = 1, 2$; $j = 1, 2$). Because the origin is chosen at random from the uniform distribution and the varieties are located at distances $1/n_i$ apart from each other along the circle, the cumulative distribution function of $d \equiv \min_{k \in N_i} z - x_i^k$ for a given z becomes $F_i(d) = 2n_i d$. The expected utility of the consumer is therefore given by

$$u_i \equiv \int_{d=0}^{m_i^*/2} r - \left(p_i^* + cd^2\right) \ \mathrm{d}F_i(d) = n_i m_i^* \left[r - p_i^* - \frac{c\left(m_i^*\right)^2}{12}\right] \tag{14}$$

(see (2) and (6)).

The right-hand side of the last equality in (14) shows that the utility level depends on four factors. The first is the reservation price. As it increases, the value of the product is enhanced, which raises the utility level: a positive *reservation-price effect.* The second factor is the price of the varieties. The higher it is, the lower the utility level is: a negative *price effect.* The third factor is the amount of the characteristics cost that a consumer needs to pay. This is captured by $c\left(m_i^*\right)^2/12$ in (14), which represents the average characteristics-cost payment. As this payment increases, the utility level declines: a negative *characteristics-cost-payment* effect. The last factor is the probability that a consumer can find a variety worth buying upon arriving at one of the two regions. This is given by $n_i m_i^*$ because the length of the circle is normalized to unity. A higher probability is associated with a higher utility level, *ceteris paribus*: a positive *range-of-varieties effect.*

The next result immediately follows from Lemma 2.

Lemma 3. *The expected utility, exclusive of the transport cost, is obtained as follows:*
i) If $n_i \geq 2$, *then* $u_i^* = u_i^C \equiv r - 13c/\left(12n_i^2\right) \qquad (i = 1, 2).$
ii) If $n_i = 1$, *then* $u_i^* = \begin{cases} u^M \equiv 4r^{\frac{3}{2}}c^{-\frac{1}{2}}/9\sqrt{3} & \text{if} \quad r < 3c/4 \\ u^B \equiv c/6 & \text{if} \quad r \geq 3c/4 \end{cases} \qquad (i = 1, 2).$

Here, note that $u_i^C > 0$ by Assumption 1. Therefore, u_i^* is always positive.

Because u_i^* plays an important role in the following analysis, it is worthwhile examining how various factors affect each of u_i^C, u^M, and u^B.

First, let us consider u_i^C. When the market area of each firm is C type, the probability of a consumer finding a variety worth buying is always 1. There is therefore no range-of-varieties effect.

We begin by explaining that u_i^C increases with n_i. As n_i rises, competition among firms becomes fiercer, which drives them to charge lower prices. This has a favorable impact on u_i^C through the price effect mentioned earlier. Furthermore, as a result of the rise in n_i, the market area of each firm shrinks. So, consumers now need to pay only a smaller amount of the characteristics cost on average, which raises u_i^C through the characteristics-cost-payment effect. Because there is no reservation-price effect or range-of-varieties effect, we are left with only these two positive effects. Such *pro-competitive effects* are not observed in NEG models with a Dixit–Stiglitz type preference, and only the first effect, namely the price effect, exists in NEG models with a quasi-linear utility. In addition, because u_i^C is concave in n_i, *the pro-competitive effects are weaker when there are more firms.*

Furthermore, u_i^C increases with r and decreases with c. The reason for the former is the positive reservation-price effect. For a change in c, two forces are at work. First, a higher c

shields firms more from competition; this gives them a greater monopoly power. This tempts them to charge a higher price. Secondly, the rise in c boosts the expected characteristics-cost payment since the size of each market area is fixed. Both of these have an adverse impact on the utility level through the price effect and the characteristics-cost-payment effect.

Secondly, u^M increases with r and decreases with c. On the one hand, as r increases, the price rises and the market area expands. The utility level rises because of the reservation-price effect and the range-of-varieties effect, but falls because of the price effect and the characteristics-cost-payment effect. Because it turns out that the first two effects dominate the last two, the utility level rises. On the other hand, an increase in c directly raises the average characteristics-cost payment but indirectly reduces it by narrowing each market area. Because the direct effect more than offsets the indirect one, the characteristics-cost-payment effect is negative. At the same time, as a result of the shrinkage of the market area, the probability of a consumer finding a variety worth buying decreases, i.e., a negative range-of-varieties effect. In the absence of a reservation-price effect and a price effect, the overall effect of a rise in c is negative.

Thirdly, and lastly, u^B increases with c but does not depend on r. Note that when the market area is B type, its size is constant. There is therefore no range-of-varieties effect. A rise in c has two effects. First, because it scales down the competitors' market areas, a firm now needs to charge a lower price to keep its market area B type. Secondly, it directly raises the characteristics-cost payment. Thus, the utility level increases through the price effect and decreases through the characteristics-cost-payment effect. It turns out that the former dominates the latter. A rise in r, on the other hand, boosts the price by the same amount. The price effect therefore just offsets the reservation-price effect and thus u^B does not change.

We are now ready to examine the consumers' decision-making.

First of all, suppose that there are some firms in each region. Since $U_{jj} = u_j^* > 0$ implies $\max\big[U_{1j}, U_{2j}\big] > 0$, *no consumer chooses the option to stay at home*, $0 \notin A_j^*$, that is, $s_{0j} = 0$ for $j = 1, 2$. Consequently, if $U_{1j} > U_{2j}$, all the consumers in region j visit a shop in region 1 ($A_j^* = \{1\}$), and $s_{1j} = 1$ and $s_{2j} = 0$. Similarly, if $U_{2j} > U_{1j}$, all the consumers in region j visit a shop in region 2 ($A_j^* = \{2\}$), and $s_{1j} = 0$ and $s_{2j} = 1$. Finally, if $U_{1j} = U_{2j}$, some consumers visit a shop in region 1 and the others visit a shop in region 2 ($A_j^* = \{1, 2\}$), and $s_{1j} + s_{2j} = 1$.

Next, suppose that firms are concentrated in region i. Then, if $U_{ij} \geq 0$ for $j \neq i$, both the consumers in region 1 and those in region 2 visit a shop in region i ($A_1^* = A_2^* = \{i\}$), and $s_{i1} = s_{i2} = 1$ and $s_{01} = s_{j1} = s_{02} = s_{j2} = 0$ for $j \neq i$. Otherwise, only the consumers in region i visit a shop in that region ($A_i^* = \{i\}$ and $A_j^* = \{0\}$), and $s_{ii} = s_{0j} = 1$ and

$s_{ij} = s_{j1} = s_{j2} = s_{0i} = 0$.

With some manipulation, we can establish the following result.

Lemma 4. *i) Suppose that there are some firms in both regions. Then, the equilibrium demand shares must satisfy*

$$(s_1, s_2) = \begin{cases} (\lambda_1, \lambda_2) & \text{if } t > u_1^* - u_2^* \text{ and } t > u_2^* - u_1^* \\ (1, 0) & \text{if } t < u_1^* - u_2^* \\ (0, 1) & \text{if } t < u_2^* - u_1^* \\ (\lambda_1 + \mu_2, \lambda_2 - \mu_2) & \text{if } t = u_1^* - u_2^* \\ (\lambda_1 - \mu_1, \lambda_2 + \mu_1) & \text{if } t = u_2^* - u_1^* \end{cases}$$

for $\mu_j \in [0, \lambda_j]$ ($j = 1, 2$).
ii) Suppose that firms are concentrated in region i. Then, the equilibrium demand shares must satisfy

$$(s_1, s_2) = \begin{cases} (1, 0) & \text{if } t \leq u_i^* \\ (\lambda_i, 0) & \text{if } t > u_i^*. \end{cases}$$

Here, μ_j is the ratio of consumers in region j who visit a shop in the foreign region.

Three observations follow. First, *the five cases in i) are mutually exclusive and exhaustive*, as well as the two cases in ii). Secondly, *at the equilibrium, there is no possibility of a "cross trip"*: the consumers in region 1 and those in region 2 never visit a shop in their respective foreign regions at the same time.[9] Finally, if n_i rises and the rise involves a change in the *pattern* of (s_1, s_2), then s_i increases or remains constant. For example, as n_1 rises, the pattern may switch from (λ_1, λ_2) to $(\lambda_1 + \mu_2, \lambda_2 - \mu_2)$ or to $(1, 0)$; in either case, s_1 increases or remains constant. However, it never switches from (λ_1, λ_2) to $(\lambda_1 - \mu_1, \lambda_2 + \mu_1)$ or to $(0, 1)$: s_1 never decreases. Consequently, *s_i is a non-decreasing function of n_i*.

