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Evaluation of the Formation of Road Networks Based on Fire-Extinction Criterion

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1. Introduction

Roads are the basic physical components of cities. Our activities are not only in buildings but also on roads. Whenever we take a trip from an origin to a destination, we often move on road networks. The functions of roads consist of traffic function and space function. The former function is related to service of passing. On the other hand, the latter function is related to forming a frame of a city, creating landscapes, and protecting us against disaster. These two functions depend on how well road networks are formed and arranged.

The formation of road networks reflects the structure of a city. There are cities whose formation of road networks are mainly a grid system like Kyoto or New York. On the other hand, there are cities whose formation of road networks are mainly radial and ring system like greater London or Paris. There are, however, road networks which are classified neither grid system nor radial and ring system. Road networks of western districts in Tokyo 23 wards, for example, seem to be apparently disorder. As Jinnai (1992) points out, however, that there is logic which focuses on the geographical features and this logic governs the road networks in these districts.

The formation of road networks needs to be evaluated not only from the point of view of the structure of a city but also from the point of view of applying a rule of the Building Standards Law, the basic law related to buildings or cities in Japan. In 2004, the Ministry of Land, Infrastructure, Transport and Tourism (MLIT) suggested that the rule that enables us to conserve narrow roads, whose width is less than four meters, could be applied if the formation of road networks is ‘well-arranged’ and if narrow roads have traditional and beautiful landscapes. It has not been proposed, however, how to evaluate the formation of road networks quantitatively and what types of spatial units to be selected in this rule.

The rule mentioned above is called ‘the 3rd paragraph rule’. This is because this rule is provided in the 3rd paragraph of the 42nd article in the Building Standards Law. The 3rd paragraph rule was enacted in 1959. On the background of this rule enacted, there were a lot of buildings on the lots which were adjacent to roads whose width was less than four meters. The roads whose width is equal to or more than 2.7 meters and less than four meters, that is called ‘narrow roads’, had been legally allowed to exist until 1950. In this year, however, the Building Standards Law was enacted as the revised law of the Urban Building Law, the past basic law related to buildings or cities enacted in 1919. This revision made the lots which were adjacent to narrow roads ‘unadjacent lots’ and made the buildings on unadjacent lots ‘illegal buildings’. While the pre-revised law allowed buildings to exist on the lots which are adjacent to narrow roads, the post-revised law basically has not allowed buildings to exist on the lots which are adjacent to narrow roads since 1950. Not to make the buildings on unadjacent lots illegal buildings, ‘the 2nd paragraph rule’, provided in the 2nd paragraph of the 42nd article in the Building Standards Law, was enforced in 1950. The 2nd paragraph rule was applied to narrow roads along which buildings existed continuously in 1950. As illustrated in figure 1, if a building on the lot which is adjacent to narrow roads, where the 2nd paragraph rule is applied, is rebuilt, the distance from the center line of narrow roads to the boundary of the lot and the front road of the lot has to be equal to or more than two meters. The difference between the 2nd paragraph rule and the 3rd paragraph rule is that, while the former rule imposes roads-widening, the latter rule does not impose roads-widening. Therefore, a building built after 1950 exists on the lot which is adjacent to the road whose width is equal to or more than four meters. On the other hand, a building built before 1950 exists not only on the lot which is adjacent to the road whose width is equal to or more than four meters but also on the lot which is adjacent to narrow roads.

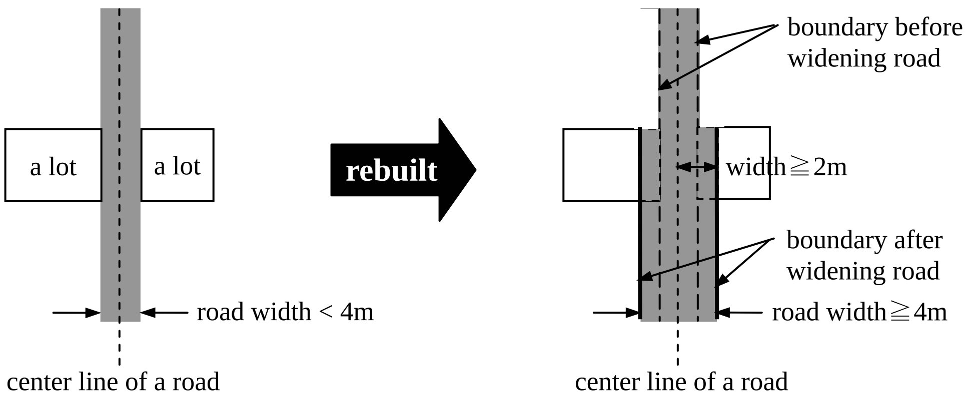


Figure 1: Widening narrow roads where the 2nd paragraph rule is applied in rebuilding

The reason why the condition of minimum road width was modified from 2.7 meters to 4 meters is to improve sanitation and ensuring passing fire engines and to prevent fire spreading. In particular, for two cars to pass each other and for fire engines to pass, road width needs to be at least four meters. Also, the longer the interval of adjacent two buildings becomes, the smaller the risk of fire spreading becomes. As Aoki (2006) suggests, if the interval of adjacent two buildings ranges from two meters to four meters, then it is possible to decrease drastically the risk of fire spreading. Thus, from the point of view of prevention of fire spreading, it is important to make the width of roads equal to or more than four meters.

While the narrowness of road width is considered as one of disadvantages from disaster prevention point of view, the narrowness of road width may be considered as one of advantages from townscape point of view. Today, the attractive points of narrow roads, called *Roji* in Japanese, are seen in a new light. In particular, the narrowness of road width is considered as the indispensable factor to create attractive landscapes and human atmosphere. From now on, the cases that the 3rd paragraph rule is applied to narrow roads which have traditional and beautiful landscapes seem to increase. To apply the 3rd paragraph rule appropriately, we need the method to evaluate the formation of road networks quantitatively.

So far, several methods for evaluating the formation of road networks have been proposed. The index using graph theory, e.g., alpha index or gamma index, is one of them (Kansky (1963), Okudaira (1976), Okuno (1977) and Noda (1996)). In the context of graph theory, road networks are regarded as the set of nodes and the set of links. An intersection and a road between two neighboring intersections on road networks correspond to a node and a link, respectively. Alpha index focuses on how many faces road networks have. The number of faces is equivalent to that of regions encircled by links. Given the number of nodes on road networks, the more the number of faces becomes, the higher the value of alpha index becomes. Also, gamma index focuses on how many links road networks have. Given the number of nodes on road networks, the more the number of links becomes, the higher the value of gamma index becomes. The difference between these two indices is whether we focus on the number of blocks or on the strength of connectivity of road networks. Two disadvantages of indices based on graph theory are difficulty in dealing with metric information and in setting criteria for judging whether the formation of road networks is well-arranged or not.

To respond to applying the rule mentioned above, we clarify the background of setting the criterion: whether the formation of road networks is well arranged or not, and clarify the interpretation of this criterion by a local administrative office. By considering this interpretation, we evaluate the formation of road networks focusing on fire-inextinguishable area and propose conditions of well arranged road networks based on fire-extinction criterion.

This paper consists of 6 sections. We discuss the backgrounds and objectives of this paper in this section. We conduct an interview to grasp how a local administrative office interprets the criterion related with the formation of road networks in section 2. In section 3, we evaluate the formation of road networks based on whether there is a ‘fire-inextinguishable area’, or not, in a region encircled by wide roads whose width is equal to or more than six meters. In section 4, we clarify the conditions of well arranged road networks based on fire-extinction criterion. In section 5, we evaluate the formation of road networks by using graph theory based index and point out some problems. In the last section, we state concluding remarks and future tasks.

2. Interpretation of the criterion about the formation of road networks by a local administrative offices

In this section, to clarify the appropriate spatial unit for evaluating the formation of road networks, we conduct an interview to grasp how a local administrative office interprets the criteria related with the formation of road networks. The local administrative office where we conduct an interview is Chuo ward office in the 23 wards of Tokyo. In Chuo ward, there are districts called Tsukishima districts which have many narrow roads whose width is less than four meters.

