Directional imbalance of transport prices and economic geography

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Abstract

This paper examines how and why transport prices become imbalanced with respect to directions of shipments and how it affects economic geography. It is shown that the equilibrium transport price of the shipment in a particular direction is a nondecreasing function of a relative size of the embarkation region. Furthermore, we show that the directional imbalance of transport prices makes the symmetric pattern more likely to be stable while the core-periphery patterns less likely to be sustainable. In short, the imbalance acts as a centripetal force.

Keywords: behavior of transport firms, break point, export price effect, import price effect, joint production, marginal cost of shipment, return constraint, substitution between imports and domestic products, sustain point

JEL Classification Numbers: F12 (Models of Trade with Imperfect Competition and Scale Economies), R13 (General Equilibrium and Welfare Economic Analysis of Regional Economies), R49 (Other: Transportation Systems)

1 Introduction

It is often observed that transport costs differ depending on directions of shipments. Many of us, for instance, know that a flight from the East coast to the West coast can cost considerably more or considerably less than the counterpart in the reverse direction. To give another example, the report of the freight rates on three major liner trade routes by UNCTAD (2008) indicates their marked difference between pairwise directions of shipments (see Table 1). In addition, Cariou and Wolff (2006) compute the freight rates of maritime transport between Europe and the Far East adjusting the BAF (bunker adjustment factor), and show that they exhibit a significant directional imbalance during the period of 2000-2004: the rate for westbound cargo is always, and twice to third times for most periods, higher than that for eastbound cargo. Finally, Jonkeren et al. (2008) report that the freight rates for trips originating from Rotterdam are 32 percent higher than those arriving there during the period of 2003-2007.

Table 1. Freight rates (market averages) per TEU on the three major liner trade routes.
($ per TEU.)

	Transpacific		Europe-Asia		Transatlantic	
	Asia-USA	USA-Asia	Europe-Asia	Asia -Europe	US-Europe	Europe-USA
2006						
First quarter	1,836	815	793	1,454	995	1,829
Second quarter	1,753	828	804	1,408	1,010	1,829
Thrid quarter	1,715	839	806	1,494	1,041	1,854
Fourth quarter	1,671	777	792	1,545	1,066	1,762
2007						
First quarter	1,643	737	755	1,549	1,032	1,692
Second quarter	1,675	765	744	1,658	1,067	1,653
Thrid quarter	1,707	780	777	1,952	1,115	1,725
Fourth quarter	1,707	794	905	2,054	1,147	1,766
2008						
First quarter	1,725	861	968	2,021	1,193	1,700
Second quarter	1,837	999	1,061	1,899	1,326	1,652

Source: UNCTAD (2008).

One of the most important reasons for this directional imbalance is a difference in volumes of trade in each direction. Jonkeren et al. (2008), using the data on inland waterways in northwest Europe, find out that the freight rates of shipments in a particular direction positively depend on the total amount of shipments in that direction. In a standard textbook, this is explained as follows (see, for example, Boyer (1997)). Consider the situation in

which the demand for transport services in one direction is larger than that in the opposite direction. The shipment in the former direction is sometimes called a 'front-haul' and that in the latter direction a 'back-haul'. Thus, the demand curve for the front-haul lies above that for the back-haul. An equilibrium is realized when the sum of the demand price (marginal evaluation) of the front-haul and that of the back-haul at a particular number of round trips equals the cost per round trip. In other words, the equilibrium is reached at the point where the curve derived by adding the two demand curves vertically cuts the curve representing the cost per round trip. The total cost of a round trip is then divided between the front-haul and the back-haul according to their demand prices. By construction, therefore, the front-haul becomes more expensive than the back-haul at the equilibrium. Now, note that it is the cost *per round trip* but not the costs of each one-way trip that matters in this explanation. This is because transport firms or 'carriers' produce transport services (shipments) in two directions *jointly*. Main reason is that they usually face a physical constraint that transport equipment such as cargo ships, freight cars and cargo trucks eventually return to a home port, namely, a 'return constraint'.

This intuitive explanation, though quite helpful to grasp the nature of the matter, has limitations. For one thing, it lacks an analysis of underlying behaviors of individual carriers. What decisions they make is not obvious, however, because their problem involves joint production with a return constraint. What is more important, it implicitly assumes that the demand for the transport services in a particular direction is independent of the price of the services in the opposite direction. This is not true in the reality. A change in a price of transport services in one direction affects the demand behaviors of consumers in a destination through a change in delivered prices of goods, which in turn gives an impact on wage rates not only in the destination but also in the region of embarkation. This further changes regional incomes as well as price indices in both regions. To take into account these effects altogether, we need to conduct a general equilibrium analysis, in which prices and quantities of transport services in each direction are all determined simultaneously. In contrary, there have been surprisingly few attempts to discuss the determination of transport prices within a general equilibrium framework. One reason might be that they have long lacked a tractable general equilibrium framework that can deal with the relationship between regional trade and transport costs. Nevertheless, we are now provided with a variety of so-called 'new economic geography' models, which has been developed by a number of researchers since Krugman (1991). The first purpose of this paper is, thus, to explain from the behaviors of individual carriers what levels transport prices are determined at and why and how they are directionally imbalanced, based on a general equilibrium model of new economic geography.

The study of the determination of transport prices, especially when the possibility of their directional imbalance is taken into consideration, is a matter of great significance in its own right. However, there is another important reason to study it. It is that transport prices affect locations of economic activities, which is a fundamental message of new economic geography. In particular, this suggests a possibility that the directional imbalance of transport prices, especially such a long-term and steady imbalance as the one observable in Table 1, gives an influence on economic geography. Suppose, for example, that the demand for the transport services from a bigger region to a smaller one is larger than that in the opposite direction. Then, the price of the services in the former direction is higher than that in the latter direction, which implies that exports of the bigger region become more expensive than those of the smaller region at the places of consumption, ceteris paribus. It is not surprising that such a systematic difference affects allocation of economic activities between two regions. The second purpose of this paper is, thus, to examine such effects of a directional imbalance of transport prices upon economic geography.

For all its importance, the directional imbalance of transport costs has seldom attracted attentions of researchers in the field of new economic geography (see Fujita et al. (1999); Fujita and Thisse (2000); and Fujita and Thisse (2002)). A notable exception is a work by Baldwin et al. (2003), who explicitly examine the imbalance. In their study, however, the imbalance is given and no transport sector is considered. Moreover, not only the directional imbalance but even the determination of a transport price has rarely been treated in the literature: in most studies, the price is regarded as an exogenous parameter and a decision making process of transport firms is abstracted away. There is one exception: Behrens et al. (2006) examine the impacts of regulation of a transport sector upon welfare levels, paying attention to incentives of transport firms, though in a very simple way.[1]

In order to explain the determination of transport prices and examine the impact of their directional imbalance upon economic geography, we construct a two region model with a transport sector. Carriers produce transport services in two directions jointly under the return constraint mentioned earlier. Their decision making process is formulated as a simple game à la Bertrand. As in the conventional models in the literature, furthermore, we characterize two types of distribution patterns, a symmetric pattern, in which a mobile factor is distributed equally in the two regions, and core-periphery patterns, in which it is concentrated in one region; and examine how the possibility of a directional imbalance of

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[1]Some works discuss an endogenous determination of transport costs but do not deal with a transport sector explicitly. Mori and Nishikimi (2002), for instance, examine the effect of economy of density on transport costs. Bougheas et al. (1999), and Mun and Nakagawa (2008) study impacts of infrastructure investment. Finally, Takahashi (2006) examines a selection of a transport technology, which determines transport costs.

transport prices affects the conditions for each type of distribution pattern to be supported by a stable equilibrium.

Main findings are summarized as follows. First, the equilibrium transport price of the services in a particular direction is a nondecreasing step-shaped function of the relative size of an embarkation region. More specifically, as the relative size of a region gradually increases from a very low level, the price of the services from that region to the other first rises, then becomes constant and then rises again. A corollary of this finding is that the transport price in a 'binding direction', the direction with a larger amount of shipment, is higher than that in the opposite direction, a 'slack direction'. Second, when two regions are sufficiently alike in size, the amounts of inter-regional shipments in respective directions are equal. When one region is much bigger than the other, instead, volumes of trade are imbalanced. Third, the directional imbalance of transport prices works to stabilize the symmetric pattern. Lastly but not the least, as we allow for the directional imbalance, the core-periphery patterns become less likely to be sustainable. The last two results demonstrate that *the directional imbalance acts as a centripetal force.*

The paper is organized as follows. In the next section, a basic model is presented. Section 3 formulates a transport sector and derives a Nash equilibrium of a game played by carriers. In the subsequent section, we explore the effects of a directional imbalance of transport costs upon economic geography. Finally, Section 5 concludes.

2   Model

As a basic framework, we use a modified version of an analytically solvable model by Forslid and Ottaviano (2003), which is presented as a 'footloose entrepreneur model' in Baldwin et al. (2003).

2.1   Basic Framework

There are two regions, denoted by 1 and 2; two sectors, agriculture and manufacturing sectors; and two factors, workers and entrepreneurs.

Workers, whose number is denoted by l, are distributed equally in two regions and cannot migrate across regions. There are, on the other hand, n entrepreneurs, of which $\lambda_1 n$ live in region 1 and $\lambda_2 n$ live in region 2 ($\lambda_1 \in [0,1]$, $\lambda_2 \equiv 1 - \lambda_1$). Instead of λ_1, a parameter $\lambda \in [0,1]$ with $\lambda \equiv \lambda_1$ will be used when it is more convenient. Entrepreneurs can freely move across regions.

The agricultural sector is competitive and produces a homogeneous product through a constant returns to scale technology. More specifically, a^A workers are used to produce 1

unit of agricultural goods. This implies that their price in region i is equal to $a^A w_i^A$, where w_i^A is a wage rate of workers in that region ($i = 1, 2$). The agricultural goods can be shipped from one region to the other with no cost so that their prices and, consequently, wage rates of workers are equalized in two regions. The wage rate is taken as a numeraire.

To the contrary, the manufacturing sector is monopolistically competitive and produces a differentiated product through an increasing returns to scale technology. Each entrepreneur owns a firm which produces a variety in the region of her residence. Thus, there are n varieties in the economy, of which $\lambda_i n$ are produced in region i ($i = 1, 2$). Each firm produces 1 unit of a variety using a^M workers and skill of an entrepreneur. All the revenue left after the payment for workers is taken by an entrepreneur. In other words, the profit of a firm is equal to 0:

$$p_i q_i - a^M q_i - w_i = 0, \qquad (i = 1, 2) \tag{1}$$

where p_i and q_i denote a price and an amount of a variety produced in region i, and w_i a wage rate received by an entrepreneur in that region.