3.3 Number of firms

In this subsection, we examine the number of firms in each region, which is determined through free entry at the first stage, given the demand shares obtained in Lemma 4. Let us denote the level of profit corresponding to the operating profit derived above as a function of s_i and n_i: $\widetilde{\Pi}^T(s_i, n_i) \equiv \pi_i^{*T} - f$ for $T \in \{C, M, B\}$ ($i = 1, 2$).[10]

[9] To see this, note that $U_{11} > U_{12}$ and $U_{22} > U_{21}$ because $t > 0$. If $U_{21} \geq U_{11}$, therefore, we have $U_{22} > U_{12}$. Consequently, $U_{21} \geq U_{11}$ and $U_{12} \geq U_{22}$ cannot hold at the same time.

[10] $\pi_i^{*M} - f$ does not actually depend on n_i. For the sake of convenience, however, we express it as a function of not only s_i, but also of n_i, as $\widetilde{\Pi}^M(s_i, n_i)$.

To begin with, we ask how many firms the regional economy can accommodate when the market area of each firm is C type. To answer the question, let us solve $\widetilde{\Pi}^C(s_i, n_i) = 0$ for n_i to obtain $n_i = \bar{n} s_i^{\frac{1}{3}}$. Here, $\bar{n} \equiv \left(c\bar{L}/f\right)^{\frac{1}{3}}$ denotes the *potential* of the economy, which gives the number of firms that earn zero profit when all the consumers in the economy visit a shop in the same region ($s_i = 1$), with the integer constraint being disregarded. The potential increases with total population, the characteristics-cost coefficient, and the inverse of the fixed cost, all of which measure the profitability of a market (recall that the characteristics-cost coefficient is positively related to the spatial monopoly power of a firm). Now, let us define $n^C(s_i)$ as the maximum integer that is no greater than $\bar{n} s_i^{\frac{1}{3}}$:

$$n^C(s_i) \in \left(\bar{n} s_i^{\frac{1}{3}} - 1, \bar{n} s_i^{\frac{1}{3}}\right] \cap \mathbf{N}_0. \tag{15}$$

Since $\widetilde{\Pi}^C(s_i, n_i)$ decreases with n_i, it represents the maximum number of firms region i's market can accommodate for a given share of demand.[11] In other words, $\widetilde{\Pi}^C\left(s_i, n^C(s_i)\right) \geq 0$ and $\widetilde{\Pi}^C(s_i, n_i) < 0$ for any $n_i \in \left(n^C(s_i), \infty\right) \cap \mathbf{N}_0$.

Using this result, we can fully characterize the equilibrium number of firms. For this purpose, let us define $\widehat{s}^C$ and $\widehat{s}^T$ as solutions to $\widetilde{\Pi}^C\left(\widehat{s}^C, 2\right) = 0$ and $\widetilde{\Pi}^T\left(\widehat{s}^T, 1\right) = 0$, respectively, for $T \in \{M, B\}$. These are the demand shares that make the profit of a firm or firms in a region equal to 0. For $\widehat{s}^C$, we consider the situation in which there are two firms in that region, and for $\widehat{s}^M$ and $\widehat{s}^B$, we consider the situation in which there is only one firm there. It turns out that $\widehat{s}^C \equiv 8/\bar{n}^3$, $\widehat{s}^M \equiv (3c)^{\frac{3}{2}} r^{-\frac{3}{2}} / \left(4\bar{n}^3\right)$, and $\widehat{s}^B \equiv 4c/\left[(4r - c)\bar{n}^3\right]$.

Now, suppose that the demand share is so high that $s_i \geq \widehat{s}^C$. Since $\widetilde{\Pi}^C(s_i, n_i)$ increases with s_i, this implies that $\widetilde{\Pi}^C(s_i, 2) \geq 0$. Therefore, at least two firms can, and indeed do, because of free entry (see (5)), operate in region i. Their market area is therefore C type, which implies that the number of firms is equal to $n^C(s_i) \geq 2$. Next, suppose that the demand share is not sufficiently high, that is, $s_i < \widehat{s}^C$. Because the inequality implies that $\widetilde{\Pi}^C(s_i, 2) < 0$, no more than one firm can operate in region i: either one firm operates or no firms operate. If, on the one hand, $r < 3c/4$, the market area of the one firm must be M type. However, $\widetilde{\Pi}^M(s_i, 1) \geq 0$ if and only if $s_i \geq \widehat{s}^M$. In this case, therefore, one firm operates if $s_i \geq \widehat{s}^M$ and no firms operate otherwise. On the other hand, if $r \geq 3c/4$, the market area of the one firm must be B type. By similar reasoning, we conclude that one firm operates if $s_i \geq \widehat{s}^B$ and no firms operate otherwise. (Note that $\widehat{s}^B > 0$ as long as $r \geq 3c/4$.) The following lemma summarizes these findings.

[11] $\partial \widetilde{\Pi}^C(s_i, n_i) / \partial n_i < 0$ follows from the fact that the entry of a firm makes the market more competitive. This corresponds to the "price-cutting effect" described by Konishi (2005). However, as n_i rises, s_i may change. Since s_i is a non-decreasing function of n_i, as mentioned above, $\widetilde{\Pi}^C(s_i, n_i)$ increases or remains unchanged as a result of the change in s_i, which is what Konishi calls the "market-size effect".

Lemma 5. *The equilibrium numbers of firms must satisfy*

$$n_i = \begin{cases} n^C\big(s_i\big) \geq 2 & \text{if} \quad s_i \geq \widehat{s}^C \\ 1 & \text{if} \quad s_i \in \big[\widehat{s}^T, \widehat{s}^C\big) \\ 0 & \text{if} \quad s_i < \widehat{s}^T, \end{cases}$$

where $T = M$ if $r < 3c/4$ and $T = B$ otherwise ($i = 1, 2$).

We focus on a non-trivial case where the fixed cost per capita is sufficiently small for *at least two firms to operate in a region whenever no fewer than $\lambda_2 \bar{L}$ consumers visit a shop in that region*, that is, $n^C(\lambda_2) \geq 2$. Because $\widetilde{\Pi}^C(s_i, n_i)$ increases with s_i and decreases with n_i, it is equivalent to $\widetilde{\Pi}^C(\lambda_2, 2) \geq 0$. A necessary and sufficient condition is that $\lambda_2 \geq \widehat{s}^C$, which we assume.

Assumption 2. (Profitable market) $\lambda_2 \geq \widehat{s}^C$.

Note that this assumption implies that $\bar{n} > 16^{\frac{1}{3}} > 2$ since $\lambda_2 \leq 1/2$.

One comment is worth adding. The above assumption guarantees that more than one firm operates in a region *at the equilibrium* if at least $\lambda_2 \bar{L}$ consumers visit a shop in that region. *Off the equilibrium*, however, it may be the case that only one firm operates, even if $\lambda_2 \bar{L}$ consumers or more visit a shop in the region. We can show that this firm can indeed earn a positive profit.[12] In other words,

$$\widetilde{\Pi}^T(s_i, 1) > 0 \quad \text{for any} \quad s_i \geq \lambda_2, \tag{16}$$

where $T = M$ if $r < 3c/4$ and $T = B$ otherwise ($i = 1, 2$).

4 Equilibrium distribution patterns

To recapitulate, we have so far obtained two relationships. Lemma 4 gives the equilibrium demand share in each region as a function of the equilibrium numbers of firms since u_i^* depends on n_i. To articulate this dependence, we hereafter denote u_i^* and u_i^C as functions of n_i, i.e., as $u(n_i) \equiv u_i^*$ and $u^C(n_i)$, respectively. Lemma 5 gives the equilibrium number

[12] On the one hand, suppose that the firm's market area is M type. Because Assumption 1 implies that $32(r/3c)^{\frac{3}{2}} > 1$, $32s_i(r/3c)^{\frac{3}{2}} > \lambda_2$ for any $s_i \geq \lambda_2$. This and Assumption 2 give $32s_i(r/3c)^{\frac{3}{2}} > \widehat{s}^C$, which is equivalent to $\widetilde{\Pi}^M(s_i, 1) > 0$. On the other hand, suppose that the market area is B type. Because this appears only if $r \geq 3c/4$, we have $r > 3c/8$, which is equivalent to $2(4r - c)/c > 1$. Consequently, $2s_i(4r - c)/c > \lambda_2$ for any $s_i \geq \lambda_2$. Hence, we conclude from Assumption 2 that $2s_i(4r - c)/c > \widehat{s}^C$, which is equivalent to $\widetilde{\Pi}^B(s_i, 1) > 0$.

of firms in each region as a function of the equilibrium demand share in that region. To complete the analysis, we need to "solve" these relationships simultaneously.