Since Chuo ward office judges that these narrow roads have traditional and beautiful landscapes, Chuo ward office have applied the 3rd paragraph rule stated in section 1 for these narrow roads to conserve attractive landscapes. The case of Tsukishima districts is the only case that this rule is actually applied since this rule was amended in 2004. This is why we choose Chuo ward office as the target of an interview.

As the result of the interview for Chuo ward office, we get the reply that, in the case of Tsukishima districts, since the road networks forming blocks are well arranged, there is no obstacle for extinguishing fire even if the 3rd paragraph rule mentioned is applied. Thus, the purpose of setting the condition about the formation of road networks is to judge whether it is possible to ensure the activity of extinguishing fire or not. Figure 2 shows the formation of road networks in Tsukishima districts. From figure 2, one of the end points of each narrow road is connected to the road whose width is equal to or more than four meters. If the width of a road is equal to or more than four meters, then fire engines are able to enter the road, which makes possible to extend fire hoses into narrow roads.

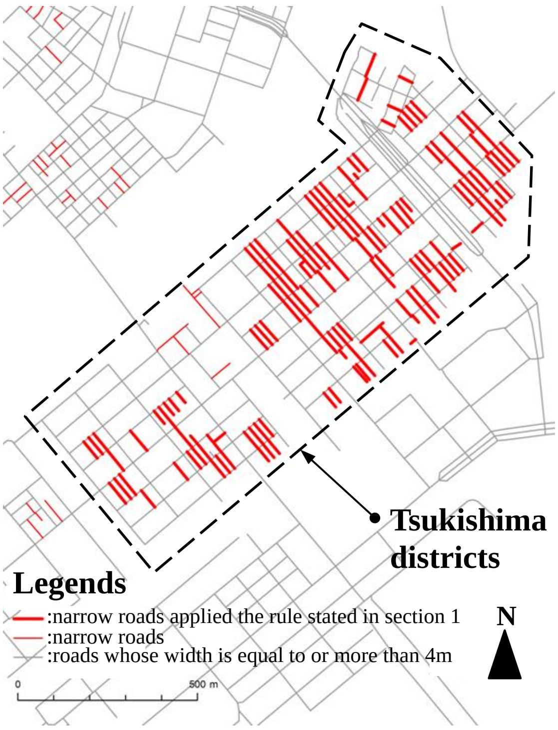


Figure 2: Road networks in Tsukishima districts

3. Evaluating the formation of road networks focusing on fire-inextinguishable areas in the 23 wards of Tokyo

In the previous section, we clarify that the purpose of setting the condition about the formation of road networks is related to the possibility of extinguishing fire. In this section, to judge whether there is the obstacle for extinguishing fire or not, we judge whether there is the area named 'the fire-inextinguishable area (FIA)' in the 23 wards of Tokyo.

FIA is defined as the area where the distance from the nearest road whose width is more than six meter is more than 140 meters. This is because the maximum length of fire hoses is able to be extended to 200 meters. Shown in figure 3, we consider the formation of road networks as grid pattern. ‘O’ means the location of a fire engine and ‘P’ means an arbitrary location. Let R be the Euclid distance from a fire engine on wide roads and let u and v be the x component of vector OP and y component of vector OP, respectively. Therefore, $u + v$ is the Rectilinear distance between O and P. In this case, the minimum R is the optimal value of the following optimization problem.

$$\begin{aligned} &\min \quad R = \sqrt{u^2 + v^2} \\ &\text{s.t} \quad u + v = 200. \end{aligned} \tag{1}$$

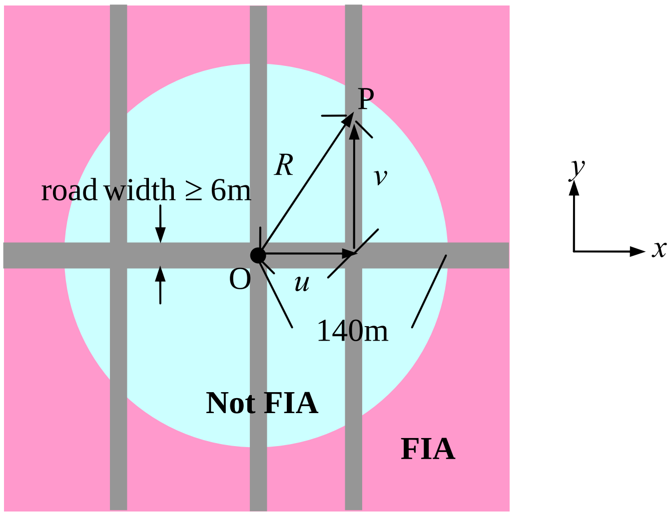


Figure 3: Definition of FIA

By solving this problem, we get the minimum R = 141 meters as the optimal value and get u = v = 100 meters as the optimal solutions. This means that it is possible to extinguish fire occurring in the interior of the circle whose radius is equal to 141 meters by extending 200 meters hoses along grid roads. This is why FIA is defined as the area where the distance from the nearest road whose width is more than six meter is more than 140 meters. From now on, we call wide roads the roads whose width is equal to or more than six meters. Figure 4 shows wide roads and FIA in the northeast area of the 23 wards of Tokyo. As shown in figure 4, we define the inner area encircled by wide roads as ‘an encircled area’.

There are two advantages for considering encircled areas. First, it is possible to grasp whether there is FIA in an arbitrary encircled area or not. By using the encircled area as the spatial unit to judge whether the formation of road networks is well arranged or not, if there is not FIA in an encircled area, we can judge the formation of road networks in this encircled area as well arranged from the point of view of the possibility of fire-extinguishing. Second, since the formation of an encircled area and that of road networks are close relationship with each other, it is possible to evaluate the formation of road networks based on the formation of an encircled area. If the formation of an encircled area is well arranged, we can judge the formation of road networks in this encircled area as well arranged.