The workers and entrepreneurs have the same preference represented by a utility function, $U = (C_M)^\mu (C_A)^{1-\mu}$. Here, C_A is a consumption of agricultural goods and $C_M = [\int_0^n x(k)^\rho \, dk]^{1/\rho}$ is an aggregate of a manufacturing consumption with $x(k)$ denoting the amount of the kth variety ($\rho \in (0, 1)$).

Entrepreneurs sell their varieties at the price that maximizes their own wages. Since not only they are subject to the same technological condition but also all the varieties enter the utility function in a symmetric manner, they charge the same price as long as they are located in the same region. Thus, we denote the prices of a variety produced in each region by p_1 and p_2.

Next, let us introduce transport costs. It takes no cost to ship a manufacturing product within the same region. In order to ship it across regions, however, consumers need to use transport services, which are provided by transport firms or 'carriers'. Carriers take a portion of manufacturing products by way of compensation for the transport services they provide. The portion is proportional to the amount of shipment and depends on its direction. Let $\tau_i \in (0, 1)$ be the amount taken by a carrier when it carries 1 unit of a manufacturing product from region i to region j ($i = 1, 2$ and $j \neq i$). (Bear in mind that the subscript of τ refers to the region of the *origin* of shipment.) That is to say, transport cost or a price of transport services from region i to region j is equal to $\tau_i p_i$.

Three comments are in order. First, in our setting, transport costs become proportional to the prices of the products to be shipped, that is, they are *ad valorem*.[2] Second, what

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[2] The assumption of the ad valorem transport costs is common among all the economic geography models based on a Dixit-Stiglitz framework. It is primarily for the sake of analytical convenience. In addition,

distinguishes our model from conventional ones is that we are taking into account the possibility that *transport costs may differ depending on directions of shipment*, that is, τ_1 is not necessarily equal to τ_2. Third and lastly, the delivered price in region j of a variety produced in region i is equal to $(1 + \tau_i)p_i$. In what follows, $t_i \equiv 1 + \tau_i$ may be used as an alternative measure of transport costs rather than τ_i whenever it is useful $(i = 1, 2)$. Note that $t_i \in (1, 2)$ by definition.

Let X_{ii} and X_{ij} be the total demands for a variety produced in region i by region i consumers and region j consumers, respectively. Applying a standard procedure to derive consumers' demand functions yields

$$X_{ii} = \mu p_i^{-\sigma} P_i^{\sigma-1} Y_i, \qquad X_{ij} = \mu t_i^{-\sigma} p_i^{-\sigma} P_j^{\sigma-1} Y_j. \qquad (i = 1, 2; j \neq i) \qquad (2)$$

Here, $\sigma \equiv 1/(1 - \rho) > 1$ is an elasticity of substitution, Y_i is an aggregate income and

$$P_i = \left[n \left(\lambda_i p_i^{1-\sigma} + \lambda_j t_j^{1-\sigma} p_j^{1-\sigma} \right) \right]^{\frac{1}{1-\sigma}} \qquad (i = 1, 2; j \neq i) \qquad (3)$$

is a price index of the manufacturing products consumed in region i. As will be shown later, all the manufacturing products taken by carriers for transport services are used up as inputs for their production. Therefore, the amount of each variety produced in region i, q_i, must be equal to the sum of demands by consumers in respective regions:

$$q_i = X_{ii} + t_i X_{ij}. \qquad (i = 1, 2; j \neq i) \qquad (4)$$

An entrepreneur in each region maximizes her wage, w_i, given by (1) $(i = 1, 2)$, setting prices at

$$p_1 = p_2 = p \equiv \frac{\sigma}{\sigma - 1} a^M. \qquad (5)$$

She therefore receives a wage equal to

$$w_i = \frac{1}{\sigma - 1} a^M q_i. \qquad (i = 1, 2) \qquad (6)$$

Moreover, because no carriers earn a positive profit as will be shown in the next section, the aggregate income consists of workers' and entrepreneurs' earnings:

$$Y_i = \frac{l}{2} + \lambda_i n w_i. \qquad (i = 1, 2) \qquad (7)$$

Finally, by choosing appropriate units, the analysis becomes simpler. First, for the agricultural goods, we take a unit so that the labor input coefficient a^A becomes equal to 1.

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a number of empirical studies have found a positive relationship between shipping prices and values of the goods shipped. A 1 % increase in the value per kilo, for instance, results in a 0.55% increase in the transport cost (import charges) for the US imports (Blonigen and Wilson (2008)) and a 0.34% increase in the transport cost in 16 Latin-American countries (Wilmsmeier et al. (2006)). The main reason for the positive relationship is that high valued products tend to demand more extensive insurance coverage, stricter security, and an additional care in the process of handling.

Second, for the manufacturing goods, we do the same so that $a^M = (\sigma - 1)/\sigma$, which implies that $p = 1$. This implies that the measure of transport costs, $t_i \equiv 1 + \tau_i$, actually gives a delivered price of the manufacturing products that are produced in region i and consumed in region j ($i = 1, 2$; $j \neq i$). Third, the number of workers, l, is normalized to 2. Lastly, the number of entrepreneurs, n, is normalized to 1. This completes a description of a basic framework.

2.2 Short-run equilibrium

A system of equations (2) to (7) determines a short-run equilibrium, for which entrepreneurs' distribution, λ, is given. Substituting (2) to (6) into the two equations in (7) and solving them simultaneously, we can obtain the equilibrium values of Y_1 and Y_2:

$$Y_i = \sigma \Gamma \left[\sigma \theta_1 \theta_2 + \mu \theta_i \left(\lambda_i \phi_i - \lambda_j \right) \right], \qquad (i = 1, 2; \; j \neq i) \tag{8}$$

where $\Gamma \equiv \left[(\sigma \theta_1 - \mu \lambda_1)(\sigma \theta_2 - \mu \lambda_2) - \sigma^2 \lambda_1 \lambda_2 \theta_1 \theta_2 \right]^{-1}$ and $\theta_i \equiv P_i^{1-\sigma}$. In addition, $\phi_i \equiv t_i^{1-\sigma}$ represents 'freeness of trade' ($i = 1, 2$). By definition, $\phi_i \in \Phi \equiv \left(2^{1-\sigma}, 1 \right)$. Equation (8), in turn, gives the equilibrium values of demand variables, X_{ii}'s and X_{ij}'s, by (2).

Furthermore, a volume of trade from region i to region j is equal to $Z_i \equiv \lambda_i n t_i X_{ij}$ ($i = 1, 2, j \neq i$). It is convenient to define their ratio:

$$\widehat{Z} \equiv \frac{Z_1}{Z_2} = \frac{\widehat{\lambda} \phi_1 \left[\widehat{\lambda} \left(\sigma - \mu \right) + \phi_2 \left(\sigma + \mu \right) \right]}{\phi_2 \left[\sigma - \mu + \widehat{\lambda} \phi_1 \left(\sigma + \mu \right) \right]}, \tag{9}$$

where $\widehat{\lambda} \equiv \lambda_1 / \lambda_2$. (In this paper, circumflexes are used to indicate ratios of the variables pertaining to two regions or two directions.) It is straightforward to see that

$$\widehat{Z} \left\{ \begin{array}{c} \geqq \\ < \end{array} \right\} 1 \quad \text{if} \quad \widehat{\lambda} \left\{ \begin{array}{c} \geqq \\ < \end{array} \right\} \left(\frac{\phi_2}{\phi_1} \right)^{\frac{1}{2}} = \left(\frac{t_1}{t_2} \right)^{\frac{\sigma-1}{2}}. \tag{10}$$

Thus, whether Z_1 exceeds Z_2 or not depends on transport costs in two directions and a distribution of entrepreneurs. More specifically, if a region is relatively bigger and/or the transport cost of an outbound trip is relatively lower compared to that of an inbound trip, then the outbound trade is more likely to outweigh the inbound trade.

2.3 Long-run equilibrium

In a long run, entrepreneurs move freely across regions seeking a higher level of indirect utility. Let $v_i(\lambda)$ be the indirect utility for an entrepreneur living in region i:

$$v_i(\lambda) = K \frac{w_i}{P_i^{\mu}} = K \sigma \Gamma \theta_i^{\frac{\mu}{\sigma-1}} \left[\sigma (\theta_j + \theta_i \phi_i) + \mu \lambda_j \left(\phi_1 \phi_2 - 1 \right) \right], \qquad (i = 1, 2; \; j \neq i) \tag{11}$$

where $K \equiv \mu^{\mu} (1 - \mu)^{1-\mu} > 0$.

This would be a good place to note that transport costs affect a relative level of indirect utility, $\widehat{v}(\lambda) \equiv v_1(\lambda)/v_2(\lambda)$, through three channels.

$$\frac{\mathrm{d}\ln\widehat{v}(\lambda)}{\mathrm{d}\ln t_i} = \left.\frac{\partial\ln\widehat{w}}{\partial\ln t_i}\right|_{\widehat{Y}\ \mathrm{const.}} + \frac{\mathrm{d}\ln\widehat{Y}}{\mathrm{d}\ln t_i}\frac{\partial\ln\widehat{w}}{\partial\ln\widehat{Y}} - \mu\frac{\mathrm{d}\ln\widehat{P}}{\mathrm{d}\ln t_i}, \qquad (i=1,2) \qquad (12)$$

where $\widehat{w} \equiv w_1/w_2$, $\widehat{Y} \equiv Y_1/Y_2$ and $\widehat{P} \equiv P_1/P_2$.

First, as a result of a change in transport costs, consumers *substitute between imports and domestic products*, which alters a relative wage rate. Suppose that t_i rises. Then, region i products become more expensive in region j $(j \neq i)$, which induces consumers in that region to shift part of their consumption from now relatively more expensive imports to now relatively less expensive domestic products. This undermines the demand for region i products by region j consumers and thus gives an adverse effect on w_i, other things being equal. This negative effect is referred to as an *export price effect*. At the same time, it stimulates the demand for domestic products in region j and therefore raises w_2. This *import price effect* gives a negative impact on the relative wage rate. These two negative effects are all together referred to as a *terms of trade effect*, which is represented by the first term in the right hand side of (12).