Note that Lemma 5 indicates that $n_i = 0$ whenever $s_i = 0$ because $\widehat{s}^M > 0$ and $\widehat{s}^B > 0$. Therefore, neither $(s_1, s_2) = (1, 0)$ nor $(s_1, s_2) = (0, 1)$ is supported by an equilibrium if there are some firms in *both* regions. Combining the two lemmas, consequently, we can readily verify the following result.

Lemma 6. *At the equilibrium, $\big((s_i, s_j), (n_i, n_j)\big)$ must be given by one of the following patterns ($i = 1, 2$; $j = 1, 2$; $j \neq i$).*
i) segregation, $\Big((\lambda_i, \lambda_j), \big(n^C(\lambda_i), n^C(\lambda_j)\big)\Big)$, only if

$$t > u\left(n^C(\lambda_1)\right) - u\left(n^C(\lambda_2)\right);^{13} \tag{17}$$

ii) full agglomeration at region i, $\Big((1, 0), \big(n^C(1), 0\big)\Big)$, only if

$$t \leq u\left(n^C(1)\right); \tag{18}$$

iii) partial agglomeration at region i, $\Big((\lambda_i, 0), \big(n^C(\lambda_i), 0\big)\Big)$, only if

$$t > u\left(n^C(\lambda_i)\right); \tag{19}$$

iv) incomplete agglomeration at region i, $(s_i, s_j) = \left(\lambda_i + \mu_j, \lambda_j - \mu_j\right)$, $n_i = n^C\left(\lambda_i + \mu_j\right)$, and

$$n_j = \begin{cases} n^C\left(\lambda_i - \mu_j\right) & \text{if} \quad \mu_j \in \left(0, \lambda_j - \widehat{s}^C\right] \\ 1 & \text{if} \quad \mu_j \in \left(\lambda_j - \widehat{s}^C, \lambda_j - \widehat{s}^T\right] \end{cases}$$

for $\mu_j \in \left(0, \lambda_j - \widehat{s}^T\right]$, only if

$$t = u\left(n^C\left(\lambda_i + \mu_j\right)\right) - u\left(n_j^*\right). \tag{20}$$

Here, $T = M$ if $r < 3c/4$ and $T = B$ otherwise.

In what follows, we elucidate the necessary conditions and sufficient conditions for each of the four patterns being supported by an equilibrium; this is done by examining the free entry conditions, (4) and (5), in addition to whichever one of (17)–(20) is relevant.

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[13] The necessary condition for segregation is that *both* (17) and $t > u\left(n^C(\lambda_2)\right) - u\left(n^C(\lambda_1)\right)$ are satisfied. However, (17) implies the latter inequality since $u(n_i)$ increases with n_i when $n_i \geq 2$ and $n^C(\cdot)$ is increasing monotonically.

4.1 Segregation

First, let us examine segregation.

For free entry conditions, it is obvious that (4) is always satisfied as a result of Assumption 2: $\widetilde{\Pi}^C\big(\lambda_i, n^C(\lambda_i)\big) \geq 0$ for $i = 1, 2$. Thus, what we need to examine is (5). Suppose that one additional firm enters the industry in region i to earn a profit equal to $\widetilde{\Pi}^C\big(s_i, n^C(\lambda_i) + 1\big)$. On the one hand, if s_i remains at λ_i, $\widetilde{\Pi}^C\big(s_i, n^C(\lambda_i) + 1\big) < 0$ by the definition of $n^C(\lambda_i)$, and, therefore, (5) is satisfied. On the other hand, if s_i changes, it increases because s_i is a non-decreasing function of n_i (recall the explanation after Lemma 4). The entrant might then succeed in obtaining a positive profit. A sufficient condition for (5) is, therefore, that the entry of an additional firm does not alter s_i. This is the case if both $t > u\big(n^C(\lambda_i) + 1\big) - u\big(n^C(\lambda_j)\big)$ and $t > u\big(n^C(\lambda_i)\big) - u\big(n^C(\lambda_j) + 1\big)$ hold for *both* $i = 1$ and $i = 2$ ($j \neq i$). The sufficient condition for this is

$$t > u\big(n^C(\lambda_1) + 1\big) - u\big(n^C(\lambda_2)\big). \tag{21}$$

Since (17) is satisfied whenever (21) holds, (21) is a sufficient condition for the segregation being supported by an equilibrium. This means the following: consumers in the smaller region patronize a shop in their home region not only when their foreign region boasts only as many firms as free entry admits, but even when one additional firm enters the latter region.

Roughly speaking, the necessary condition, (17), and the sufficient condition, (21), indicate that *if t is sufficiently high, the segregation is supported by an equilibrium; otherwise, it "breaks"*.[14] More precisely, we can obtain two critical levels, or " break points", of transport costs, which can be used to judge whether the segregation breaks or not.

Proposition 1. (Break points for segregation)
There are critical values $\underline{t}^{BR}$ and $\bar{t}^{BR}$ with $\underline{t}^{BR} < \bar{t}^{BR}$ such that the segregation breaks if $t \leq \underline{t}^{BR}$, but it does not break if $t > \bar{t}^{BR}$. The values are given by the following equations:[15]

$$\begin{aligned}
\underline{t}^{BR} &\equiv u^C\left(\bar{n}\lambda_1^{\frac{1}{3}} - 1\right) - u^C\left(\bar{n}\lambda_2^{\frac{1}{3}}\right) = \frac{13c}{12}\left[\frac{1}{\bar{n}^2\lambda_2^{\frac{2}{3}}} - \frac{1}{\left(\bar{n}\lambda_1^{\frac{1}{3}} - 1\right)^2}\right] \\
\bar{t}^{BR} &\equiv u^C\left(\bar{n}\lambda_1^{\frac{1}{3}} + 1\right) - u^C\left(\bar{n}\lambda_2^{\frac{1}{3}} - 1\right) = \frac{13c}{12}\left[\frac{1}{\left(\bar{n}\lambda_2^{\frac{1}{3}} - 1\right)^2} - \frac{1}{\left(\bar{n}\lambda_1^{\frac{1}{3}} + 1\right)^2}\right].
\end{aligned} \tag{22}$$

[14] The segregation in our model and the dispersion in the NEG models have the common feature that there are some firms in both regions. This is why we use the term "break" here, which is widely used in the NEG literature to express the situation in which dispersion fails to be supported by a stable equilibrium.

[15] Because of Assumption 2, $\bar{n}\lambda_1^{\frac{1}{3}} - 1 > 0$ and $\bar{n}\lambda_2^{\frac{1}{3}} - 1 > 0$.

The proof is tedious and is relegated to the Appendix.

It is important to note that *the reason for the segregation breaking is that the consumers in the smaller region are tempted to visit a shop in their foreign region if the transport cost is too low.* This is a clue to understanding how various factors affect the break point transport costs.

First, an inspection of (22) reveals that both $\underline{t}^{BR}$ and $\bar{t}^{BR}$ increase with λ_1 for $\lambda_1 \in \left[1/2, 1 - \widehat{s}^C\right]$; that is, the segregation becomes more likely to break as the population distribution becomes more biased. The reason is simple. As λ_1 rises, the distribution of firms also becomes more biased toward the larger region; this, through the pro-competitive effect, makes the option for the consumers in the smaller region to visit a shop in their foreign region relatively more attractive compared to the option of visiting a shop in their home region.