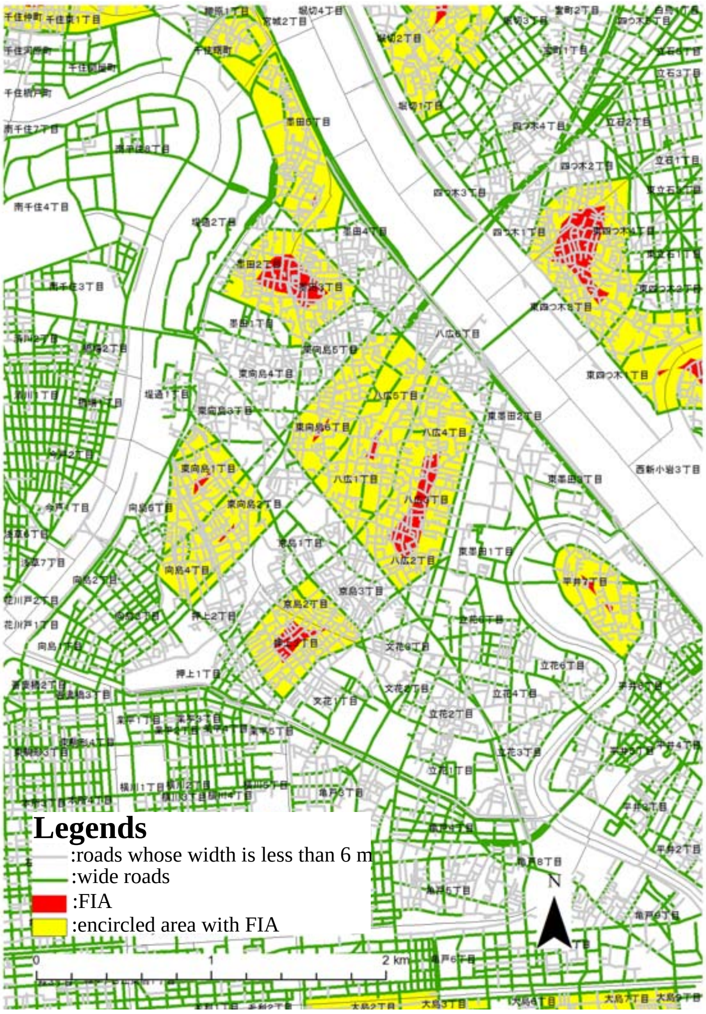


Figure 4: Networks of wide roads and FIA

Figure 5 illustrates the distribution of FIA and that of encircled areas with narrow roads and with FIA. In figure 5, the black-colored encircled areas indicate FIA. The number of encircled areas with FIA is 383. The dark gray-colored encircled areas with large FIA indicate the encircled areas which satisfy the following two conditions: 1). the encircled areas with FIA, 2). the encircled area with narrow roads which satisfy the following two conditions for applying the 3rd paragraph rule: a). not cul-de-sacs, b). the length of a narrow road is less than 60 meters. The conditions a) and b) are two of three conditions mentioned in the 3rd paragraph rule. The rest condition is whether the formation of road networks is well arranged or not. The dark gray-colored encircled areas whose area is large are distributed in Nakano ward, the southeast area of Suginami ward and the north area of Setagaya ward. In the 23 wards of Tokyo, the number of such encircled areas is 371. On the other hand, the light gray-colored and dotted encircled areas do not have FIA and have narrow roads satisfying the condition a) and b) mentioned above. In the 23 wards of Tokyo, the number of such encircled areas is 1646. Thus, from the point of view of the possibility of fire-extinction, the gray-colored and dotted encircled areas are judged as well arranged areas. In this case, if there are narrow roads with attractive landscapes in the gray-colored and dotted encircled areas, it is possible to apply the 3rd paragraph rule for these narrow roads.

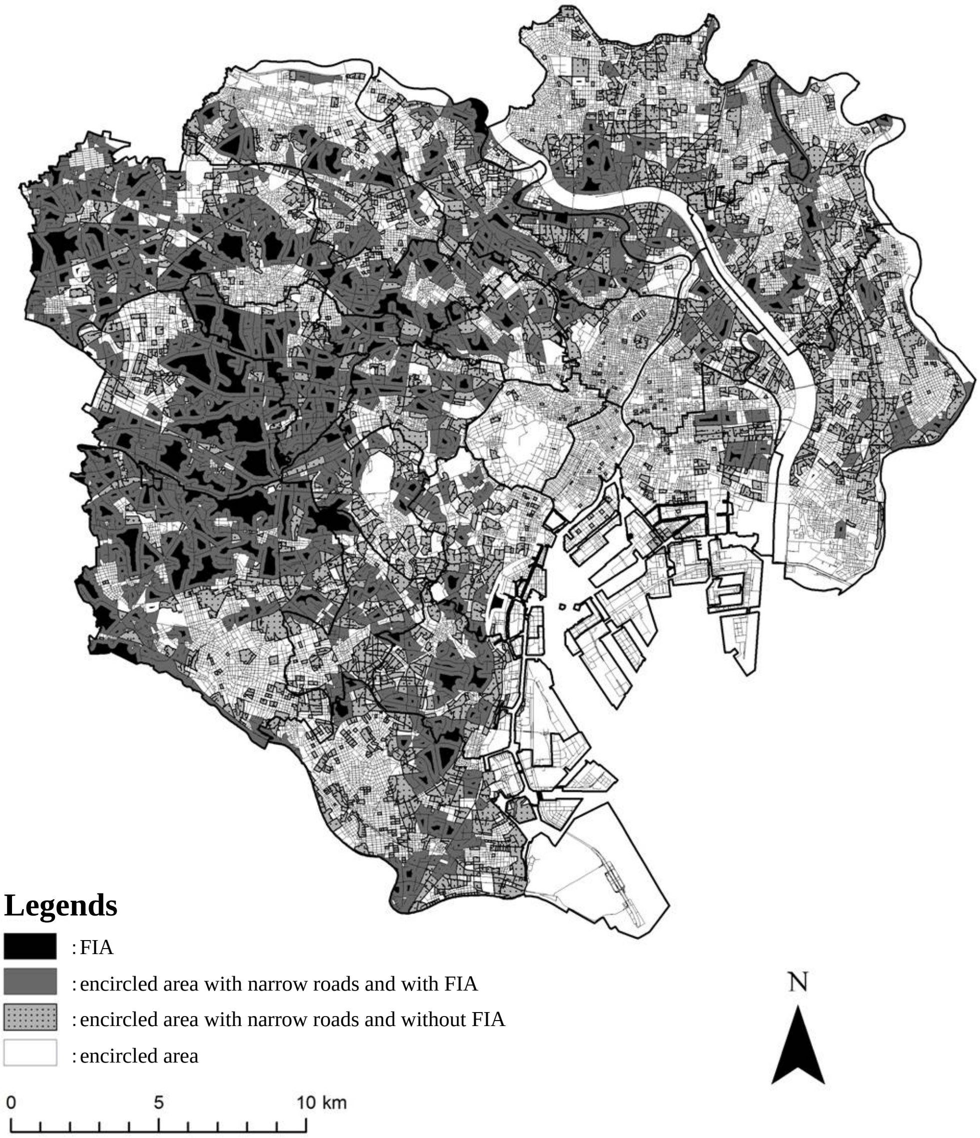


Figure 5: FIA and the encircled area with well arranged formation of road networks in the 23 wards of Tokyo

4. The condition of well arranged formation of road networks based on fire-extinction criterion

In the previous section, we grasp the encircled areas with well arranged formation of road networks focusing on whether there is FIA in an arbitrary encircled area or not. As we find out from figure 5, whether FIA exists in an arbitrary encircled area depends on the formation of wide road networks. The knowledge about the formation of road networks which generate FIA in an arbitrary encircled region is indispensable to carry out widening narrow roads reasonably. In this section, we discuss the formation of road networks such that there is no FIA in an arbitrary encircled region.

First of all, we discuss the condition that there is not FIA in a rectangular encircled area whose length of the longer edge and that of the shorter edge are a and b, respectively. In this case, the necessary and sufficient condition so that there is not FIA in a rectangular encircled area is as follows:

$$a \leq 280\,[\mathrm{m}] \text{ or } b \leq 280\,[\mathrm{m}]. \tag{2}$$

This condition means that the rectangular encircled area with large area does not always have FIA. Thus, it is not possible to judge whether there is FIA in a rectangular encircled area or not, based on the area of a rectangular encircled area.

Well arranged road networks depend not only on whether there is FIA in an arbitrary encircled area but also on the amount of cul-de-sacs in the encircled area. As Maki et al. (1980) points out, the larger the area of an encircled area becomes, the more the number of cul-de-sacs tends to become. To discuss this relation more quantitatively, let the number of cul-de-sacs and the area of an encircled area be $e_{\mathrm{dge}=1}$ and S[ha], respectively. By linear regression analysis, using S as the explanatory variable and $e_{\mathrm{dge}=1}$ as the explained variable, we get the regression equation as follows:

$$\hat{e}_{\mathrm{deg}=1} = 1.01S, \qquad (3)$$

where the coefficient of determination, R^2, is 0.90. Also, let the summation of length of cul-de-sacs in an encircled area be $L_{\mathrm{dge}=1}$[m]. By linear regression analysis, using S as the explanatory variable and $L_{\mathrm{dge}=1}$ as the explained variable, we get the regression equation as follows:

$$\hat{L}_{\mathrm{deg}=1} = 31.9S, \qquad (4)$$

where the coefficient of determination, R^2, is 0.92. From equation (3) and equation (4), if the area of an encircled area increases by one hectare, it is found that a cul-de-sac generates and the average length of generating cul-de-sac is about 32 meters. This relation is simple and easy to understand.