Second, the second term describes an effect through regional incomes. The fall in w_i and the rise in w_j due to the terms of trade effect bring about a decrease in Y_i and an increase in Y_j, respectively. For w_i, the decrease in Y_i works adversely but the increase in Y_j works favorably. The overall direction of a change of w_i depends on the sizes of the changes in Y_i and Y_j and the sensitivities of w_i to these changes. Unless region j is much 'bigger' than region i, that is, unless the former region has a quite large population and/or a big advantage in transport costs (t_j is much lower than t_i), however, the effect of the decrease in Y_i dominates that of the increase in Y_j.[3] Then, w_i declines. Otherwise, we have the opposite: changes in regional incomes result in a rise in w_i.

Third and finally, we have a channel through price indices, represented by the last term of (12). As t_i goes up, the delivered price of region i products and, therefore, a price index increase in region j, which raises a relative price index. This gives an adverse impact on the relative level of indirect utility.

Now, migration behaviors of entrepreneurs follow a simple equation, which is common in the literature:

$$\dot{\lambda} = \zeta[v_1(\lambda) - v_2(\lambda)], \qquad (13)$$

where $\zeta > 0$ denotes speed of adjustment.

A distribution of entrepreneurs is at a long-run equilibrium if it is a stationary point of (13). That is to say, an interior distribution ($\lambda \in (0,1)$) is supported by a long-run

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[3]Note that this applies to the case where two regions are of equal size. It is because a wage rate responds more to the income of a home region than to that of a foreign region, ceteris paribus.

equilibrium if $v_1(\lambda) = v_2(\lambda)$. The core-periphery pattern with $\lambda_i = 1$ is so if $v_i(1) \geq v_j(1)$ $(i = 1, 2; j \neq i)$.

In addition, equation (13) implies that an equilibrium distribution, denoted by λ^*, is stable if

$$\begin{cases} \dfrac{\mathrm{d}\widehat{v}(\lambda^*)}{\mathrm{d}\lambda} \leq 0 \text{ holds when } \lambda^* \in (0, 1), \\[2ex] \text{at least either } \dfrac{\mathrm{d}\widehat{v}(1)}{\mathrm{d}\lambda} \leq 0 \text{ or } \widehat{v}(1) > 1 \text{ holds when } \lambda^* = 1, \text{ and} \\[2ex] \text{at least either } \dfrac{\mathrm{d}\widehat{v}(0)}{\mathrm{d}\lambda} \leq 0 \text{ or } \widehat{v}(0) < 1 \text{ holds when } \lambda^* = 0. \end{cases}$$

3 Determination of transport prices

In this section, we examine determination of transport prices paying a special attention to how they become divergent depending on directions of shipments.

To begin with, let us normalize the unit of transport services so that 1 unit of them is necessary to ship 1 unit of a manufacturing product across regions. This implies that the total demand for the transport services from region i to region $j \neq i$ is equal to the volume of trade in that direction, i.e., Z_i. On the other hand, the total supply of the services is given by $\sum_{h=1}^{m} z_i^h$. Here, z_i^h is the amount of services from region i to region $j \neq i$ provided by an h-th carrier; and m is the number of carriers, which is given. Therefore, the markets of transport services clear if $Z_1 = \sum_{h=1}^{m} z_1^h$ and $Z_2 = \sum_{h=1}^{m} z_2^h$.

Each carrier must incur two kinds of costs to produce transport services. One is the costs that depend on the total amounts of shipment in a round trip. They include the costs to stow and discharge a cargo and a rental cost of containers, for instance. For simplicity, I assume that a fixed amount, $c > 0$, of manufacturing products are necessary to handle 1 unit of shipment. Thus, carrier h incurs this type of costs equal to $c\left(z_1^h + z_2^h\right)$ (remember that $p = 1$). The other is the costs related to a 'capacity' of shipment. Suppose that carriers face the return constraint mentioned earlier, which requires that transport equipment such as cargo ships, freight cars and cargo trucks eventually return to a home port region. In this case, part of costs depend on the scale of the equipment dispatched for a round trip, or, we may call it, a capacity of shipment, which is determined by the size of a front-haul, namely, $\max[z_1^h, \ z_2^h]$. Here again we limit our attention to the simplest case: carriers need to use manufacturing products whose amount is proportional to $\max[z_1^h, \ z_2^h]$. Thus, this 'capacity cost' is given by $k\max[z_1^h, \ z_2^h]$ ($k \geq 0$). In sum, the total costs to produce transport services is equal to $c\left(z_1^h + z_2^h\right) + k\max[z_1^h, \ z_2^h]$.[4]

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[4]Alternatively, we might assume that carriers use labor instead of manufacturing products in the production of transport services. Under this assumption, however, the manufacturing products taken by carriers by way of compensation for transport costs are to be placed in a market and eventually consumed by someone.

It is important to emphasize that when the amounts of shipments diverges between the two directions, marginal (and average) costs of shipment differ depending on its direction. To see this, suppose that $z_1^h > z_2^h$, without loss of generality. In order to increase a front-haul, namely, a shipment from region 1 to region 2, it is necessary for carrier h to rent an additional cargo ship, a freight car or a cargo truck. Such an expansion of transport equipment or capacity induces the carrier to pay a marginal cost of k as well as c. However, it does not have to incur such a cost as long as the addition is made to a back-haul, namely, a shipment from region 2 to region 1. Therefore, *the marginal cost of a front-haul is $c + k$ while that of a back-haul is c.*

Moreover, it is assumed that when there is an imbalance, the marginal costs do not exceed the price of manufacturing products, i.e., 1,. In other words, we assume that $c < 1$ and $c + k < 1$. Since $k \geq 0$, only the latter inequality matters:

Assumption 1. $c + k < 1$.

This assumption, as will become clear, guarantees that transport prices are lower than the price of manufacturing products, 1.

Now, recalling that each carrier receives τ_i when it carries 1 unit of manufacturing products from region i to region $j \neq i$, we can write down an h-th carrier's profit as

$$\pi^h \equiv (\tau_1 - c)\, z_1^h + (\tau_2 - c)\, z_2^h - k \max[z_1^h,\ z_2^h]. \tag{14}$$

Carriers choose the prices and the quantities of transport services that maximize the profit given by (14), playing a two stage game, given total demands Z_1 and Z_2. In the first stage, they simultaneously quote prices of transport services, τ_1^h's and τ_2^h's ($h = 1, \cdots, m$). The carriers who quote the lowest price of τ_i are entitled to sell in the next stage any amount of transport services up to $Z_i/\overline{m}_i$ where $\overline{m}_i$ is the number of such carriers. Those who fail to quote the lowest price cannot sell any services. Carriers are thus put in a competitive environment in the sense that each of them faces an infinitely high elasticity of demand.[5] In the second stage, they decide the amounts of services to produce, z_1^h and z_2^h.

This modification involves two problems. First, because the structure of a model changes, we can no longer consider a conventional model as a special case of our model. As a result, the comparison of results becomes difficult. Second, technically speaking, it means that t_i is to be removed from (4). It turns out that this change, though it seems minor, makes the analysis much more complicated and prevents us from deriving clear-cut results.

[5]It is often maintained that linear shipping, which is one of the two distinct markets of ocean transport for non-oil cargo, has been characterized by collusive agreements, or shipping conference (see Sjostrom (2004) and the studies cited there). However, their effectiveness is still an open question (see Hummels (2007)). Other industries such as tramp shipping, trucking and charter plane industries are allegedly more competitive. In this paper, we concentrate on a benchmark case of a competitive market.

We focus on the Nash equilibrium of this transport game that meets the following three requirements. First, each carrier earns a nonnegative profit. Second, the markets of transport services clear. Third and lastly, behaviors of carriers are symmetric, that is, all of them quote the same prices and produce the same amounts. In what follows, the term 'equilibrium' will refer to the Nash equilibrium satisfying all these requirements. To characterize the equilibrium, it is convenient to distinguish two cases: the case with $\widehat{Z} \neq 1$ and that with $\widehat{Z} = 1$.

First, suppose that $\widehat{Z} \neq 1$. Let us designate the direction in which total demand for transport services is larger as a 'binding direction' and the opposite direction as a 'slack direction'. Furthermore, we denote the embarkation region of the shipment in a binding direction by b and that in a slack direction by s. Thus, $Z_b > Z_s$ ($b = 1, 2; s \neq b$). The market clearing and the symmetric behaviors of carriers demand that each of them produce Z_1/m and Z_2/m of respective services. Each carrier then earns a profit equal to $\sum_{l=1,2} (\tau_l - c) Z_l/m - kZ_b/m$. The first point to make is that *as long as $\widehat{Z} \neq 1$, price markups of transport services in each direction must be 0 at the equilibrium*: $\tau_b - c - k = 0$ and $\tau_s - c = 0$. If the price markups were positive, a carrier could get a higher profit by charging a slightly lower price than its competitors. For instance, if $\tau_b - c - k > 0$, a carrier could lower its quote a little from τ_b and increase the production from Z_b/m to Z_b, which gives a higher profit. If the price markups were negative, on the other hand, it could enhance its profit by slightly reducing its supply. If $\tau_b - c - k < 0$, for example, a carrier could contract the production of transport services in a binding direction so that its amount gets smaller than Z_b/m (but still larger than Z_s/m).

This implies that the equilibrium price equals $c + k$ in a binding direction while c in a slack direction. Several comments are in order. First, because $Z_b/m > Z_s/m$ by definition, the shipment in a binding direction is a front-haul and that in a slack direction is a back-haul for each carrier. Therefore, we can say that *each carrier sets the prices equal to the marginal (and average) costs of shipment in each direction*. This indicates that our transport game is a variant of the *Bertrand price setting game*. Second, each carrier earns 0 profit at the equilibrium, which is a typical result of the Bertrand price setting game. Third and lastly, the next result immediately follows:

Proposition 1. *The transport price in a binding direction is no lower than that in a slack direction.*

This result is consistent with an empirical finding by Jonkeren et al. (2008).

Now, suppose that $b = 1$, that is, the direction from region 1 to region 2 is binding. Because $\widehat{Z} > 1$ and $(\tau_1, \tau_2) = (c + k, c)$ in this case, (10) implies that $\widehat{\lambda} > \widehat{\lambda}^U \equiv \left[(c + k + \right.$

$1)/(c+1)]^{\frac{\sigma-1}{2}}$. Similarly, $b = 2$ $(\widehat{Z} < 1)$ occurs if and only if $\widehat{\lambda} < \widehat{\lambda}^L \equiv [(c+1)/(c+k+1)]^{\frac{\sigma-1}{2}}$. Consequently, we conclude that if there is an equilibrium with $\widehat{Z} \neq 1$, then the equilibrium pair of transport prices is given by

$$(\tau_1^*, \tau_2^*) \equiv \begin{cases} (c+k, \ c) & \text{if} & \widehat{\lambda} > \widehat{\lambda}^U \\ (c, \ c+k) & \text{if} & \widehat{\lambda} < \widehat{\lambda}^L. \end{cases} \tag{15}$$

Furthermore, we can easily show that (τ_1^*, τ_2^*) is indeed supported by a Nash equilibrium of the transport game.