The result can be illustrated by a figure. In Figure 3, the two solid lines depict the utility levels associated with $\underline{t}^{BR}$ as functions of λ_1: the $\underline{u}_1$ curve represents $u^C\left(\bar{n}\lambda_1^{\frac{1}{3}} - 1\right)$, and the $\underline{u}_2$ curve represents $u^C\left(\bar{n}\lambda_2^{\frac{1}{3}}\right)$. Note that the former expression is the lower limit of the utility level that a consumer receives by visiting a shop in region 1, and the latter is the upper limit of the utility level that she receives by visiting a shop in region 2. Because the utility level increases with the number of varieties provided, the $\underline{u}_1$ curve slopes upward and the $\underline{u}_2$ curve slopes downward. Furthermore, $\underline{t}^{BR}$ is equal to the vertical distance between the two curves, i.e., $\underline{u}_1 - \underline{u}_2$. Therefore, $\underline{t}^{BR}$ increases with λ_1. We can similarly explain $\bar{t}^{BR}$ as the vertical distance between the two dotted lines that represent the utility levels associated with it.

Figure 3: Break point

Secondly, the effects of $\bar{n}$ on $\underline{t}^{BR}$ and $\bar{t}^{BR}$ are, in general, ambiguous. However, they are unambiguously negative in the special case where $\bar{n}$ is sufficiently large. There are two reasons why we pay attention to that case. First, in the context of our discussion of the location of commercial activities within a city, $\bar{n}$ corresponds to the number of stores at, say, a central commercial district in a city when retail stores are concentrated there. That number is usually quite large. Secondly, a large $\bar{n}$ alleviates the cumbersome effects of the integer constraint: by focusing on an economy with a large $\bar{n}$, and, therefore, large $n^C(s_i)$, one can obtain more or less "smooth" or "well-behaved" results. Now, we can show that when $\bar{n}$ is sufficiently large and $\lambda_1 > \lambda_2$, the break point transport costs are inversely related to $\bar{n}$.[16]

[16] Note that $\partial\underline{t}^{BR}/\partial\bar{n} = \underline{K}\underline{\Gamma}/\underline{\Delta}$ and $\partial\bar{t}^{BR}/\partial\bar{n} = \bar{K}\bar{\Gamma}/\bar{\Delta}$, where $\underline{K}$ and $\bar{K}$ are positive constants, $\underline{\Gamma} \equiv \bar{n}^3\lambda_1^{\frac{1}{3}} - \lambda_2^{-\frac{2}{3}}\left(\bar{n}\lambda_1^{\frac{1}{3}} - 1\right)^3$, $\bar{\Gamma} \equiv \lambda_1^{\frac{1}{3}}\left(\bar{n}\lambda_2^{\frac{1}{3}} - 1\right)^3 - \lambda_2^{\frac{1}{3}}\left(\bar{n}\lambda_1^{\frac{1}{3}} + 1\right)^3$, $\underline{\Delta} \equiv \bar{n}^3\left(\bar{n}\lambda_1^{\frac{1}{3}} - 1\right)^3 \geq 0$, and $\bar{\Delta} \equiv \left(\bar{n}\lambda_1^{\frac{1}{3}} + 1\right)^3\left(\bar{n}\lambda_2^{\frac{1}{3}} - 1\right)^3 \geq 0$. Suppose that $\lambda_1 > \lambda_2$. Then, as $\bar{n}$ tends to positive infinity, $\underline{\Gamma}$ and $\bar{\Gamma}$

This is explained as follows. As $\bar{n}$ grows, the numbers of firms in the respective regions at segregation tend to increase, which raises the utility levels through the pro-competitive effect. Recall that this effect is stronger when there are fewer firms. The utility level is, therefore, improved more in region 2 than in region 1 unless $\lambda_1 = \lambda_2$. Consequently, a smaller transport cost suffices to discourage consumers in region 2 from visiting a shop in region 1. Thus, the break points decrease. In Figure 3, the change is represented by a relatively small upward shift of the $\underline{u}_1$ curve and a relatively large upward shift of the $\underline{u}_2$ curve; this shortens the vertical distance between the two curves, $\underline{u}_1 - \underline{u}_2$, and therefore reduces $\underline{t}^{BR}$.

Finally, the impact of a change in c is also ambiguous. The direct effect is one that would be observed if $\bar{n}$ remained unchanged. As is shown in (22), this is positive. To understand the reason, remember that a rise in c makes consumers worse off through the price effect and the characteristics-cost-payment effect. However, it has a more devastating impact on consumers who visit a shop in the smaller region than it does on those who visit a counterpart in the larger region.[17] In other words, the larger region becomes relatively more attractive for consumers. To counteract this effect, the break points increase. We also have an indirect effect that is transmitted through a change in $\bar{n}$. As c rises, $\bar{n}$ increases. We have seen that this has a negative impact on the break point transport costs as long as $\bar{n}$ is sufficiently large. In this case, therefore, the direction of the indirect effect is opposite to that of the direct effect; consequently, the direction of the overall change depends on the relative magnitudes of the two effects.

These results are summarized in the following proposition.

Proposition 2. (Factors affecting the break points)
The break points rise, and the segregation becomes more likely to break, as
i) the population distribution gets more biased, and/or
ii) the potential of the economy declines (given that it is sufficiently high and the population distribution is not symmetric).

4.2 Full agglomeration

We next examine full agglomeration, in which firms are concentrated in one region, i.e., a "core", and all the consumers in the economy visit a shop in that region. Let region i be the core. Region $j \neq i$ is referred to as a "periphery". The pattern is supported by an

approach negative infinity. Consequently, $\partial \underline{t}^{BR}/\partial \bar{n} < 0$ and $\partial \bar{t}^{BR}/\partial \bar{n} < 0$ for sufficiently large $\bar{n}$.

[17] Note that $\mathrm{d}^2 u^C(n_i)/\mathrm{d}c\ \mathrm{d}n_i = 13/(6n_i^3) > 0$.

equilibrium, or is "sustainable" only if (18) is satisfied. If this condition did not hold, the consumers in the periphery would stop going shopping.

For free entry conditions, (4) is satisfied because $s_i = 1 > \widehat{s}^C$ by Assumption 2. Next, let us examine (5). Since $s_i = 1$ for $(n_i, n_j) = \left(n^C(1) + 1, 0\right)$ and $\widetilde{\Pi}^C\left(1, n^C(1) + 1\right) < 0$ by the definition of $n^C(\cdot)$, the first inequality of (5) is satisfied. The second inequality, on the other hand, demands that no single firm be willing to launch a business in the periphery. To begin with, suppose that

$$t < u\left(n^C(1)\right) - u(1). \tag{23}$$

Then, even if one firm starts a business in the periphery, the consumers in that region, as well as those in the core, continue to visit a shop in the core. Therefore, (23) is a sufficient condition for no single firm to be willing to operate in the periphery. On the other hand, a necessary condition is that

$$t \leq u\left(n^C(1)\right) - u(1). \tag{24}$$

Suppose that this is not the case. If a firm began a business in the periphery, consumers in that region would prefer visiting it to visiting a counterpart in the core. The shop in the periphery would therefore attract at least $\lambda_2 \bar{L}$ consumers and obtain a positive profit because of (16). Hence, (24) is a necessary condition.

Now, note that (23) implies (18) because $u(1) > 0$. Therefore, (23) is a sufficient condition for full agglomeration being sustainable. However, (24) is a necessary condition for this. These conditions indicate that *if t is sufficiently low, full agglomeration is sustainable; otherwise, it is not.* The following proposition gives two critical levels of transport costs, i.e., "sustain points", which are yardsticks in judging the sustainability of full agglomeration.

Proposition 3. (Sustain points for full agglomeration)
There are critical values $\underline{t}^{SS}$ and $\bar{t}^{SS}$ with $\underline{t}^{SS} < \bar{t}^{SS}$ such that full agglomeration is sustainable if $t \leq \underline{t}^{SS}$, but it is not sustainable if $t \geq \bar{t}^{SS}$. The values are given by the following equations:

$$\begin{array}{rcccl} \underline{t}^{SS} & \equiv & u\left(\bar{n} - 1\right) - u(1) & = & \underline{t}_T^{SS} \equiv r - \Psi^T - \underline{\Phi}, \\ \bar{t}^{SS} & \equiv & u\left(\bar{n}\right) - u(1) & = & \bar{t}_T^{SS} \equiv r - \Psi^T - \bar{\Phi}, \end{array} \tag{25}$$

where $T = M$ if $r < 3c/4$ and $T = B$ otherwise; $\Psi^M \equiv 4r^{\frac{3}{2}}c^{-\frac{1}{2}}/9\sqrt{3}$; $\Psi^B \equiv c/6$; $\underline{\Phi} \equiv 13c/\left[12(\bar{n} - 1)^2\right]$; and $\bar{\Phi} \equiv 13c/\left(12\bar{n}^2\right)$.