Figure 6 illustrates the scatter plot of S and $e_{\mathrm{dge}=1}$. Figure 6 is illustrated in the case of the encircled areas with FIA or without FIA. By comparing left hand and right hand of figure 6, we find out the following tendency. First, if there is not FIA in encircled areas, there exists upper bound about the number of cul-de-sacs. Second, there does not exist the threshold related to whether there is FIA in encircled areas. If there exists FIA in an encircled area, the minimum area of the encircled area is 4.8 hectare. On the other hand, if there dose not exist FIA in an encircled area, the maximum area of the encircled area is about 200 hectare.

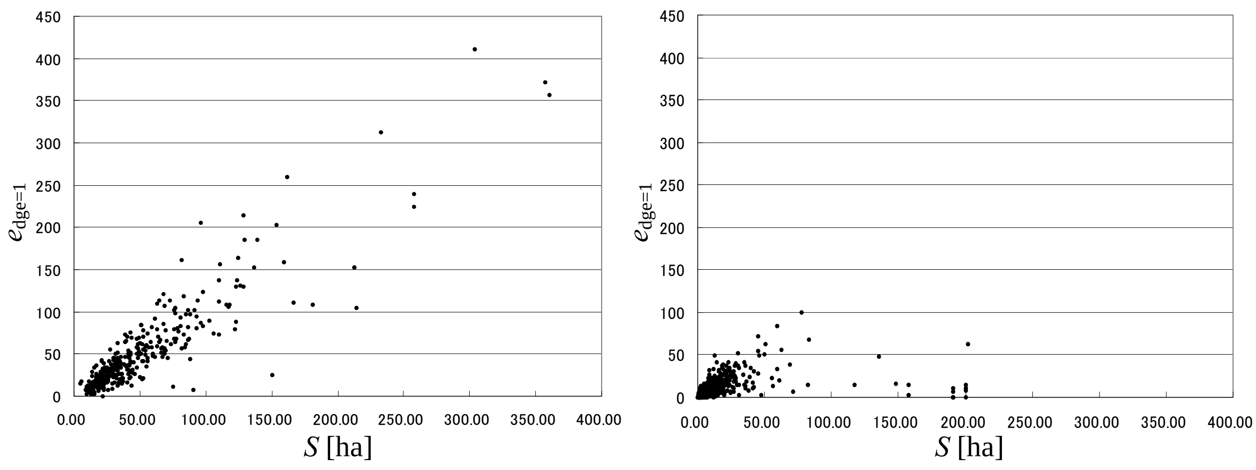


Figure 6: The scatter plot of S and $e_{\mathrm{dge}=1}$ (left: with FIA, right: without FIA)

Thus, if the area of a encircled area is more than 4.8 hectare and less than 200 hectare, it is not possible to judge whether there exist FIA in an encircled area or not, based on the area of the encircled area. This indicates that, to judge whether there exist FIA in an encircled area or not, we need to evaluate the formation of an encircled area.

The necessary and sufficient condition that there is no FIA in a rectangular encircled area is equation given by (2). In general, however, since the formation of the encircled area is close to a multi polygon, it is not possible to discuss as easily as in the case of a rectangular encircled region. In this case, what is the condition corresponding to equation (2)? As shown in figure 7, we consider an encircled region as a multi convex polygon. Let one of vertices of multi polygon with n vertices be P_i (i=1,…,n). In the case of figure 7, n equals to 5. Let the interior angle at P_i be $\angle P_{i-1}P_iP_{i+1} = \theta_i$, where, if $i = n$, then $\angle P_{n-1}P_nP_1 = \theta_n$ and if $i = 1$, then $\angle P_nP_1P_2 = \theta_1$. Let the vertex of FIA corresponding to P_i of an encircled region be Q_i. By drawing the perpendicular line from Q_i to $P_{i-1}P_i$ and P_iP_{i+1}, let the intersection of this perpendicular line and $P_{i-1}P_i$ or that of this perpendicular line and P_iP_{i+1} be H_{i1} and H_{i2}, respectively. In this case, for an arbitrary edge, P_iP_{i+1}, there does not exist FIA in an encircled area if the inequality satisfies as follows:

$$P_iP_{i+1} \le P_iH_{i2} + P_{i+1}H_{i+1,1}\,. \tag{5}$$

Moreover, by drawing the line from P_i to Q_i, we get two right triangles which are congruent to each other: $\triangle P_iQ_iH_{i1} \equiv \triangle P_iQ_iH_{i2}$. Therefore, P_iQ_i is the bisector of θ_i. In this case, the length of P_iH_{ik} (k = 1, 2) is as follows:

$$P_iH_{ik} = 140\tan\left(\frac{\pi}{2} - \frac{\theta_i}{2}\right). \tag{6}$$

Thus, the condition that there does not exist FIA in an encircled area is, for an arbitrary P_iP_{i+1}, to satisfy the following inequality:

$$P_iP_{i+1} \le 140\left\{\tan\left(\frac{\pi}{2} - \frac{\theta_i}{2}\right) + \tan\left(\frac{\pi}{2} - \frac{\theta_{i+1}}{2}\right)\right\}, \text{ for all } i = 1,\dots, n. \tag{7}$$

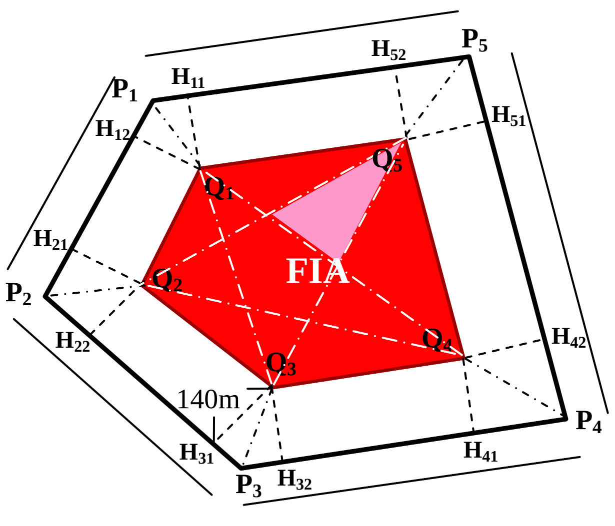


Figure 7: The encircled area depicted as a convex multi polygon (n = 5)

If $\theta_i = \theta_{i+1} = \pi/2$, then inequality (7) corresponds to inequality (2). By comparing inequality (2) with inequality (7), if the formation of an encircled region is not rectangular, it is found that the whether there exist FIA in an encircled region or not depends on the degree of θ_i.

The mean value of θ_i is $(n-2)\pi/n$. As n increases, the mean value of θ_i also increases and the value of the right hand of inequality (7) decreases. This means that the more n becomes, the higher the possibility that there is FIA in an encircled area unless the length of P_iP_{i+1} is made shorter. Conversely, the less n becomes, the more the condition is relaxed. Certainly, the minimum value of n is three. From the point of view of easiness to turn right or left at intersections, however, a square formed region is superior to a triangular formed region. Thus, by considering inequality (7) at the same time, the formation of grid pattern road networks is the best arranged formation.

For all P_iP_{i+1}, inequality (7) does not have to be satisfied so that there does not exist FIA in an encircled region. In the case of the encircled area in figure 7, if, for arbitrary three of five edges of an encircled area, inequality (7) is satisfied, there does not exist FIA in this area. For example, if P_1P_2 and P_3P_4 satisfy inequality (7), the area of triangle $Q_1Q_2Q_j$ (j=3, 4, 5) and that of $Q_3Q_4Q_j$ (j=1, 2, 5) are 0. In this case, the light gray colored area lasts as FIA. To get rid of FIA from this area, an arbitrary edge of the last three edges which do not satisfy inequality (7) at least satisfies inequality (7).

5. Evaluating road networks based on graph theory

In the previous sections, we evaluate the formation of road networks by focusing on whether there exist FIA in an arbitrary encircled region or not and discuss the formation of road networks such that there is no FIA in an arbitrary encircled region. Certainly, we evaluate the formation of wide roads networks by focusing on FIA. We, however, are not able to evaluate the formation of narrow roads networks.