Lemma 1. *The strategy profile in which all the carriers quote the same pair of prices, (τ_1^*, τ_2^*), is a Nash equilibrium in the transport game.*

The proof is tedious and relegated to the Appendix.

Next, consider the remaining case with $\widehat{Z} = 1$. First of all, (10) requires that a pair of transport prices satisfy

$$\left(\frac{\tau_1 + 1}{\tau_2 + 1} \right)^{\frac{\sigma-1}{2}} = \widehat{\lambda}. \tag{16}$$

Second, notice that the profit of each carrier at the equilibrium is now equal to $(\tau_1 + \tau_2 - 2c - k)Z_1/m$. The nonnegative profit requires that $\tau_1 + \tau_2 - 2c - k \geq 0$. If $\tau_1 + \tau_2 - 2c - k > 0$, however, a carrier would make an attempt to undercut either or both prices to get a higher profit. Therefore, we must have

$$\tau_1 + \tau_2 - 2c - k = 0. \tag{17}$$

This implies that the profit of each carrier becomes 0 at the equilibrium.

Solving (16) and (17) for τ_1 and τ_2, we conclude that if there is an equilibrium with $\widehat{Z} = 1$, then the equilibrium pair of transport prices is given by

$$(\tau_1^o, \tau_2^o) \equiv \left(\frac{\widehat{\lambda}^{\frac{2}{\sigma-1}}(2c+k+1) - 1}{1 + \widehat{\lambda}^{\frac{2}{\sigma-1}}}, \ \frac{2c+k+1 - \widehat{\lambda}^{\frac{2}{\sigma-1}}}{1 + \widehat{\lambda}^{\frac{2}{\sigma-1}}} \right). \tag{18}$$

It is readily verified that τ_1^o is increasing in $\widehat{\lambda}$ (and therefore, in λ) while τ_2^o is decreasing in $\widehat{\lambda}$ (and therefore, in λ). Transport prices coincide with each other at the symmetric pattern $(\widehat{\lambda} = 1)$ and diverge more and more as the distribution goes farther away from it.

We need a set of conditions for this pair of transport prices to be supported by a Nash equilibrium. Suppose that a price of transport services in a certain direction is lower than c. Then, each carrier has an incentive to cut the service in that direction and earn a higher profit. Therefore, a Nash equilibrium requires that $\tau_1^o - c \geq 0$ and $\tau_2^o - c \geq 0$. Formally, this is shown as follows.

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Lemma 2. *The strategy profile in which all the carriers quote the same pair of prices,* (τ_1^o, τ_2^o), *is a Nash equilibrium in the transport game, if and only if* $\tau_1^o - c \geq 0$ *and* $\tau_2^o - c \geq 0$.

The proof of this lemma, which is relegated to the Appendix, is similar to that of Lemma 1 with some minor changes. By (18), the two conditions $\tau_1^o - c \geq 0$ and $\tau_2^o - c \geq 0$ can be rewritten as follows:

$$\widehat{\lambda}^L \leq \widehat{\lambda} \leq \widehat{\lambda}^U. \tag{19}$$

Because these two cases with $Z \neq 1$ and $Z = 1$ are exhaustive, we have established the following result.

Proposition 2. *The equilibrium transport prices are given by* (τ_1^*, τ_2^*) *in (15) when (19) does not hold, and* (τ_1^o, τ_2^o) *in (18) when (19) holds. At the equilibrium, all the carriers earn 0 profit.*

The first panel of Fig. 1 shows transport prices as functions of λ for $\sigma = 3$, $c = 0.2$ and $k = 0.3$. The solid line represents τ_1 and the dotted line τ_2. Because τ_1^o increases and τ_2^o decreases with λ, the transport prices exhibit step-like shapes. As σ rises, furthermore, an increasing part of τ_1 becomes flatter and the range of λ associated with it widens.

— Insert Fig. 1 around here. —

Three remarks are worth adding. First, the 0 profit result implies that the manufacturing products taken by carriers by way of compensation for the transport services are just used up as inputs for their production. This enables us to analyze the transport sector and the manufacturing sector separately, which greatly simplifies the analysis. Second, as has been postulated, the derived transport prices do not exceed the price of manufacturing products, namely, 1. When they are given by (τ_1^*, τ_2^*), it is self-evident. When they are given by (τ_1^o, τ_2^o), on the other hand, it may need some explanation. Note that τ_1^o reaches its maximum at $\widehat{\lambda} = \widehat{\lambda}^U$ because it increases with $\widehat{\lambda}$. However, we can verify by a tedious computation that $\tau_1^o < 1$ at that point as long as Assumption 1 holds. Hence, $\tau_1^o < 1$ for any $\widehat{\lambda} \in [\widehat{\lambda}^L, \widehat{\lambda}^U]$. A parallel reasoning proves that $\tau_2^o < 1$ for any $\widehat{\lambda} \in [\widehat{\lambda}^L, \widehat{\lambda}^U]$. Third and finally, the equilibrium prices are always positive.

To close this section, notice that (9) implies that $\partial \widehat{Z}/\partial \widehat{\lambda} > 0$ (for constant ϕ_1 and ϕ_2). Therefore, whenever (τ_1, τ_2) is given by (τ_1^*, τ_2^*), $\widehat{Z}$ increases with λ. This establishes the following result.

Proposition 3. *When* $\widehat{\lambda} < \widehat{\lambda}^L$, *the volume of trade from region 1 to region 2 is smaller than that in the opposite direction. When* $\widehat{\lambda} > \widehat{\lambda}^U$, *to the contrary, it is larger. In both*

cases, it increases with a size of region 1. Finally, when $\widehat{\lambda}^L \leq \widehat{\lambda} \leq \widehat{\lambda}^U$, the volumes of trade are equal for two directions and they are independent of sizes of two regions.

Diagram (b) of Fig. 1 shows $\widehat{Z}$ as a function of λ (again $\sigma = 3$, $c = 0.2$ and $k = 0.3$). It indicates that $\widehat{Z}$ is continuous but that its derivative is not.

4 Impacts of the directional imbalance of transport prices upon economic geography

In this section, we study how the directional imbalance of transport prices affects economic geography. Two types of distribution patterns, core-periphery patterns and a symmetric pattern are discussed in order. Then, the effects of changes in parameters, in particular, the two parameters related to the costs to produce transport services, upon economic geography are briefly discussed.

4.1 Core-periphery pattern

To begin with, we consider a core-periphery pattern in which entrepreneurs are concentrated in region 1, that is, $\lambda = 1$. Thus, region 1 is a 'core' region while region 2 is a 'periphery' region. The other core-periphery patten with $\lambda = 0$ can be similarly analyzed.

First of all, since $\widehat{\lambda}$, which becomes infinitely large at $\lambda = 1$, exceeds $\widehat{\lambda}^U$, $(\tau_1,\ \tau_2)$ is given by $(\tau_1^*,\ \tau_2^*) = (c + k,\ c)$.

Proposition 4. *When entrepreneurs are concentrated in region 1, transport prices are given by $(\tau_1,\ \tau_2) = (c + k,\ c)$.*

As has been mentioned earlier, the core-periphery pattern with $\lambda = 1$ is supported by a long-run equilibrium if $\widehat{v}(1) \geq 1$; and if so, it is stable when at least either $\mathrm{d}\widehat{v}(1)/\mathrm{d}\lambda \leq 0$ or $\widehat{v}_1(1) > 1$ holds. Recall that the condition $\mathrm{d}\widehat{v}(1)/\mathrm{d}\lambda \leq 0$ matters only when v_1 and v_2 *happen* to become equal to each other at $\lambda = 1$. Because there is no reason to believe that such a measure-0 occurrence has a particular importance in the real world, one would rather dismiss it as a singular case. Thus, we focus on the other condition, $\widehat{v}(1) > 1$, namely,

$$\Upsilon \equiv \phi_1^{\frac{\mu}{\sigma-1}} \left[\frac{\phi_2(\sigma + \mu)}{2} + \frac{\sigma - \mu}{2\phi_1} \right] - \sigma < 0, \tag{20}$$

which is a sufficient condition for the core-periphery pattern to be supported by a stable long-run equilibrium, or a condition for it to be *sustainable* according to a terminology in literature.

The first point to make is that (20) is always satisfied if

$$\sigma < 1 + \mu.^{[6]} \tag{21}$$

When (21) holds, therefore, *the core-periphery pattern becomes sustainable whatever transport prices are.* (21) is known as a *black hole condition* in the literature. When σ is sufficiently high, to the contrary, Υ becomes positive. Indeed, we can verify that Υ approaches $(1 + \phi_1 \phi_2)/2\phi_1 - 1 > 0$ as σ goes to the positive infinity.[7]

Next, to explore the sustainability further, let us draw the locus of $\Upsilon = 0$ in a $\phi_1 - \phi_2$ plane. Because it corresponds to a *sustain point* of freeness of trade in a conventional model with no directional imbalance of transport prices, we call it a *sustain curve*. Fig. 2 depicts such curves for four different values of σ, namely, $\sigma = 1.5$, $\sigma = 1.9$, $\sigma = 3$ and $\sigma = 4$, with μ being fixed at 0.8. Now, (20) indicates that Υ increases with ϕ_2. At the area below each curve, therefore, Υ is negative, that is, the core-periphery pattern is sustainable. At the area above it, in contrast, the pattern is unsustainable.

— Insert Fig. 2 around here. —

The slope of a sustain curve is equal to

$$\left. \frac{\mathrm{d}\phi_2}{\mathrm{d}\phi_1} \right|_{\Upsilon = 0} = \frac{\Delta}{\phi_1^2 (\sigma - 1)(\sigma + \mu)},$$

where $\Delta \equiv (\sigma - \mu)(\sigma - \mu - 1) - \phi_1 \phi_2 \mu (\sigma + \mu)$. When the black hole condition, (21), is satisfied, Δ apparently becomes negative; and therefore, the sustain curve has a downward slope at any $(\phi_1, \phi_2) \in \Phi^2$ (remember that $\Phi \equiv (2^{1-\sigma}, 1)$). In Fig. 2, the curve for $\sigma = 1.5$ represents this case. When the black hole condition is not satisfied, to the contrary, it is not straightforward to determine the sign of Δ. From the figure, however, one would conjecture that the sustain curve is upward sloping for *any relevant pair* of (ϕ_1, ϕ_2), that is, for any $(\phi_1, \phi_2) \in \Phi^2$. The following lemma, whose proof is relegated to the Appendix, confirms that the conjecture is true.