The proof is tedious and is relegated to the Appendix.

Figure 4 depicts $u\left(\bar{n} - 1\right)$, $u\left(\bar{n}\right)$, and $u(1)$ as functions of r.[18] $\underline{t}^{SS}$ is measured by the vertical distance between the $u\left(\bar{n} - 1\right)$ curve and the $u(1)$ curve, whereas $\bar{t}^{SS}$ is measured

[18]It is readily verified that both the $u\left(\bar{n} - 1\right)$ curve and the $u\left(\bar{n}\right)$ curve cut $u(1)$ at $r < 3c/4$ since $\bar{n} > 16\frac{1}{3}$ from Assumption 2.

by that between the $u(\bar{n})$ curve and the $u(1)$ curve. Full agglomeration is sustainable if t is smaller than the former distance, and only if it is smaller than the latter distance.

Figure 4: Sustain points

Equation (25) enables us to examine how various factors affect the sustain points. The key observation is that *the sustain points give the maximum transport cost that induces consumers in the periphery to visit a shop in the core when one firm happens to launch a business in the periphery.*

First, $\underline{t}^{SS}$ and $\bar{t}^{SS}$, or the two vertical distances mentioned above, increase with r. This is explained as follows. Suppose that r rises. The utility level of a consumer who visits a shop in the core increases only through the reservation-price effect; it therefore increases by the same amount as r. Consequently, the slopes of the $u(\bar{n}-1)$ curve and the $u(\bar{n})$ curve are equal to 45°. Now, suppose that a firm opens a shop in the periphery. The utility level of a consumer who visits that shop may or may not increase. Two cases can be distinguished. On the one hand, when the market area of the only firm is M type, the utility level increases since u^M increases with r. In addition, it can be shown that the amount of the increase is smaller than that of r. Therefore, in the range of r associated with the M type ($r < 3c/4$), the $u(1)$ curve slopes upward but is flatter than the 45° line. On the other hand, when the market area of the only firm is B type, the utility level remains unchanged since u^B is independent of r. Therefore, the $u(1)$ curve is horizontal in the range of r associated with the B type ($r \geq 3c/4$). These arguments establish that the vertical distance between the $u(\bar{n}-1)$ curve and the $u(1)$ curve, and that between the $u(\bar{n})$ curve and the $u(1)$ curve, increase with r.

Secondly, $\underline{t}^{SS}$ and $\bar{t}^{SS}$ increase with $\bar{n}$. As $\bar{n}$ grows, more firms come to operate in the core. This raises the utility level of a consumer who visits a shop in that region through the pro-competitive effect. Thus, the $u(\bar{n}-1)$ curve and the $u(\bar{n})$ curve shift upward. In the periphery, in contrast, nothing is affected by the change in $\bar{n}$ because only one firm operates in that region. Consequently, the $u(1)$ curve does not move. Hence, the vertical distances increase with $\bar{n}$.

Thirdly, and finally, both $\underline{t}_B^{SS}$ and $\bar{t}_B^{SS}$ decrease with c, but its effects on $\underline{t}_M^{SS}$ and $\bar{t}_M^{SS}$ are ambiguous.[19] We can explain this with the help of Figure 5. On the one hand, as c rises, the $u(\bar{n}-1)$ curve and the $u(\bar{n})$ curve shift downward. In the figure, the $u(\bar{n}-1)$ curves before and after the change are depicted by the solid line and the dotted line, respectively

[19]Tedious computation yields $\mathrm{d}\underline{t}_M^{SS}/\mathrm{d}c = (\Psi^M - 2\underline{\Phi})/(2c)$, $\mathrm{d}\underline{t}_B^{SS}/\mathrm{d}c = -(\Psi^B + \underline{\Phi})/c < 0$, $\mathrm{d}\bar{t}_M^{SS}/\mathrm{d}c = (\Psi^M - 2\bar{\Phi})/(2c)$, and $\mathrm{d}\bar{t}_B^{SS}/\mathrm{d}c = -(\Psi^B + \bar{\Phi})/c < 0$ (note that $\bar{n}$ depends on c).

(the $u(\bar{n})$ curves are not shown). On the other hand, the quadratic part of the $u(1)$ curve rotates clockwise since u^M decreases with c, whereas the horizontal part shifts upward since u^B increases with c. In the figure, the $u(1)$ curves before and after the change are also depicted by the solid line and the dotted line, respectively. Hence, for $r > 3c/4$, the rise in c necessarily shortens the vertical distances: it lowers the sustain points. For $r \leq 3c/4$, on the other hand, it may shorten or strengthen the vertical distances because the $u(\bar{n}-1)$ curve and the $u(1)$ curve (and $u(\bar{n})$ curve and the $u(1)$ curve) move in the same direction: the direction of the change in the sustain points is ambiguous.

Figure 5: Effect of a change in c on the sustain points

We have established the following proposition.

Proposition 4. (Factors affecting the sustain points)
The sustain points rise, and full agglomeration becomes more likely to be sustainable, as
i) the reservation price goes up,
ii) the potential of the economy grows, and/or
iii) the characteristics-cost coefficient declines when $r \geq 3c/4$.

4.3 Partial agglomeration

We also examine partial agglomeration, where firms are concentrated in region i, i.e., the core, but attract only consumers living in that region ($i = 1, 2$), that is, the consumers in the other region, the periphery, refrain from going shopping. It is straightforward to see, by reasoning similar to that used in the previous subsections, that the necessary condition for no single firm having an incentive to set up in the periphery is given by $t \leq u\left(n^C(\lambda_i)\right) - u(1)$. However, this contradicts (19) since $u(1) > 0$. Hence, there is no possibility of partial agglomeration being supported by an equilibrium.

Proposition 5. (Partial agglomeration)
Partial agglomeration is not an equilibrium pattern.

4.4 Incomplete agglomeration

Finally, let us consider incomplete agglomeration. All the consumers in region i, i.e., the core, go to a shop in their home region, and some of the consumers in the periphery visit a shop in their home region but the rest go to the counterpart in the core ($i = 1, 2$). We do not pay any further attention to this pattern for two reasons.

First, it is "unstable".[20] As the number of firms in one region diverges slightly from the equilibrium number by accident, the equality in (20) breaks: consumers in the periphery are now not indifferent between visiting a shop in the core and visiting a counterpart in the periphery. The economic geography therefore turns to either full agglomeration or segregation. There is no force that is able to restore it to incomplete agglomeration.

More importantly, equilibrium with incomplete agglomeration does not exist, except for a singular case. Here, one needs to recall that the equilibrium number of firms must be an integer. Now, suppose that (n_1^0, n_2^0) satisfies (20). Then, *both* n_1^0 and n_2^0 are integers only in a singular case: at least either of them is almost always a non-integer, in which case the equilibrium does not exist. In addition, suppose that n_1^0 is not an integer, without loss of generality. Even if n_1 is an integer that is quite close to n_1^0, incomplete agglomeration cannot brestorere realized. This is because, however close the integer is to n_1^0, the consumers in the periphery are not indifferent between visiting a shop in the core and visiting a counterpart in the periphery, which results in either full agglomeration or segregation.

4.5 Effects of parameter changes on economic geography

We are now ready to study how economic geography depends on various parameters. The most important parameter is transport cost. In Figure 6, the three thick lines represent the equilibrium patterns, i.e., segregation ($s_1 = \lambda_1$), full agglomeration at region 1 ($s_1 = 1$), and full agglomeration at region 2 ($s_1 = 0$). These show that segregation does not break for $t > \bar{t}^{BR}$, and that full agglomeration is sustainable for $t \leq \underline{t}^{SS}$. The figure shows a remarkable resemblance to that in conventional NEG models. When the transport cost is too high, only segregation, corresponding to dispersion in the NEG models, is supported by an equilibrium. When the transport cost is medium, both segregation and full agglomeration become equilibrium patterns. Finally, when it is sufficiently low, only full agglomeration is supported by an equilibrium. Thus, we can reiterate a common narrative in the NEG literature: as the transport cost declines over a long period of time, the economic geography shifts from segregation (dispersion) to (full) agglomeration.

Figure 6: Transport costs and economic geography

[20] In order to discuss the stability in a rigorous manner, it would be necessary to formulate the behaviors of firms and consumers *off* the equilibrium as a dynamical system. However, expanding the model in such a direction does not add much insight, although it involves considerable elaboration. Therefore, we believe that it is better to treat this matter informally.