In this section, we evaluate the formation of road networks by using indices based on graph theory. From the point of view of graph theory, we can consider road networks as a plane graph. A plane graph is a planar graph together with a given imbedding in the plane (Jonathan & Jay (2004)). A planar graph is a graph of minimum genus 0, without any edge-crossings except on nodes. Figure 8 shows the examples of plane graph and non-plane graph. There are mainly two indices based on graph theory to evaluate road networks: alpha index and gamma index. These indices are proposed in the discipline of quantitative Geography. Besides, Noda (1996) proposes grid-tree-proportion index (GTP index) which is the extension of these two indices. First of all, we consider the features of these three indices.

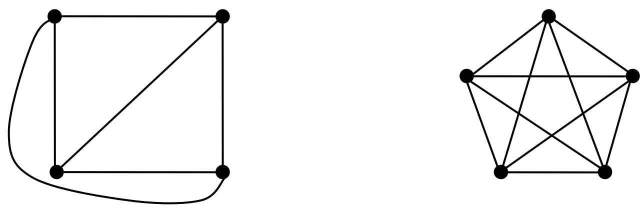


Figure 8: Plane graph (left) and non-plane graph (right)

5.1. Alpha index and gamma index

Alpha index is the ratio of the number of faces to the maximum number of faces which are possible to be made given the number of nodes on a graph. A face is defined as the region encircled by links (Jonathan & Jay (2004)). In the context of city planning, a face is equivalent to a block. Figure 9 illustrates faces in a network.

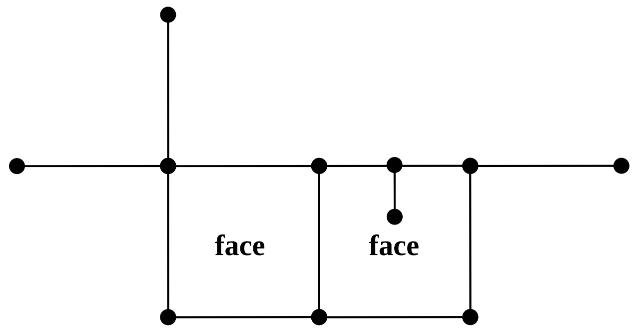


Figure 9: Faces in a network

We denote the number of nodes, the number of links and the number of independent graphs by *v*, *e* and *p*, respectively. Given the number of nodes, the maximum number of cycles is $2v-5$. Also, by using Euler's formula, the number of faces of graphs, *f*, is $e-v+p$. Hence, alpha index is defined as follows:

$$\alpha = \frac{e-v+p}{2v-5} \in [0,1], \text{ for } v \geq 3. \quad (8)$$

Since the number of faces is equivalent to that of blocks, it is possible to judge that the regions, where the value of alpha index is high, have many blocks. This indicates that, if the area of a region is fixed, the more the number of blocks becomes, the smaller the average area of a block becomes. In the context of fire-extinguishing activity, the smaller the area of a block becomes, the lower the possibility that a block has fire-inextinguishable area becomes.

Gamma index is the ratio of the number of links to the maximum number of links which are possible to be made given the number of nodes on a graph. Given the number of nodes, the maximum number of links is $3(v-2)$. Hence, gamma index is defined as follows:

$$\gamma = \frac{e}{3(v-2)} \in [0,1], \text{ for } v \geq 3. \quad (9)$$

Gamma index focuses on the connectivity of road networks. In the context of city planning, the road networks with high value of gamma index are well arranged in terms of redundancy of choosing roads. Also, from the point of view of the possibility of two-way escape, we can say that road networks with high value of gamma index are well arranged.

The difference between these two indices is whether we focus on the number of blocks or on the strength of connectivity of road networks. As Okuno (1977) points out, while alpha index does not accurately indicate the strength of connectivity of road networks, gamma index conquers this disadvantage.

5.2. GTP index

GTP index is derived by considering both typical two types of formation of road networks: grid pattern and tree pattern, and formation of general road networks which are not classified neither grid pattern nor tree pattern. As Noda (1996) points out, by calculating the value of both alpha index and gamma index of general road networks, it is found that there are two properties as follows:

(a). The value of both alpha index and gamma index of general road networks is usually less than that of square lattice pattern road networks with the same number of nodes as that of general road network.

(b). The value of both alpha index and gamma index of general road networks is more than that of tree pattern road networks with the same number of nodes as that of general road network.

From these two properties, GTP index is defined as follows:

$$GTP = \frac{x_S - x_T}{x_G - x_T} \in [0,2], \qquad (10)$$

where x_G is the value of each index of square lattice pattern road networks, x_T is the value of each index of tree pattern road networks, and x_S is the value of each index of actual road networks.

GTP index can be expressed as the function of x_S : $f(x_S)$. If a network pattern is complete tree pattern, then $x_S = x_T$. In this case, the value of GTP index is equal to 0. Also, if a network pattern is square lattice pattern, then $x_S = x_G$. In this case, the value of GTP index is equal to 1. Since, in the case of actual road networks, the formation of road networks is usually between tree pattern and square lattice pattern, the value of GTP index is between 0 and 1.

One of the features of GTP index is that, if we substitute equation (8) or (9) for equation (10), we get the following equation:

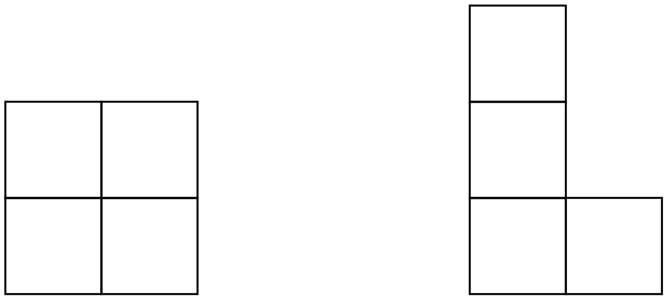
$$GTP = \frac{e - v + p}{(\sqrt{v} - 1)^2} \in [0,2], \qquad (11)$$

where the denominator of the right hand of equation (11) is the number of faces which a network with a square lattice pattern has. Thus, GTP index is considered as a new index which put alpha index and gamma index together.

If we apply GTP index for evaluating the formation of road networks, we have to consider the validity that we consider the ideal formation of road networks as a square lattice pattern. As shown in figure 10, the value of GTP index of a square lattice pattern is different from that of a grid pattern. In the context of city planning, however, is it possible to find out the difference between a square lattice pattern and a grid pattern? Probably, we should not consider that a road network with grid pattern is inferior to a road network with square lattice pattern, but consider that a road network with grid pattern is as well arranged as a road network with square lattice pattern.

5.3. Criteria for well arranged road networks

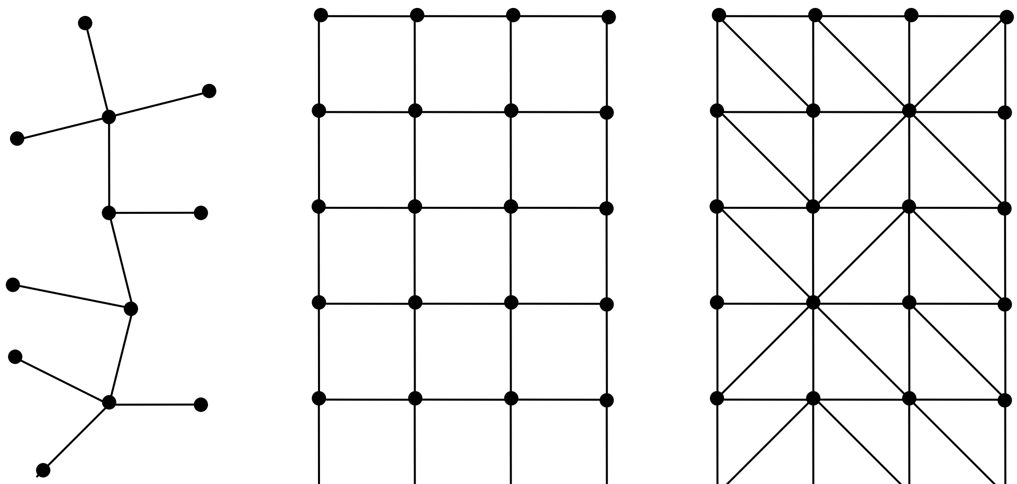
To evaluate the formation of road networks, we need not only indices but also the criterion for judging whether the formation of a road network is well arranged or not. Okuno (1977) considers, as basic formation patterns of road networks, three types of network patterns shown in figure 11: a tree pattern, a grid pattern and a delta pattern. These basic formation patterns are frequently used in the field of traffic engineering. In the context of city planning, the delta pattern is hardly considered as well arranged because it has many intersections whose degree is equal to or more than 5.