Lemma 3. *When the black hole condition is not satisfied, the sustain curve is upward sloping for any $(\phi_1, \phi_2) \in \Phi^2$.*

We are now ready to discuss the relationship between a directional imbalance of transport prices and the sustainability of the core-periphery pattern. Notice that the size of the imbalance is measured by parameter k since $(\tau_1, \ \tau_2) = (c + k, \ c)$.

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[6]Note that $\Upsilon = \sigma(\Upsilon' \Upsilon'' - 1)$ where $\Upsilon' \equiv \phi_1^{\frac{-(\sigma - \mu - 1)}{\sigma - 1}}$ and $\Upsilon'' \equiv [\phi_1 \phi_2 (\sigma + \mu) + \sigma - \mu]/(2\sigma)$. For one thing, $\Upsilon' < 1$ if (21) holds, because $\phi_1 < 1$. Moreover, $\Upsilon'' < 1$ since $\phi_1 \phi_2 (\sigma + \mu) + \sigma - \mu < (\sigma + \mu) + \sigma - \mu = 2\sigma$ for $\phi_1 < 1$ and $\phi_2 < 1$. Hence, $\Upsilon < 0$.

[7]Because $\phi_2 \geq \phi_1$ and $\phi_1 > 2 - 1/\phi_1$, we have $\phi_2 > 2 - 1/\phi_1$, which is equivalent to $(1 + \phi_1 \phi_2)/2\phi_1 - 1 > 0$.

When σ is sufficiently low that the black hole condition is satisfied, on the one hand, a change in k has no impact on the sustainability. This is because the core-periphery pattern is sustainable no matter what the levels of transport prices are.

When σ is so high that the black hole condition does not hold, on the other hand, parameter k does affect the sustainability. Since it does so only through ϕ_1, we have

$$\left.\frac{\mathrm{d}\Upsilon}{\mathrm{d}k}\right|_{\Upsilon=0} = \frac{\mathrm{d}\phi_1}{\mathrm{d}k}\frac{\partial\Upsilon}{\partial\phi_1} = \frac{t_1^{\sigma-\mu-2}}{2\sigma}\Delta,$$

with all variables being evaluated at $\Upsilon = 0$. Now, because Lemma 3 implies that $\Delta > 0$ on the sustain curve, $\mathrm{d}\Upsilon/\mathrm{d}k|_{\Upsilon=0}$ is positive: Υ decreases as k falls. Consequently, a fall in k makes the core-periphery pattern more likely to be sustainable. Thus, we have established the following proposition.

Proposition 5. *i) When σ is sufficiently low that the black hole condition is satisfied, the core-periphery pattern is sustainable irrespective of the size of directional imbalance.*
ii) When σ is so high that the black hole condition does not hold, less divergent transport prices are associated with higher sustainability of the core-periphery pattern.

To understand *ii)* of the proposition intuitively, let us return to the identity (12) that decomposes the impact of a change in t_1 on $\widehat{v}(1)$, given that t_2 is fixed, into the terms of trade effect, regional income effect and price index effect. Suppose that k falls. Then, t_1 declines but t_2 remains unchanged, which scales down the directional imbalance of transport prices. For one thing, as a result of the decline in t_1, consumers in region 2 would try to substitute imports for domestic products, if they had been consuming domestic products. This import price effect reduces the wage rate of a *potential* entrepreneur in region 2, which raises the relative wage rate, $\widehat{w}$. On the other hand, there is no export price effect. This is because no consumers in region 2 *actually* make the substitution: in fact, no manufacturing production is conducted in that region. Consequently, the terms of trade effect solely consists of the import price effect, and is negative. What is more, as long as the core-periphery pattern is concerned, relative regional income is independent of transport prices: there is no regional income effect. Finally, as t_1 declines, varieties become less expensive in region 2. Because t_1 does not affect the price index in region 1, $\widehat{P}$ rises, and thus a positive price index effect.[8] It turns out that the import price effect dominates the relative price index effect for any $(\phi_1, \phi_2) \in \Phi^2$. As a result, the relative indirect utility increases and the core-periphery pattern becomes more likely to be sustainable as k falls.

Another way to see the relationship between the directional imbalance of transport prices and the sustainability is examining directly the effects of changes in parameters on the

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[8] The terms of trade effect and the price index effect are given by $-(\sigma-1)(1-b)/[1-b+\phi_1\phi_2(1+b)]$ and μ, respectively.

sustainability. Fig. 3 describes the loci of $\Upsilon = 0$ in a $k - c$ plane for four different values of σ with μ again being fixed at 0.8. It is easily verified that $\Upsilon = 0$ when $k = c = 0$ or $\phi_1 = \phi_2 = 0$. Therefore, each locus passes through the origin. If the black hole condition holds, furthermore, we can prove that a locus has a negative slope.[9] This case is represented by the locus for $\sigma = 1.5$. Here, Υ becomes negative when (k, c) lies above the locus and positive when it lies below it. For any admissible pair of (k, c), therefore, $\Upsilon < 0$ and the core-periphery pattern is sustainable. If the black hole condition is not met, instead, the sign of the slope of a curve depends on parameters. For relatively low σ, it has a downward slope for any $k \in [0, 1)$. In the figure, the locus for $\sigma = 1.9$ describes this case. The sign of Υ at a particular point in the plane can be determined in the same manner as in the previous case. As σ rises, the locus rotates counterclockwise around the origin. For a high level of σ, the locus has a shape of a reverse image of character "C". That is to say, it is downward sloping for high values of c and upward sloping for low values of it (see the locus for $\sigma = 4$ in Fig. 3). In this case, Υ is negative "inside" the character "C", and positive "outside" it.

— Insert Fig. 3 around here. —

From the figure, several observations can be obtained. In the first place, it is apparent that for a given value of c, a fall in k makes the core-periphery pattern more likely to be sustainable, which reflects the result obtained earlier (see *ii)* of Proposition 5). In the second place, let us examine the effect of a fall in c when k is given. To begin with, we consider a special case with $k = 0$, where our model is reduced to a conventional model with no directional imbalance of transport prices. In this case, there is a critical point, a 'sustain point', of c: the core-periphery pattern is sustainable if c is lower than it, whereas it is unsustainable if c is higher than it. It is because lower c implies lower transport costs, which enable entrepreneurs in the core to deliver their products to the workers in the periphery at less expense. Now, let us turn our attention to the case with $k > 0$, the case with the directional imbalance. For sufficiently high c, the core-periphery pattern is unsustainable. As c falls, it eventually reaches a critical point corresponding to the sustain point mentioned above; and thereafter, the pattern becomes sustainable. What is happening here is virtually the same as in the case of no directional imbalance. As c falls further, however, it hits another critical point. For c lower than it, the core-periphery pattern again becomes unsustainable. In sum, as k gradually falls, the pattern becomes sustainable, unsustainable, and then again sustainable. Such a non-monotonic behavior is not observed in a conventional model. Indeed, the second critical point approaches 0 as k goes to 0; and therefore, the lower

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[9]Note that $\mathrm{d}c/\mathrm{d}k\big|_{\Upsilon=0} = \Delta(c+1)\big/\big[\phi_1\phi_2(\sigma-1)(\sigma+\mu)(c+k+1) - \Delta(c+1)\big]$. Because $\Delta < 0$ when the black hole condition holds, the derivative is negative.

'unsustainability range' of c disappears at $k = 0$. The reason for this novel result is that as c falls, a *relative size* of the directional imbalance of transport prices expands although its *absolute size*, which is equal to k, remains unchanged. Consequently, when c becomes too low, the negative effect of imbalance on the sustainability grows to the extent that it more than offsets the positive effect of low transport costs brought about by low c. Finally, we have observed that as σ rises, the locus rotates counterclockwise. This implies that the area for which Υ is negative shrinks. In this sense, the core-periphery pattern becomes less likely to be sustainable as σ rises. This result is the same as the one derived in a conventional model with no directional imbalance of transport prices.

4.2 Symmetric pattern

Next, we consider a symmetric pattern ($\lambda = 1/2$ or $\widehat{\lambda} = 1$).

Because $\widehat{\lambda}^L < 1 < \widehat{\lambda}^U$, transport prices are given by $(\tau_1^o, \tau_2^o) = (c + k/2, \ c + k/2)$ at that pattern. Furthermore, the inequality implies that a distribution sufficiently close to the symmetric pattern also satisfies (19). More formally, there exists $\varepsilon > 0$ such that condition $\widehat{\lambda}^L < \widehat{\lambda} < \widehat{\lambda}^U$ is satisfied for any $\widehat{\lambda} \in (1 - \varepsilon, \ 1 + \varepsilon)$. These findings are summarized as follows.

Proposition 6. *When entrepreneurs are distributed equally or nearly equally in two regions, transport prices are given by* $(\tau_1, \ \tau_2) = (c + k/2, \ c + k/2)$.

What is more, the symmetric pattern is supported by a long-run equilibrium because $\widehat{v}(1/2) = 1$. In addition, the equilibrium is stable if $\mathrm{d}\widehat{v}(1/2)/\mathrm{d}\lambda \leq 0$, which is equivalent to $\mathrm{d}\ln\widehat{v}(1/2)/\mathrm{d}\ln\widehat{\lambda} \leq 0$. Notice that $\widehat{\lambda}$ affects $\widehat{v}$ both directly and indirectly through a change in transport prices:

$$
\begin{aligned}
\frac{\mathrm{d}\ln\widehat{v}(1/2)}{\mathrm{d}\ln\widehat{\lambda}} &= \left.\frac{\partial\ln\widehat{v}(1/2)}{\partial\ln\widehat{\lambda}}\right|_{(t_1,t_2)\ \text{const.}} + \frac{\mathrm{d}\ln t_1^o}{\mathrm{d}\ln\widehat{\lambda}}\frac{\partial\ln\widehat{v}(1/2)}{\partial\ln t_1} + \frac{\mathrm{d}\ln t_2^o}{\mathrm{d}\ln\widehat{\lambda}}\frac{\partial\ln\widehat{v}(1/2)}{\partial\ln t_2} \\
&= \left.\frac{\partial\ln\widehat{v}(1/2)}{\partial\ln\widehat{\lambda}}\right|_{(t_1,t_2)\ \text{const.}} + 2\frac{\mathrm{d}\ln t_1^o}{\mathrm{d}\ln\widehat{\lambda}}\frac{\partial\ln\widehat{v}(1/2)}{\partial\ln t_1},
\end{aligned}
\tag{22}
$$

where $t_i^o \equiv \tau_i^o + 1$ ($i = 1, 2$). Here, the last equality follows from the fact that $\mathrm{d}\ln t_1^o/\mathrm{d}\ln\widehat{\lambda} = -\mathrm{d}\ln t_2^o/\mathrm{d}\ln\widehat{\lambda}$ and $\partial\ln\widehat{v}(1/2)/\partial\ln t_1 = -\partial\ln\widehat{v}(1/2)/\partial\ln t_2$. If t_1^o were to be equal to t_2^o for any $\widehat{\lambda}$, then $\mathrm{d}\ln t_1^o/\mathrm{d}\ln\widehat{\lambda}$ would become equal to $\mathrm{d}\ln t_2^o/\mathrm{d}\ln\widehat{\lambda}$; and the second term in the second line of (22) would disappear. This means that *the term represents the effect due to a directional imbalance of transport prices.*