One might wonder if the sustain points are greater than the break points, as the figure describes. We can show that this is indeed the case, as long as $\bar{n}$ is sufficiently large.

Proposition 6. (Relationship between break points and sustain points)
If $\bar{n}$ is sufficiently large, $\bar{t}^{BR} < \underline{t}^{SS}$.

The proof is tedious and is relegated to the Appendix.

This result guarantees that as long as $\bar{n}$ is sufficiently large, there is an *overlap* in the range of t where both segregation and full agglomeration are supported by an equilibrium. In this range, there are three multiple equilibria, and which of them is selected is indeterminate. This may be determined by a historical accident, in which case hysteresis is present, or by the expectations of firms and consumers. As is well known in the NEG literature, the existence of this overlap has many important implications, especially on economic policies (see Baldwin et al. (2003), for example).

The pitchfork diagram also shows by a thin dotted line the locus of $\bar{t}^{BR}$ when λ_1, measured by the vertical axis, changes. The line, which we refer to as a BR line, slopes upward since $\bar{t}^{BR}$ increases with λ_1. Thus, segregation is more likely to break when the population distribution is more biased, which we have already seen in i) of Proposition 2.

It is also straightforward to examine the effects on the economic geography of changes in other parameters. First, as r rises, the sustain points increase (see i of Proposition 4), but it makes no impact on the break points. Therefore, full agglomeration becomes more likely to be sustainable and nothing changes for segregation. Secondly, as c declines, the sustain points increase as long as c is sufficiently low (see iii of Proposition 4). Because the break points remain unchanged, in this case, the change only makes full agglomeration more likely to be sustainable. Thirdly, and lastly, the effect of a change in $\bar{n}$, when $\bar{n}$ is sufficiently large, is a little more complicated. As it rises, the break points decrease *and* the sustain points increase. Therefore, the rise makes segregation less likely to break *and* full agglomeration more likely to be sustainable. Thus, both patterns become more likely to be an equilibrium pattern. Consequently, the range of the overlap expands and more indeterminacy is involved in the economic geography.

5 Concluding remarks

This paper has studied the economic geography realized when consumers are heterogeneous in their tastes and are uncertain about the characteristics of the available varieties in respective commercial areas. We have verified that such a combination of taste heterogeneity and imperfect information can be a source of spatial agglomeration. Thus, it can substitute for the role played by the co-movement of production and income in NEG models. Furthermore, it has been shown that only two distribution patterns, i.e., segregation and full agglomeration, can be supported by an equilibrium. We have identified critical values in the transport cost: if the transport cost is higher than the (upper) break point, the segregation does not break; if it is lower than the (lower) sustain point, full agglomeration is sustained. Thus, the results exhibit a remarkable resemblance to those of standard NEG models; this indicates that the NEG models provide merely one of several, or possibly many, explanations for agglomeration phenomena.

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Appendix

Proof of Lemma 1.

First, we show that *it is an equilibrium that every firm charges* p^M *if* $r < r_i^M$. Note that $r < r_i^M$ guarantees that the market area of each firm is indeed M type when every firm charges p^M. Furthermore, suppose that a firm charges p' but all the other firms charge p^M. The market area of the deviating firm can be no greater than $m^M(p')$, whatever p' is. Therefore, its profit does not exceed $\pi_i^M(p')$. However, since p^M maximizes $\pi_i^M(\cdot)$, $\pi_i^M(p') \leq \pi_i^M(p^M)$. Hence, there is no incentive for such a deviation.

Next, we show that *it is an equilibrium that every firm charges* p_i^C *if* $r > r_i^C$. Note that $r > r_i^C$ guarantees that the market area of each firm is indeed C type when every firm charges p^C. Furthermore, suppose that all the firms except firm k charge p_i^C. By construction, firm k also charges p_i^C, given that its market area becomes C type. Thus, what we need to show is that the firm cannot obtain a greater profit by charging p' that does not make its market area C type. First, suppose that firm k charges such a low price that the subjective price of its variety becomes lower than p_i^C at the location of firm $k+1$. Because $p' + c/n_i^2 > p_i^C$ for any $p' > 0$, no such undercutting is feasible. Secondly, suppose that firm k charges such a high price that its market area becomes M type. If $p_i^C + c/n_i^2 \geq r$, no such over-cutting is profitable because firm k has to charge $p' \geq r$. Then, suppose that $p_i^C + c/n_i^2 < r$. Three observations follow: i) The market area of firm k becomes M type if and only if $p' > \rho_i(p_i^C)$. However, $p_i^C + c/n_i^2 < r$ implies that $\rho_i(p_i^C) > p^M$, as long as $r > r_i^C$. Because function $\pi_i^M(\cdot)$ is concave and p^M gives its maximum, $p' > \rho_i(p_i^C) > p^M$ implies that $\pi_i^M(p') \leq \pi_i^M\big(\rho_i(p_i^C)\big)$. ii) $\pi_i^M\big(\rho_i(p_i^C)\big) = \pi_i^C\big(\rho_i(p_i^C), p_i^C\big)$ by construction. iii) $\pi_i^C\big(\rho_i(p_i^C), p_i^C\big) \leq \pi_i^C(p_i^C, p_i^C)$ since p_i^C maximizes $\pi_i^C(\cdot, p_i^C)$. These three observations imply that $\pi_i^M(p') \leq \pi_i^C(p_i^C, p_i^C)$. Hence, no such over-cutting is profitable. Thirdly, and lastly, suppose that firm k charges a price equal to $\rho_i(p_i^C)$, so that its market area becomes B type. Because $\pi_i^B\big(\rho_i(p_i^C)\big) = \pi_i^C\big(\rho_i(p_i^C), p_i^C\big) \leq \pi_i^C(p_i^C, p_i^C)$, such over-cutting is not profitable either.

Furthermore, we show that *it is an equilibrium that every firm charges* p_i^B *if and only if* $r_i^M \leq r \leq r_i^C$. Suppose that all the firms except firm k charge p_i^B. Note that $\mathrm{d}\pi_i^M(p)/\mathrm{d}p\big|_{p=p_i^B} \lesseqqgtr 0$ if $r \gtreqqless r_i^M$. If $r \geq r_i^M$, the firm does not benefit from a slight increase in its price, which will change its market area to the M type. If $r < r_i^M$, in contrast, the firm benefits from such over-cutting. Similarly, we can see that $\mathrm{d}\pi_i^C(p, p_i^B)/\mathrm{d}p\big|_{p=p_i^B} \gtreqqless 0$ if $r \lesseqqgtr r_i^C$. Therefore, if $r \leq r_i^C$, the firm does not become better off as a result of a price reduction that makes its market area C type. Otherwise, it does become better off. Hence, $r_i^M \leq r \leq r_i^C$ is a necessary and sufficient condition for p_i^B constituting an equilibrium.

Now, suppose that $r < r_i^M$. We have seen that p^M is an equilibrium price but p_i^B is not. Furthermore, since $r < r_i^M$ implies that $r < r_i^C$, p_i^C is not an equilibrium price either. Consequently, p^M is a *unique* equilibrium price. Similarly, p^C is a *unique* equilibrium price if $r > r_i^C$. Lastly, if $r_i^M \leq r \leq r_i^C$, neither the M type nor the C type is realized (recall the discussion regarding (12) or (13)), and, therefore, p_i^B is a *unique* equilibrium.

Proof of Proposition 1.

First, suppose that $t \leq \underline{t}^{BR}$. Because $n^C(\lambda_1) > \bar{n}\lambda_1^{\frac{1}{3}} - 1$ and $n^C(\lambda_2) \leq \bar{n}\lambda_2^{\frac{1}{3}}$ (see (15)),

$u^C\left(n^C(\lambda_1)\right) - u^C\left(n^C(\lambda_2)\right) \geq \underline{t}^{BR}$. Therefore, $t \leq u^C\left(n^C(\lambda_1)\right) - u^C\left(n^C(\lambda_2)\right) = u\left(n^C(\lambda_1)\right) - u\left(n^C(\lambda_2)\right)$ since $n^C(\lambda_1) \geq 2$ and $n^C(\lambda_2) \geq 2$ by Assumption 2. Consequently, the necessary condition, (17), does not hold. Hence, the pattern breaks. Secondly, let us suppose that $t > \bar{t}^{BR}$. Because $n^C(\lambda_1) \leq \bar{n}\lambda_1^{\frac{1}{3}}$ and $n^C(\lambda_2) > \bar{n}\lambda_2^{\frac{1}{3}} - 1$, $u^C\left(n^C(\lambda_1)+1\right) - u^C\left(n^C(\lambda_2)\right) \leq \bar{t}^{BR}$. Therefore, $t > u^C\left(n^C(\lambda_1)+1\right) - u^C\left(n^C(\lambda_2)\right) = u\left(n^C(\lambda_1)+1\right) - u\left(n^C(\lambda_2)\right)$ and, consequently, the sufficient condition, (21), holds. Hence, the pattern does not break.