square lattice pattern
(GTP=1)

grid pattern
(GTP=0.85)

Figure 10: Validity of considering a square lattice pattern as an ideal pattern



tree pattern

grid pattern

delta pattern

Figure 11: Basic formation patterns of road networks

Table1 shows the relation of the basic formation patterns and the value of alpha index, gamma index and GTP index. Suppose that a grid pattern is well arranged. Then, the formation of road networks is well arranged if more than one of the following three conditions are satisfied: the value of alpha index is between 1/4 and 1/2, the value of gamma index is between 1/2 and 2/3, or the value of GTP index is between 0 and 1. These values are the criteria for judging whether the formation of a road network is well arranged or not. Detail calculation of these criteria is shown in Appendix 1.

Table1: Relation of the value of each index and the basic formation patterns

	gamma index	alpha index	GTP index
tree pattern	$1/3 \leqq \gamma \leqq 1/2$	$0 \leqq \alpha < 1/4$	$0 < GTP < 1/2$
grid pattern	$1/2 < \gamma < 2/3$	$1/4 \leqq \alpha < 1/2$	$1/2 \leqq GTP < 1$
delta pattern	$2/3 \leqq \gamma \leqq 1$	$1/2 \leqq \alpha \leqq 1$	$1 \leqq GTP \leqq 2$

5.4. Calculating value of each index by using GIS –in case of Bunkyo ward, Tokyo-

In this subsection, we evaluate the formation of road networks in Bunkyo ward, one of the central regions of the 23 wards of Tokyo, by using alpha index, gamma index and GTP index. Figure 12 shows the road networks of Bunkyo ward. As shown in figure12, Bunkyo ward has various types of the formation of road networks. This is why we select the road networks of Bunkyo ward for evaluating the formation of road networks.

Figure 12: Road networks of Bunkyo ward

To calculate the value of each index, we have to determine a spatial unit of data aggregation. In other words, we have to count *v*, *e* and *p* in a region. Noda (1995) selects as a spatial unit of data aggregation a square region whose length of an edge is 500 meters. Certainly, since square regions are regular segmentations, it is easy to analyze the spatial distribution of data. When we judge whether the formation of road networks is well arranged or not, however, is a square region an appropriate spatial unit of data aggregation? Noda (1995) refers to the property stated above as an advantage of using a square region as a spatial unit. On the other hand, he also points out a disadvantage of using a square region as a spatial unit. If road networks are segmented based on square regions, a lot of 'cul-de-sacs' are generated because a road intersecting an edge of square regions are divided into 2 parts. In fact, these 'cul-de-sacs' are not actual cul-de-sacs. Certainly, this problem tends to be improved if the area of a square region is made larger. If the area of a square region is made larger, however, it is impossible to evaluate the formation of road networks in a narrow scope. In particular, when we judge whether the 3rd paragraph rule stated in section 1 could be applied or not, we need to evaluate the formation of road networks in a district scale. In this case, it is not appropriate to make the area of a square region larger. It is considered that there is a trade-off relation between the accuracy of evaluation and the scale of the area of a square region. In this paper, to resolve this problem, we try to use as a spatial unit of data aggregation a district whose boundary almost corresponds to roads.

Table 2 shows the result of counting *v*, *e* and *p* and of calculating the value of alpha index, the value of gamma index and the number of GTP index in each district of Bunkyo ward. While it is possible to count *v* and *e* by using GIS application, it is difficult to count *p* by using GIS application. Instead of using GIS application, we count *p* by manual. Also, figure 13 shows the choropleth map of alpha index and that of GTP index, respectively, in the districts of Bunkyo ward. From these table and figures, it is found that the district with the highest value of both alpha and gamma index is Hongo 2 district. The value of both alpha and that of gamma index are 0.16 and 0.38 respectively. According to Table 1, the formation of the road network in Hongo 2 district is judged as a grid pattern based on alpha index or GTP index. On the other hand, the formation of the road network in this district is judged as a tree pattern based on gamma index.

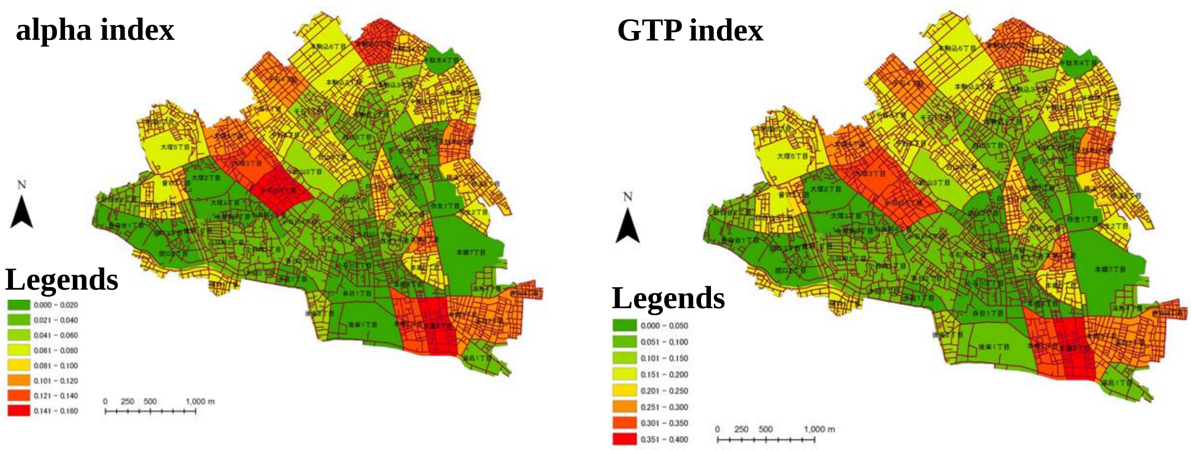


Figure 13: Choropleth map (left: alpha index, right: GTP index)

5.5. Problems of using indices based on graph theory

This contradiction stated in section 5.4 is attributed to boundary effects. As shown in figure 14, although the formation of the road network of Hongo 2 district seems to be almost a grid pattern, the value of alpha index and that of gamma index are lower than we evaluate intuitively. This is because a lot of ‘quasi-cul-de-sacs’ are generated by segmenting road links by the boundary of districts. This means that the number of faces is underestimated. Also, according to table 1, there is no district with a grid pattern formation of the road network in Bunkyo ward because the value of gamma index of all districts in Bunkyo ward is less than 0.5. This result is much different from the impression by watching figure 14.