Since $t_1 = t_2$ and $\phi_1 = \phi_2$ at the symmetric equilibrium, their equilibrium values are denoted simply by t and ϕ, respectively, in the rest of this section. With evaluating all the

derivatives at $\widehat{\lambda} = 1$, we obtain from (11) and (18)

$$
\begin{aligned}
\left.\frac{\partial \ln \widehat{v}(1/2)}{\partial \ln \widehat{\lambda}}\right|_{(t_1,t_2) \text{ const.}} &= \Psi \equiv \frac{(1-\phi)\Psi'}{(\sigma-1)(1+\phi)\left[\sigma-\mu+\phi(\sigma+\mu)\right]} \quad \text{and} \\
2\frac{\mathrm{d}\ln t_1^o}{\mathrm{d}\ln \widehat{\lambda}}\frac{\partial \ln \widehat{v}(1/2)}{\partial \ln t_1} &= \Omega \equiv \frac{2t^{-\sigma}\Omega'}{(1+\phi)\left[\sigma-\mu+\phi(\sigma+\mu)\right]},
\end{aligned}
$$

where $\Psi' \equiv \phi(\sigma+\mu)(\sigma+\mu-1) - (\sigma-\mu)(\sigma-\mu-1)$ and $\Omega' \equiv \mu\left[\sigma-\mu+\phi(\sigma+\mu)\right] - 2\sigma(\sigma-1)$.

Here, Ψ represents the direct effect that arises provided that transport prices remain constant. It is what one can find in a conventional model with exogenous and non-divergent transport prices (see Baldwin et al. (2003), among others). Because Ψ' is increasing in ϕ, there is a critical value, $\phi^B \equiv (\sigma-\mu)(\sigma-1-\mu)/(\sigma+\mu)(\sigma-1+\mu)$ such that $\Psi \gtreqless 0$ if $\phi \gtreqless \phi^B$. This is a *break point* in a conventional model: if freeness of trade is higher than ϕ^B, the symmetric equilibrium 'breaks' and becomes unstable. Notice that when the black hole condition given by (21) is satisfied, ϕ^B becomes negative; and therefore, $\Psi > 0$ for any $\phi \in \Phi$: the symmetric pattern becomes always unstable in this case.

In our model, however, there is an additional term, Ω, which represents the indirect effect through a change in transport prices, or more specifically, the effect due to a directional imbalance of transport prices. Notice that Ω' increases with ϕ and becomes negative at $\phi = 1$ as long as the black hole condition does not hold. Hence, in that case, for any admissible value of transport price, $\phi \in \Phi$, we have $\Omega' < 0$ and therefore $\Omega < 0$. The negativity of Ω means that the symmetric equilibrium becomes more likely to be stable if we allow for the endogeneity and the imbalance of transport prices. Formally, we have the following proposition:

Proposition 7. *As long as the black hole condition is not satisfied, the range of ϕ for which the symmetric equilibrium becomes stable is wider when transport prices are endogenously determined and allowed to be imbalanced with respect to directions than when they are not.*

To understand why $\Omega < 0$, recall that the effect of a change in transport prices on the relative level of indirect utility is decomposed into three parts (see (12)). Suppose that $\widehat{\lambda}$ increases a little from 1 so that region 1 becomes slightly bigger than region 2. Then, the transport price in the binding direction, i.e., the direction from region 1 to region 2, rises while that in the slack condition falls. Indeed, a 1 % increase in $\widehat{\lambda}$ results in a $(\sigma-1)^{-1}$ % increase in t_1^o and a $(\sigma-1)^{-1}$ % decrease in t_2^o. Let us focus on the increase in t_1^o. First, the relative wage rate decreases by the terms of trade effect, which contains the export price effect and the import price effect. Second, the regional income effect works in the same direction as the terms of trade effect, because we are now dealing with the case where the two regions are sufficiently alike in size. Third and lastly, the relative price index declines. It

turns out that the sum of the terms of trade effect and the regional income effect dominates the price index effect; and consequently, the relative level of indirect utility declines.[10] This works as a centripetal force and the symmetric equilibrium becomes more likely to be stable. The effect of the decrease in t_2^o is similarly explained.

The question which is not yet answered is how the 'break point' behaves in our model with the additional term of Ω. Since $\Psi + \Omega$ is nonlinear, it is difficult to analytically obtain further consequences: we need to rely on numerical simulations. Now, fixing μ at 0.8, we can draw the value of $\Psi + \Omega$ as a function of ϕ. Diagram (a) of Fig. 4 describes such curves for four different levels of σ. Now, suppose that σ gradually falls. For sufficiently high σ ($\sigma = 3$ and $\sigma = 4$, for example), $\Psi + \Omega$ is negative and the symmetric equilibrium is stable for any $\phi \in \Phi$. As σ rises, the curve rotates counterclockwise or shifts upward. When σ reaches a critical value ($\sigma = 2.221$), its top touches the horizontal axis. Subsequently, the curve cuts the horizontal axis twice. Then, for $\sigma = 1 + \mu$ ($= 1.8$), the curve cuts the horizontal axis at $\phi = 0$ and $\phi = 1$. Thereafter, the black hole condition becomes satisfied. The curve lies above the axis for an entire range of $\phi \in \Phi$.

— Insert Fig. 4 around here. —

One could understand the fact that the '$\Psi + \Omega$ curve' is bell-shaped with the help of Diagrams (b) and (c), which describe Ψ and Ω, respectively, as functions of ϕ for given levels of σ. As long as ϕ is not too high, Ω is negative; and therefore, the $\Psi + \Omega$ curve lies below the 'Ψ curve'. At the same time, Diagram (c) shows that an absolute value of Ω increases with ϕ unless ϕ is too high. Consequently, the $\Psi + \Omega$ curve lies farther away from the Ψ curve for higher values of ϕ.

Moreover, it is a novel outcome that ϕ has a non-monotonic impact on $\Psi + \Omega$ when σ is of medium level. Suppose that σ is greater than $1 + \mu$ but sufficiently close to it, as represented by $\sigma = 1.9$. For such σ, $\Psi + \Omega$ becomes negative at low values and high values of ϕ and positive at intermediate values of it (see Diagram (a) of Fig. 4). Therefore, the symmetric equilibrium becomes stable when ϕ is either low or high and becomes unstable when it is in-between. A conventional model does not exhibit such a non-monotonic property: it prescribes that the symmetric equilibrium become stable when ϕ is lower than a break point and become unstable when ϕ exceeds it. To give an intuitive explanation for this non-monotonicity, we again consider a situation where region 1 happens to become slightly bigger than region 2, or, $\widehat{\lambda}$ infinitesimally increases from 1. As to the result that the symmetric equilibrium is stable for sufficiently *low* freeness of trade, nothing new enters the picture:

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[10]The terms of trade effect, the regional income effect, and the price index effect are given by $-\phi(\sigma - 1)/(1 + \phi)$, $-2\mu\phi(\sigma - 1)(1 - \phi)/(1 + \phi)^2 [\sigma - \mu + \phi(\sigma + \mu)]$, and $\mu\phi/(1 + \phi)$, respectively.

relocating to the smaller region is profitable for entrepreneurs, because by doing so, they can acquire the domestic demand in the smaller region, whose amount is considerably large due to a high delivered price of imports. To the contrary, the result that the equilibrium is stable for sufficiently *high* freeness of trade is a novel feature of our model. If there were no directional imbalance of transport prices, entrepreneurs would have an incentive to relocate to the larger region (Ψ is positive); but only a weak incentive (Ψ is small) for sufficiently high freeness of trade. (Indeed, when manufacturing products are traded freely ($\phi = 1$), this incentive disappears ($\Psi = 0$).) Now, the increase in $\widehat{\lambda}$ brings about a directional imbalance by raising a transport price in a binding direction and reducing that in a slack direction. As has been explained, this weakens the incentive of entrepreneurs to relocate to the larger region through the terms of trade effect, regional income effect and the price index effect. For high freeness of trade, the intrinsic incentive to relocate to the larger region is so weak that it is overturned by the effect of the directional imbalance. That is, they actually have an incentive to relocate to the *smaller* region. Therefore, the symmetric equilibrium becomes stable when freeness of trade is very high.

Finally, we can draw the locus of the break point as a function of σ. In Fig. 5, which is drawn for $\mu = 0.8$ as before, the solid line represents the break point in a conventional model with no endogenous determination of transport prices nor the possibility of their directional imbalance, that is, the locus of $\Psi = 0$. The symmetric equilibrium is stable when a pair of parameters lies below the line, that is to say, when freeness of trade is lower than the break point value. Instead, it is unstable for a pair above the line, namely, a pair with freeness of trade exceeding the break point value. Furthermore, the line is upward sloping: the higher σ is, the more likely the symmetric equilibrium is to be stable. On the other hand, the dotted line represents the break point in our model with the endogenous determination of transport prices and the possibility of their directional imbalance, namely, the locus of $\Psi + \Omega = 0$. The symmetric equilibrium is stable for a pair of parameters at the right of the line, and unstable for that at the left of it. Notice that both lines cut the horizontal axis at $\sigma = 1 + \mu$, the critical value of σ for the black hole condition. Furthermore, the maximum value of σ on the dotted line ($\sigma = 2.221$) is the value for which the top of the $\Psi + \Omega$ curve touches the horizontal axis in Diagram (a) of Fig. 4.

— Insert Fig. 5 around here. —

4.3 Overall impacts of changes in parameters

One of the greatest achievements of new economic geography is that it has revealed the impacts of changes in transport costs upon economic geography. As is asserted in the

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literature, transport costs have been declining significantly over the centuries. Researchers in the field have proved through a rigorous analysis that this is expected to strengthen a centripetal force and reduce a centrifugal force, and, as a result, tend to make economic activities concentrate in a fewer regions, which has indeed happened in the real world. In this paper, however, transport costs are not exogenous but determined through each carrier's decision makings. What is exogenous is the cost parameters, k and c. In this section, thus, we compete the analysis by examining in a spirit of the literature of new economic geography what happens to economic geography if those parameters gradually decline over time.