Proof of Proposition 3.
First, suppose that $t \leq \underline{t}^{SS}$. Because $n^C(1) > \bar{n} - 1$, $u\left(n^C(1)\right) - u(1) > \underline{t}^{SS}$. Therefore, $t < u\left(n^C(1)\right) - u(1)$ and, consequently, the sufficient condition, (23), holds. Hence, full agglomeration is sustainable. Secondly, let us suppose that $t \geq \bar{t}^{SS}$. Because $n^C(1) \leq \bar{n}$, $u\big(n^C(1)\big) - u(1) \leq \bar{t}^{SS}$. Therefore, $t \geq u\big(n^C(1)\big) - u(1)$ and, consequently, the necessary condition, (24), does not hold. Hence, full agglomeration is not sustainable.

Proof of Proposition 6.
If $r < 3c/4$, $\lim\limits_{\bar{n}\to\infty} \underline{t}^{SS} - \bar{t}^{BR} = \lim\limits_{\bar{n}\to\infty} \underline{t}_M^{SS} - \bar{t}^{BR} = r^{\frac{3}{2}}\left[\frac{1}{r^{\frac{1}{2}}} - \frac{4}{9(3c)^{\frac{1}{2}}}\right] > 0$, because $r < 3c/4$ implies that $r < 273c/16$. Instead, if $r \geq 3c/4$, $\lim\limits_{\bar{n}\to\infty} \underline{t}^{SS} - \bar{t}^{BR} = \lim\limits_{\bar{n}\to\infty} \underline{t}_B^{SS} - \bar{t}^{BR} = r - c/6 > 0$. By continuity, therefore, $\bar{t}^{BR} < \underline{t}^{SS}$ for sufficiently large $\bar{n}$.

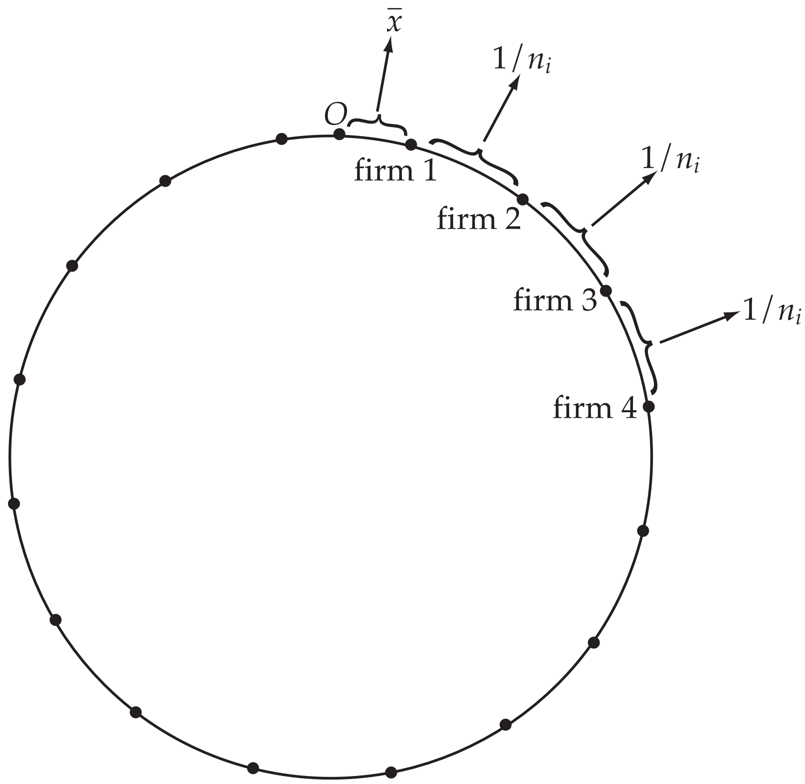
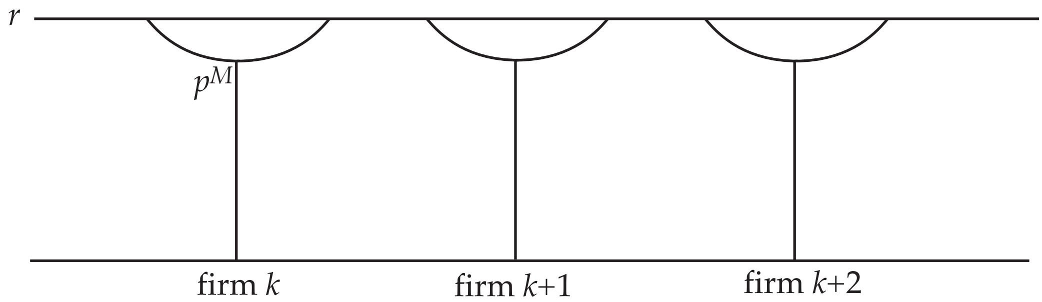
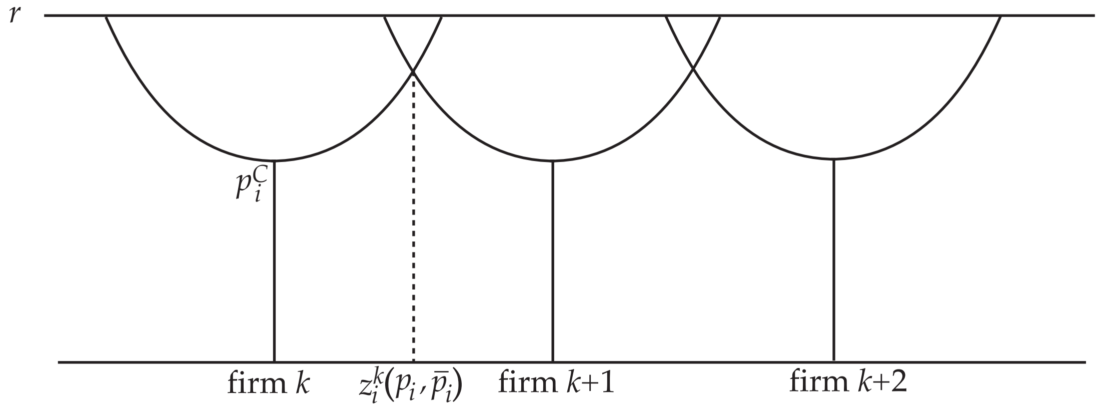