Table 2: The result of counting v , e and p and of calculating the value of alpha, gamma and GTP

name of a district	the area of a district	the number of buildings	v	e	p	alpha index	gamma index	GTP index
Hongo 2	20.1	667	106	136	3	0.16	0.44	0.38
Koishikawa 5	21.0	639	113	142	2	0.14	0.43	0.33
Otsuka 3	21.9	475	96	118	2	0.13	0.42	0.31
Hongo 1	19.0	375	82	100	3	0.13	0.42	0.32
Hongo 6	8.3	399	74	90	1	0.12	0.42	0.29
Hon-komagome 5	17.6	1002	176	215	5	0.13	0.41	0.29
Yushima 3	16.9	751	189	229	4	0.12	0.41	0.27
Sengoku 4	19.3	1218	120	144	2	0.11	0.41	0.26
Yushima 2	12.8	558	103	122	2	0.10	0.40	0.25
Hongo 3	16.6	614	118	140	4	0.11	0.40	0.27
Nezu 1	9.6	410	64	74	2	0.10	0.40	0.25
Sendagi 2	14.4	914	150	175	8	0.11	0.39	0.26
Hakusan 1	14.3	841	151	175	5	0.10	0.39	0.23
Otsuka 1	17.4	639	131	151	8	0.11	0.39	0.26
Nezu 2	11.1	942	107	122	2	0.08	0.39	0.20
Mejirodai 3	17.4	719	91	103	1	0.07	0.39	0.18
Sengoku 3	18.3	773	129	146	4	0.08	0.38	0.20
Sendagi 3	21.8	1165	208	234	7	0.08	0.38	0.18
Sengoku 2	17.5	809	118	131	4	0.07	0.38	0.18
Otowa 2	6.6	170	49	53	4	0.09	0.38	0.22
Hon-komagome 4	11.9	639	142	157	9	0.09	0.37	0.20
Sendagi 5	21.2	1291	191	211	10	0.08	0.37	0.18
Sekiguchi 1	16.6	579	123	135	6	0.08	0.37	0.18
Hon-komagome 6	31.8	594	92	100	5	0.07	0.37	0.18
Koraku 2	10.4	386	47	50	2	0.06	0.37	0.15
Sengoku 1	15.7	730	134	146	3	0.06	0.37	0.13
Hakusan 4	20.4	961	164	179	7	0.07	0.37	0.16
Otsuka 6	12.9	841	113	122	7	0.07	0.37	0.17
Hon-komagome 2	27.9	868	130	140	7	0.07	0.36	0.16
Otsuka 5	30.9	926	115	123	9	0.08	0.36	0.18
Hon-komagome 3	23.0	885	177	190	4	0.05	0.36	0.11
Hongo 5	13.5	733	97	99	10	0.06	0.35	0.15
Koraku 1	33.5	113	27	26	2	0.02	0.35	0.06
Nishikata 2	16.3	789	127	128	10	0.04	0.34	0.10
Hakusan 5	23.0	716	133	133	10	0.04	0.34	0.09
Koishikawa 4	20.8	457	112	111	9	0.04	0.34	0.09
Mukougaoka 2	21.1	836	122	121	10	0.04	0.34	0.09
Hon-komagome 1	15.9	728	130	129	7	0.02	0.34	0.06
Koishikawa 3	18.7	843	142	141	12	0.04	0.34	0.09
Nishikata 1	10.9	344	82	80	6	0.03	0.33	0.06
Hakusan 2	19.9	852	123	121	7	0.02	0.33	0.05
Sendagi 1	9.5	435	77	75	3	0.01	0.33	0.02
Hongo 4	16.4	812	132	129	6	0.01	0.33	0.03
Kohinata 2	17.5	629	96	92	11	0.04	0.33	0.09
Yusima 1	14.3	187	49	46	6	0.03	0.33	0.08
Hakusan 3	21.1	312	62	58	10	0.05	0.32	0.13
Koishikawa 1	11.9	399	87	82	7	0.01	0.32	0.03
Otowa 1	11.1	310	83	78	11	0.04	0.32	0.09
Mejirodai 1	18.6	418	83	77	7	0.01	0.32	0.02
Koishikawa 2	14.9	539	98	91	8	0.01	0.32	0.01
Kasuga 1	18.7	194	56	51	6	0.01	0.31	0.02
Suidou 2	11.2	442	70	64	9	0.02	0.31	0.06
Yayoi 2	11.1	352	52	47	11	0.06	0.31	0.16
Kohinata 4	6.6	164	50	45	6	0.01	0.31	0.03
Kohinata 3	8.0	364	64	58	7	0.01	0.31	0.02
Kasuga 2	15.4	400	88	79	14	0.03	0.31	0.07
Mukougaoka 1	14.2	587	82	73	11	0.01	0.30	0.03
Mejirodai 2	13.4	470	92	82	14	0.02	0.30	0.05
Kohinata 1	18.5	547	136	122	20	0.02	0.30	0.05
Suidou 1	9.2	239	52	44	9	0.01	0.29	0.03
Sekiguchi 2	16.5	130	54	45	10	0.01	0.29	0.03
Sekiguchi 3	9.8	189	68	57	13	0.02	0.29	0.04
Yayoi 1	14.0	198	40	32	8	0.00	0.28	0.00
Otsuka 2	19.2	308	55	43	12	0.00	0.27	0.00
Hongo 7	42.9	230	38	29	9	0.00	0.27	0.00
Yushima 4	10.4	276	43	32	13	0.03	0.26	0.07
Otsuka 1	14.0	194	51	29	22	0.00	0.20	0.00
Sendagi 4	8.2	485	0	0	0	0.00	0.00	0.00

Figure 14: Formation of the road network in Hongo 2 district

Besides, the existence of boundary effect is obstacle in comparing the formation of road networks of more than two districts. The boundary effect makes the evaluation of the formation of road networks in the wrong order. A road network with better arranged formation than others is judged as not better arranged than others or vice-versa.

5.6. Methods for avoiding boundary effects

Boundary effects occur since we use districts as spatial unit to aggregate *e*, *v* and *p*. Certainly, the boundary of districts almost corresponds to roads. The boundary of districts, however, does not always correspond to road links. A little difference between the location of a district's boundaries and that of road links triggers boundary effects. In particular, if the data source of district boundaries is different from that of road links, this difference may occur. Therefore, as one of the solutions to avoid triggering boundary effects, we should use spatial unit data, e.g., districts data, which are consistent with the data of road links. Another solution is to make spatial unit data, e.g., the encircled areas mentioned in section 3, by segmenting a finite plane based on road links.

In this subsection, we evaluate the formation of road networks in encircled areas by using indices based on graph theory, which is the complementary of evaluating the formation of road networks based on fire extinction criterion. Figure 15 shows a large encircled area and roads whose width is less than six meters. By aggregating *e* and *v* in the encircled area, it is considered to avoid triggering boundary effects. There is, however, one problem: counting the number of nodes which are connected with roads which are not in this encircled area. If the number of these nodes is counted to calculate indices, boundary effects occur. To avoid this problem, the number of nodes on wide roads and connected with roads in this encircled area is only counted. From now on, we call these nodes 'main nodes' of an encircled area. Let the number of nodes and that of links in encircled area be v_{in} and e_{in}, respectively. Also, let the number of main nodes be v_m and let the number of links between adjacent two main nodes be e_m. Without loss of generality, e_m is equal to v_m. Besides, it is possible to suppose that the number of independent graphs, p, in an encircled area is one. This property is important because it is difficult to count p by using GIS. Therefore, the number of faces, f, is calculated by the following equation:

$$f = (e_{in} + e_m) - (v_{in} + v_m) + 1 = e_{in} - v_{in} + 1 . \quad (12)$$

Equation (12) indicates that it is enough to count only v_{in} and e_{in} for calculating f.