In the first place, let us consider a decline in k for given c. For sufficiently high σ, the core-periphery patterns are unsustainable while the symmetric equilibrium is stable for any $k \in [0, 1)$. Instead, for sufficiently low σ that the black hole condition is satisfied, the core-periphery patterns are sustainable while the symmetric equilibrium is unstable for any $k \in [0, 1)$. Thus, an interesting case occurs for intermediate levels of σ. Fig. 6 shows so-called 'pitchfork diagrams' for two values of σ when μ and c are fixed at 0.8 and 0.2, respectively. The horizontal axis measures k while the vertical axis λ.

— Insert Fig. 6 around here. —

For $\sigma = 3$ (Diagram (a)), the core-periphery patterns are sustainable only for low values of k ($k < 0.553$). This is because a decline in k results in a fall in the transport price in a binding direction, which enhances the relative indirect utility in the core through a positive and dominating terms of trade effect and a negative but dominated price index effect. A 'sustainability range' of k, namely, the range of k for which the core-periphery patterns become sustainable, furthermore, appears at a lowermost part of k for σ lower than a critical value (5.048), and expands as σ falls. On the other hand, the symmetric equilibrium is stable for any $k \in [0, 1)$. Therefore, when k is high, it is expected that economic activities are dispersed between two regions. As k declines, the dispersed pattern probably continues but there emerges a possibility that economic geography switches to the agglomeration by accident.

For $\sigma = 1.9$ (Diagram (b)), the core-periphery patterns are sustainable for any $k \in [0, 1)$. The symmetric equilibrium is, on the other hand, stable only for low values of k ($k \leq 0.466$). We have explained this by comparing levels of Ψ and Ω as follows (see Fig. 4). When k is low, freeness of trade is high at the symmetric equilibrium. For high freeness of trade, entrepreneurs would have a small incentive to relocate to the larger region if there were no directional imbalance of transport prices. The directional imbalance, however, reduces the incentive. It turns out that the latter effect of the imbalance more than offset the former so that they actually have an incentive to relocate to the *smaller* region. In addition, a

'stability range' of k, that is, the range of k for which the symmetric equilibrium becomes stable, contracts as σ decreases and disappears for σ lower than a critical value (1.837). Finally, one would conceive a following scenario for this configuration of sustainability and stability. When k is high, manufacturing activities are concentrated in one region. As k becomes sufficiently low, however, the possibility of dispersion arises in addition to the agglomeration.

Furthermore, notice that in each diagram of Fig. 6, two dotted arcs are drawn. They represent equilibrium distributions which are interior but not symmetric. The upper arc represents the distribution supported by an equilibrium with $(\tau_1, \tau_2) = (c + k, c)$. We can verify that it actually lies above the line depicting $\widehat{\lambda}^U$, which is not shown in the diagrams (see (15)). Similarly, the lower arc, which lies below the line depicting $\widehat{\lambda}^L$, represents the distribution supported by an equilibrium with $(\tau_1, \tau_2) = (c, c + k)$ (see (15)). It follows from numerical computation that these equilibria with an asymmetric interior distribution are unstable. Thus, we can disregard them.[11]

Lastly, let us take a look at a change in c, fixing the levels of μ and k at 0.8 and 0.3, respectively. First, when σ is sufficiently high or sufficiently low, the result is similar to the one obtained for a change in k. Thus, let us focus on intermediate levels of σ. For one thing, Diagram (a) of Fig. 7 depicts a pitchfork diagram for $\sigma = 3$. The symmetric equilibrium is stable for any $k \in [0, 1)$. The sustainability range of c is, on the one hand, an interval, $(0.019, 0.797)$, which we can also observe in Fig. 3. As has been explained, the core-periphery patterns are not sustainable for too low c, because the *relative* size of the directional imbalance of transport prices, which works against the sustainability, becomes too large. The sustainability range emerges for σ lower than a critical value (3.332) and expands toward both sides as σ declines. What is more, when $\sigma = 1.9$ (see Diagram (b)), the core-periphery patterns are sustainable for any $k \in [0, 1)$. The symmetric equilibrium is stable for low values of c ($c \leq 0.283$) and unstable for high values of it. The stability range of c contracts as σ decreases and disappears for σ smaller than a critical value (1.825). Finally, two dotted arcs in each diagram again represent unstable equilibrium distributions, associated with the transport prices given by $(\tau_1, \tau_2) = (c + k, c)$ and $(\tau_1, \tau_2) = (c, c + k)$, respectively.

— Insert Fig. 7 around here. —

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[11] As long as $\sigma \neq 1 + \mu$, in addition, there is no asymmetric interior distribution supported by an equilibrium with $(\tau_1, \tau_2) = (\tau_1^o, \tau_2^o)$. This is explained as follows. Because $\phi_2 = \phi_1 \widehat{\lambda}^2$ for these prices, we have $\widehat{v}(\lambda) = \widehat{\lambda}^{-\frac{\sigma - \mu - 1}{\sigma - 1}}$. Unless $\sigma = 1 + \mu$, therefore, $\widehat{v}(\cdot)$ is an increasing or a decreasing function of λ. Consequently, $\widehat{v}(\lambda) = 1$ has a unique solution, $\lambda = 1/2$, as long as $\sigma \neq 1 + \mu$.

5   Concluding Remarks

In this paper, we have explained from the behaviors of individual transport firms how and why transport prices become imbalanced with respect to directions of shipments, and examined how it affects economic geography in a general equilibrium framework of new economic geography. It is found that the equilibrium transport price of the shipments in a particular direction is a nondecreasing step-shaped function of a relative size of the embarkation region, which implies that the transport price in a binding direction is higher than that in the slack direction. Furthermore, it is shown that volumes of trade are equal between two directions when regions are sufficiently alike in size, whereas they are different when one region is much bigger than the other. In addition, the directional imbalance of transport prices makes the symmetric pattern more likely to be stable and the core-periphery patterns less likely to be sustainable. To put it in other words, the imbalance acts as a centripetal force.

References

[1] Baldwin, R., R. Forslid, P. Martin, G. Ottaviano and F. R.-Nicoud. (2003) *Economic Geography and Public Policy*, Princeton University Press, Princeton.

[2] Behrens, K., C. Gainé and J.-F. Thisse. (2006) "Is the regulation of the transport sector always detrimental to consumers?" mimeo.

[3] Blonigen, B. A. and W. W. Wilson. (2008) "Port efficiency and trade flows," *Review of International Economics*, 16, 21-36.

[4] Bougheas, S., P. O. Demetriades and E. L. W. Morgenroth. (1999) "Infrastructure, transport costs and trade," *Journal of International Economics*, 47, 169-189.

[5] Boyer, K.. (1997) *Principles of Transportation Economics*, Addison-Wesley, Reading.

[6] Cariou, P. and F.-C. Wolff. (2006) "An analysis of Bunker Adjustment Factors and freight rates in the Europe/Far East market (200-2004)," *Maritime Economics and Logistics*, 8, 187-201.

[7] Forslid R. and G. I. P. Ottaviano. (2003) "An analytically solvable core-periphery model," *Journal of Economic Geography*, 3, 229-40.

[8] Fujita, M., P. Krugman and A. J. Venables. (1999) *The Spatial Economy: Cities, Regions, and International Trade*, The MIT Press, Cambridge.

[9] Fujita, M. and J.-F. Thisse. (2000) "The formation of economic agglomerations: old problems and new perspectives," in J.-M. Huriot and J.-F. Thisse (eds.), *Economics of Cities*, Cambridge University Press, Cambridge, pp. 3-73.

[10] Fujita, M. and J.-F. Thisse. (2002) *Economics of Agglomeration*, Cambridge University Press, Cambridge.

[11] Hummels, D. (2007) "Transport costs and international trade in the second era of globalization," *Journal of Economic Perspectives*, 21, 131-154.

[12] Jonkeren, O., E. Demirel, J. van Ommeren, and P. Rietveld. (2008) "Endogenous transport prices and trade imbalances," *Tinbergen Institute Discussion Paper*, 2008-088/3.

[13] Mori T. and K. Nishikimi. (2002) "Economies of transport density and industrial agglomeration," *Regional Science and Urban Economics*, 32, 167-200.

[14] Mun S. and S. Nakagawa. (2008) "Cross-border transport infrastructure and aid policies," *Annals of Regional Science*, 42, 465-486.

[15] Sjostrom, W. (2004) "Ocean shipping cartels: A survey," *Review of Network Economics*, 3, 107-134.

[16] Stopford, M. (2009) *Maritime Economics*, Routledge, Oxon.

[17] Takahashi, T. (2006) "Economic geography and endogenous determination of transportation technology," *Journal of Urban Economics*, 60, 498-518.

[18] UNCTAD. (2008) *Review of Maritime Transport 2008*, United Nations Publications, New York.

[19] Wilmsmeier, G., J. Hoffmann, and R. J. Sanchez. (2006) "The impact of port characteristics on international maritime transport costs," in K. Cullinane and W. K. Talley (eds.), *Port Economics*, Elsevier, Amsterdam, 16, 117-140.

Appendix

Proof of Lemma 1.

In this proof, we examine whether carrier h can obtain a positive profit by deviating from the Nash strategy, given that every other carrier sticks to it. Suppose that $z_i^h \geq z_j^h$ $(i, j = 1, 2; i \neq j)$. Then, the profit of the deviator is written as

$$\sum_{l=1,2} \left(\tau_l^h - c \right) z_l^h - k z_i^h = - \left[\left(\tau_j^* - c \right) \left(z_i^h - z_j^h \right) + \sum_{l=1,2} \left(\tau_l^* - \tau_l^h \right) z_l^h \right],$$

where the equality follows from the fact that $\tau_1^* + \tau_2^* - 2c - k = 0$. Because $\tau_j^* - c \geq 0$ (see (15)), $\left(\tau_j^* - c \right) \left(z_i^h - z_j^h \right) \geq 0$. Furthermore, if the deviator charges a price higher than τ_l^*, it will get $z_l^h = 0$ and, therefore, $\left(\tau_l^* - \tau_l^h \right) z_l^h = 0$. On the other hand, if it undercuts τ_l^*, it may receive a positive demand and $\left(\tau_l^* - \tau_l^h \right) z_l^h \geq 0$. Therefore, $\sum_{l=1,2} \left(\tau_l^* - \tau_l^h \right) z_l^h \geq 0$. Hence, the deviating carrier can never earn a positive profit.