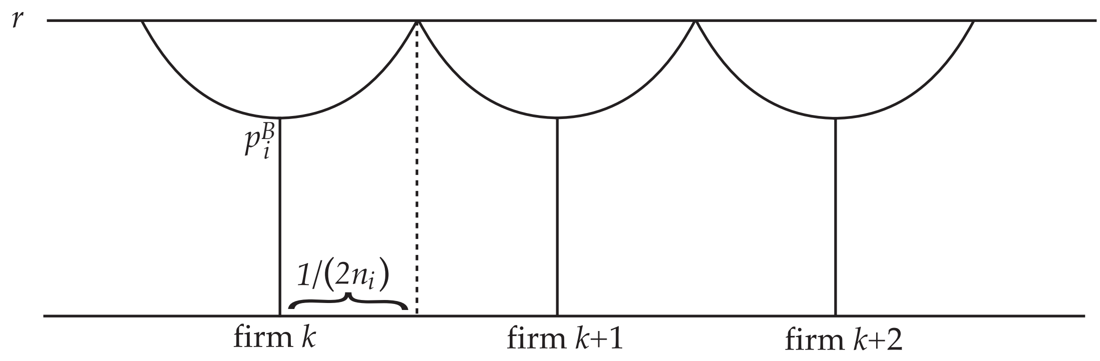
Figure 1. Characteristics space



(a) Monopoly type



(b) Competitive type



(c) Borderline type

Fig. 2. Three types of market area

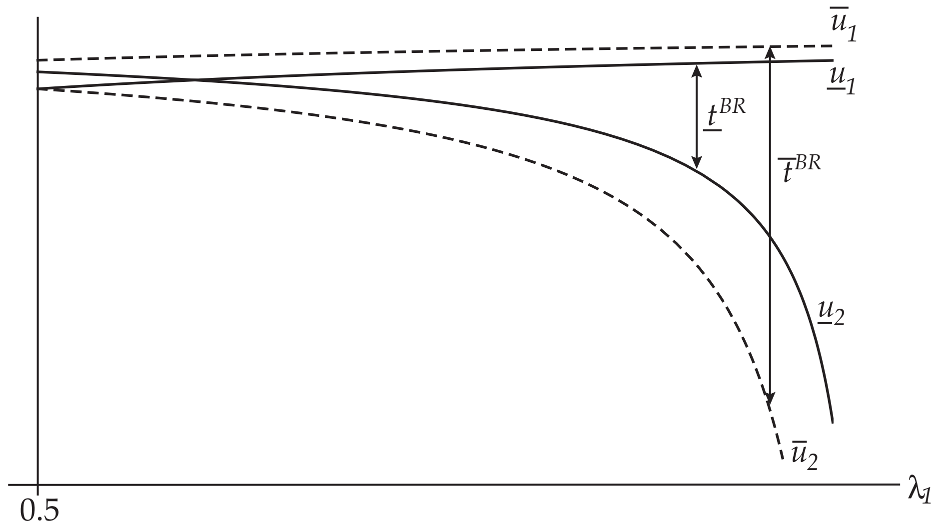


Figure 3. Break points

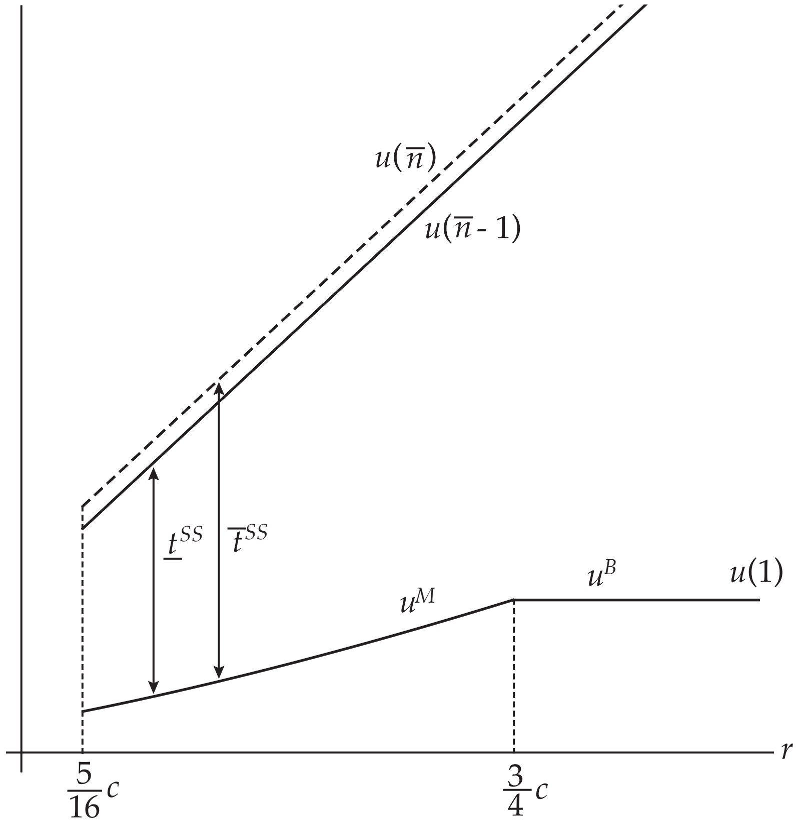


Figure 4. Sustain points

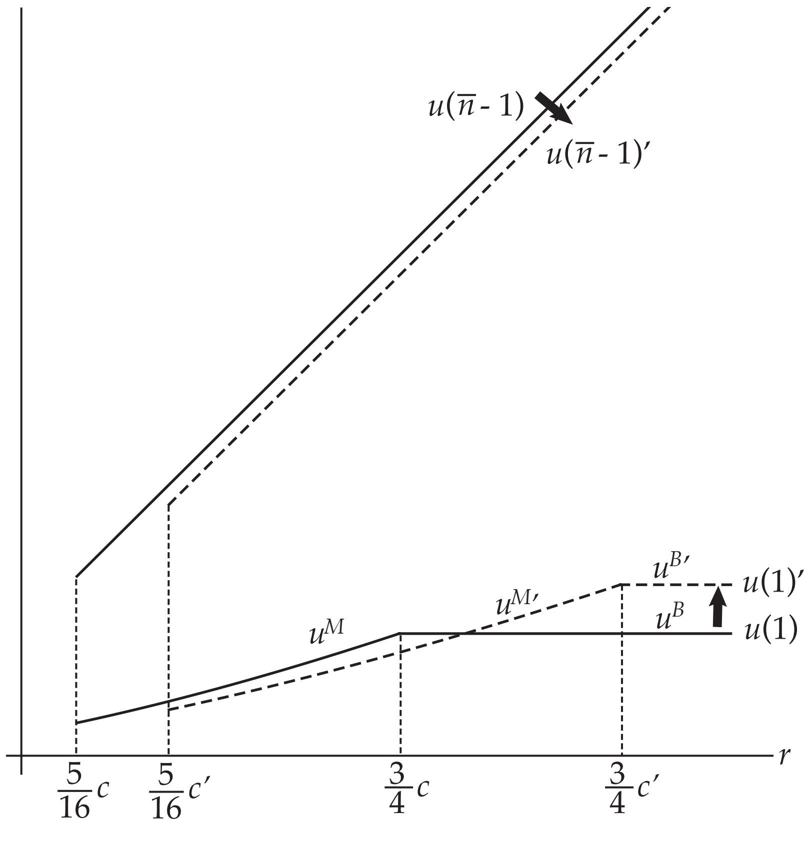


Figure 5. Effect of a change in c on the sustain points

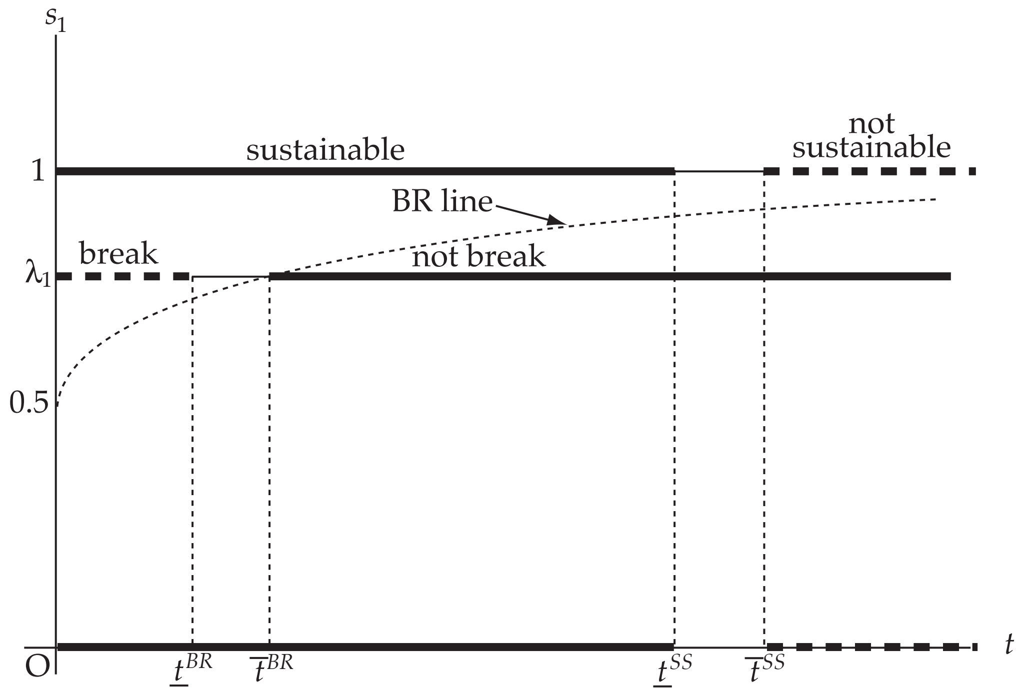


Figure 6. Transport cost and economic geography