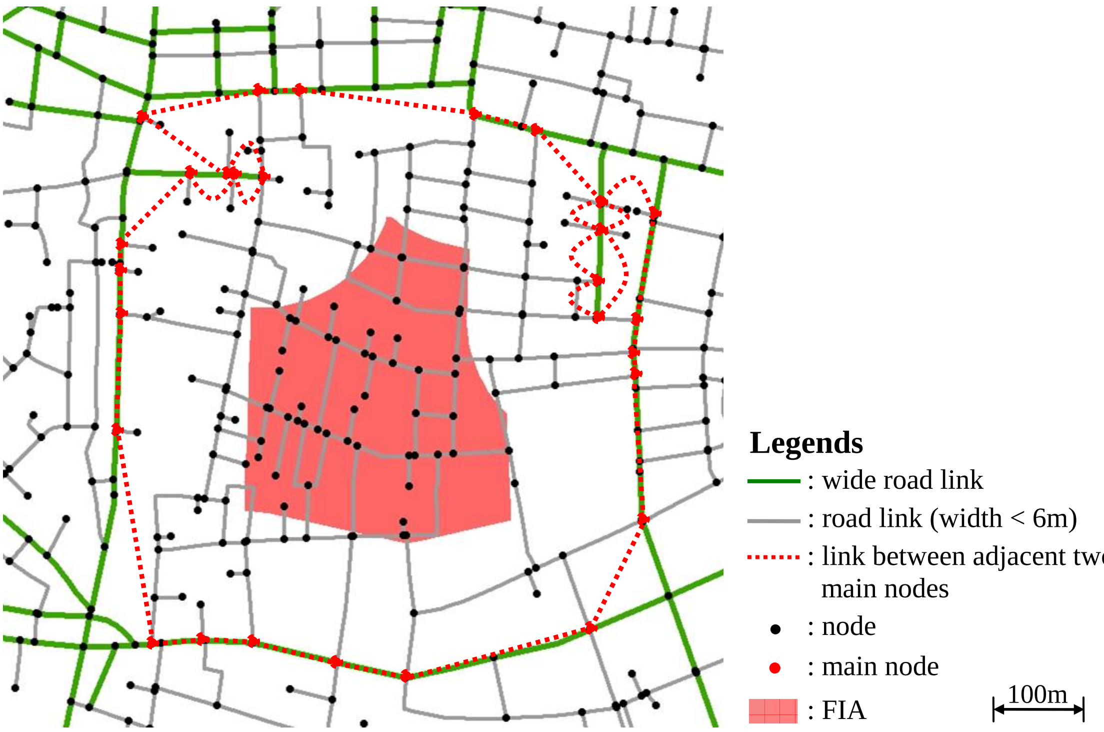


Figure 15: Road networks in an encircled area

Also, when we calculate the value of alpha index, gamma index, we have to count v and e in an encircled area including the perimeter of the encircled area. Let the number of nodes on the perimeter of an encircled area be v_{peri} and the number of links on the perimeter of an encircled area be e_{peri}. Without loss of generality, e_{peri} is equal to v_{peri}. This property is important because it is difficult to count e. Compared with counting e_{peri}, counting v_{peri} is easy if we use the following equation:

$$v_{peri} = v - v_{in} . \quad (13)$$

Therefore, e in an encircled area including the perimeter of the encircled area is calculated by the following equation:

$$e = e_{in} + e_{peri} = e_{in} + v_{peri} = e_{in} + (v - v_{in}) . \quad (14)$$

Therefore, we can calculate the value of alpha index and that of gamma index by the following equations:

$$\alpha = \frac{e_{\text{in}} - v_{\text{in}} + 1}{2v - 5}, \text{ for } v \geq 3, \tag{15}$$

$$\gamma = \frac{e_{\text{in}} + (v - v_{\text{in}})}{3(v - 2)}, \text{ for } v \geq 3. \tag{16}$$

Figure 16 shows the choropleth map of alpha index and that of gamma index, respectively, in Bunkyo ward, by using an encircled area as the spatial unit for aggregating v and e. As stated in subsection 5.3, if the value of alpha index is close to 0.5, then the formation of road networks in an encircled area is close to grid pattern. Also, the higher the value of gamma index is, then the formation of road networks in an encircled area are well arranged in terms of redundancy of choosing roads and possibility of two-way escape.

Figure 16: Choropleth map (left: alpha index, right: gamma index)

*Spatial unit for aggregating v and e: an encircled area.

**Green colored links and black colored links mean wide roads and roads in an encircled area, respectively.

6. Conclusions

In this paper, we discuss how to evaluate the formation of road networks to apply the 3rd paragraph rule stated in section 1 for narrow roads. In section 2 and 3, we ask the local administrative office for how to interpret the criteria related with the formation of road networks in the rule and find out that the formation of road networks is closely related to the possibility of fire-extinguishing. Therefore, we evaluate the formation of road networks based on fire-extinction criterion and we grasp the distribution of FIA and that of encircled region with or without FIA. If a narrow road with attractive landscapes exists in the encircled area without FIA, since we judge the formation of road networks of this area as well arranged, it is possible to apply the rule for this narrow road. In section 4, we discuss the condition that there does not exist FIA in an encircled area and we get inequality (7) as the sufficient condition. From inequality (7), we conclude that, in the context of city planning, it is the way so that there does not exist FIA, to make shorter the length of the interval between two neighboring intersections. In section 5, we try to evaluate the formation of road networks by using index based on graph theory and point out some problems in using indices based on graph theory as evaluating the formation of road networks. It is found that the difference between the data source of districts data and that of road links triggers the difference between the location of a district's boundaries and that of road links, which causes boundary effects. By making spatial unit data, e.g., encircled areas, to aggregate v and e, however, we avoid triggering boundary effects. Therefore, we could use indices based on graph theory as the complementary method for evaluating the formation of road networks in an arbitrary encircled area.

There are some aspects to be resolved. In section 4, we suppose that an encircled region is a convex multi polygon. There are, however, many encircled regions which do not satisfies this supposition. So, we have to clarify the condition so that there does not exist FIA in a non-convex encircled region.

Appendix 1: calculation of criteria of each index for judging whether the formation of a road network is classified as grid pattern or not

a). alpha index

The criteria of alpha index for judging whether the formation of a road networks is classified as grid pattern or not are derived as the infimum of the value of alpha index in case of grid pattern network and the infimum of the value of alpha index in case of delta pattern network. Let alpha index in case of tree pattern and alpha index in case of delta pattern be α_{G} and α_{D}, respectively. The value of α_{G} and α_{D} is calculated by the following equations:

$$\alpha_{\mathrm{G}} = \frac{\left(\frac{3}{2}v-1\right)-v+1}{2v-5} = \frac{\frac{1}{2}v}{2v-5}, \text{ for } v \geq 3, \tag{17}$$

$$\alpha_{\mathrm{D}} = \frac{\{3+2(v-2)-1\}-v+1}{2v-5} = \frac{v}{2v-5}, \text{ for } v \geq 3. \tag{18}$$

Therefore, the infimum of the value of α_{G} and that of α_{D} are as follows:

$$\inf \alpha_{\mathrm{G}} = \lim_{v\to\infty} \frac{\frac{1}{2}v}{2v-5} = \frac{1}{4}, \tag{17-1}$$

$$\inf \alpha_{\mathrm{D}} = \lim_{v\to\infty} \frac{v}{2v-5} = \frac{1}{2}. \tag{18-1}$$

Also, the infimum of the value of GTP index in case of grid pattern network, GTP_{G} and the infimum of the value of GTP index in case of delta pattern network, GTP_{D} are calculated by the following equations:

$$\inf GTP_{\mathrm{G}} = \lim_{v\to\infty} \frac{\frac{1}{2}v}{(\sqrt{v}-1)^2} = \frac{1}{2}, \tag{19}$$

$$\inf GTP_{\mathrm{D}} = \lim_{v\to\infty} \frac{v}{(\sqrt{v}-1)^2} = 1. \tag{20}$$

b). gamma index

The criteria of gamma index for judging whether the formation of a road networks is classified as grid pattern or not are derived as the supremum of the value of gamma index in case of tree pattern network and the infimum of the value of alpha index in case of delta pattern network. Let gamma index in case of tree pattern and gamma index in case of delta pattern be γ_T and γ_D, respectively. The value of γ_T and γ_D is calculated by the following equations:

$$\gamma_T = \frac{v-1}{3(v-2)}, \text{ for } v \geq 3 \tag{21}$$

$$\gamma_D = \frac{2v-3}{3(v-2)}, \text{ for } v \geq 3. \tag{22}$$

Therefore, the supremum of the value of γ_T and the infimum of the value of γ_D are as follows:

$$\sup \gamma_T = \frac{4-1}{3(4-2)} = \frac{1}{2}, \tag{21-1}$$

$$\inf \gamma_D = \lim_{v \to \infty} \frac{2v-3}{3(v-2)} = \frac{2}{3}. \tag{22-1}$$

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