Proof of Lemma 2.

The profit of a deviator, carrier h, is now represented by

$$\sum_{l=1,2} \left(\tau_l^h - c \right) z_l^h - k z_i^h = - \left[\left(\tau_j^o - c \right) \left(z_i^h - z_j^h \right) + \sum_{l=1,2} \left(\tau_l^o - \tau_l^h \right) z_l^h \right]$$

with $z_i^h \geq z_j^h$ $(i = 1, 2; j \neq i)$, where the equality follows from (17). By the same reason explained in the proof of Lemma 1, $\sum_{l=1,2} \left(\tau_l^o - \tau_l^h \right) z_l^h \geq 0$. First, let us prove the sufficient condition. If $\tau_1^o - c \geq 0$ and $\tau_2^o - c \geq 0$, then $\tau_j^o - c \geq 0$. Therefore, $\left(\tau_j^o - c \right) \left(z_i^h - z_j^h \right) \geq 0$. Hence, the deviating carrier cannot earn a positive profit. Second, in order to prove the necessary condition, suppose that $\tau_2^o - c < 0$. Then, carrier h can choose z_1^h and z_2^h arbitrarily so that $z_1^h > z_2^h$ while keeping the prices at $\left(\tau_1^h, \tau_2^h \right) = \left(\tau_1^o, \tau_2^o \right)$. By doing so, it earns a positive profit because $\left(\tau_2^o - c \right) \left(z_1^h - z_2^h \right) + \sum_{l=1,2} \left(\tau_l^o - \tau_l^o \right) z_l^h < 0$. This is a contradiction. Hence, it must be the case that $\tau_2^o - c \geq 0$ at the equilibrium. We can similarly prove that the condition $\tau_1^o - c \geq 0$ is also necessary.

Proof of Lemma 3.

Because $\Upsilon = 0$ at $(\phi_1, \phi_2) = (1, 1)$, the sustain curve passes through $(1, 1)$. Note that at that point, $\Delta = (\sigma - \mu)(\sigma - \mu - 1) - \mu(\sigma + \mu) \gtreqqless 0$ if $\sigma \gtreqqless \sigma^0 \equiv (1 + 3\mu + \sqrt{1 + 2\mu + 9\mu^2})/2$. Three cases are distinguished. First, if $\sigma > \sigma^0$, then $\Delta > 0$ for any $(\phi_1, \phi_2) \in \Phi^2$ because $\phi_1 < 1$ and $\phi_2 < 1$. (The sustain curve for $\sigma = 4$ in Fig. 2 represents this case since $\sigma^0 = 3.146$ for $\mu = 0.8$.) Second, suppose that $\sigma < \sigma^0$. Then, at $(1, 1)$, the sustain curve is downward sloping. Note that $\mathrm{d}^2 \phi_2 / \mathrm{d}\phi_1^2 < 0$ when evaluated at $\Delta = 0$. That is, the sustain curve is concave at its stationary points. This, together with the continuity of $\mathrm{d}\phi_2 / \mathrm{d}\phi_1 |_{\Upsilon=0}$, implies that it has at most one stationary point. Consequently, there are two possibilities. One is the possibility that the curve is downward sloping at any $\phi_1 \in \Phi$. (The curve for $\sigma = 1.9$ in Fig. 2 represents this case.) In this case, it passes through $\phi_2 > 1$, that is, $\phi_2 \notin \Phi$, for any $\phi_1 \in \Phi$. Hence, we do not have to consider this case. The other is the possibility that the curve is upward sloping for any $\phi_1 \in \left(2^{1-\sigma}, \overline{\phi}_1 \right)$ and downward sloping for $\phi_1 \in \left(\overline{\phi}_1, 1 \right)$ where $\overline{\phi}_1 \in \Phi$ gives a stationary point. (The curve for $\sigma = 3$ in Fig. 2 represents this case.) By construction, the curve takes $\phi_2 > 1$ at interval $\phi_1 \in [\overline{\phi}_1, 1)$. Therefore, if the curve

passes through $(\phi_1, \phi_2) \in \Phi^2$, we must have $\phi_1 < \overline{\phi}_1$. Hence, the curve is upward sloping at such a point. Third and finally, suppose that $\sigma = \sigma^0$. In this case, the sustain curve becomes stationary at $(1, 1)$. Because it is concave at that point, the curve is upward sloping for any $\phi_1 \in \Phi$.
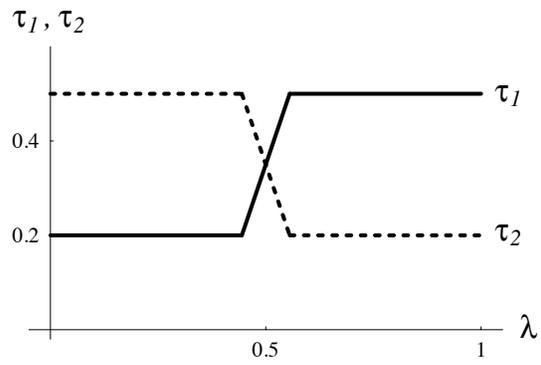
(a) Transport prices.
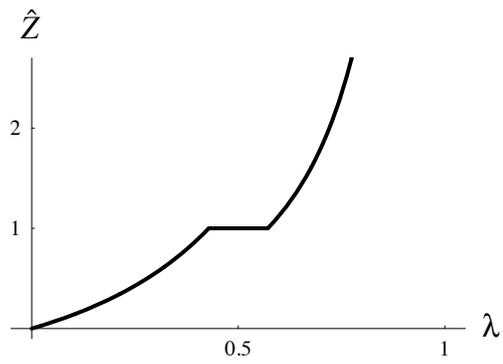
(b) Relative volume of trade.

Fig. 1. Effect of entrepreneurs' distribution on equilibrium variables when $\sigma = 3$, $\mu = 0.8$, $c = 0.2$ and $k = 0.3$.
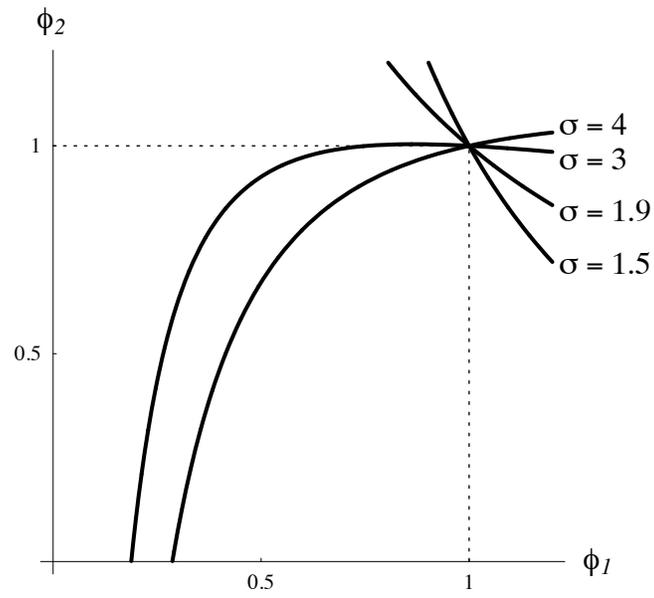
Fig. 2. Sustain curves when $\mu = 0.8$.
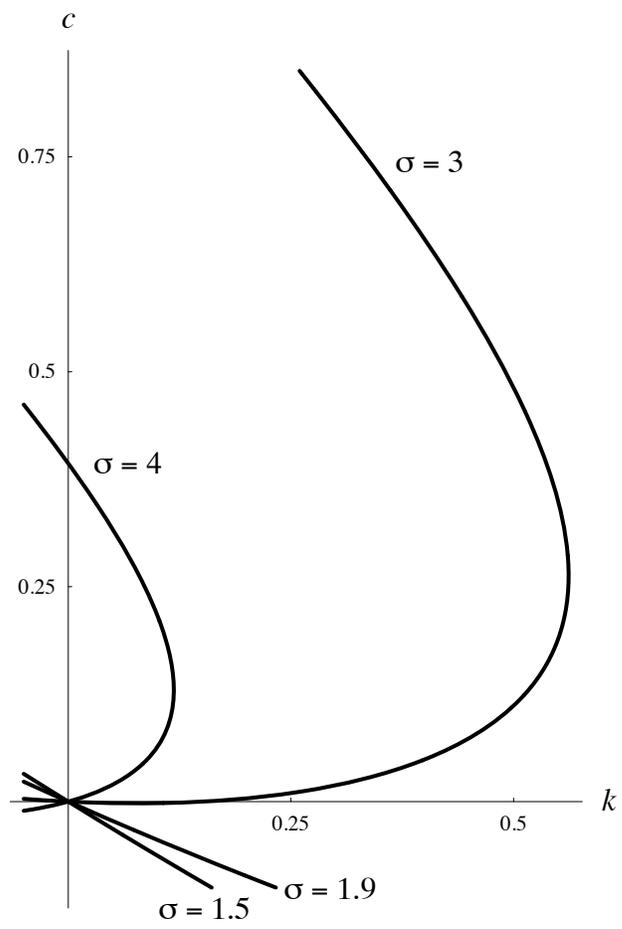
Fig. 3. Loci of $\Upsilon = 0$ when $\mu = 0.8$.

(a) $\Psi + \Omega$.

(b) Ψ.

(c) Ω.
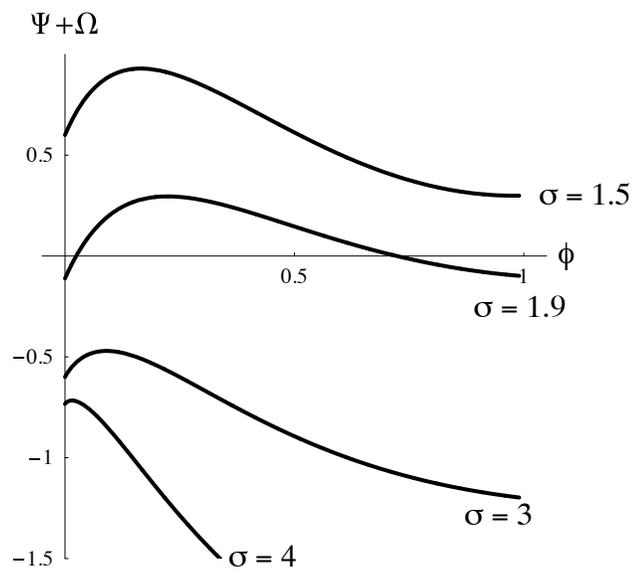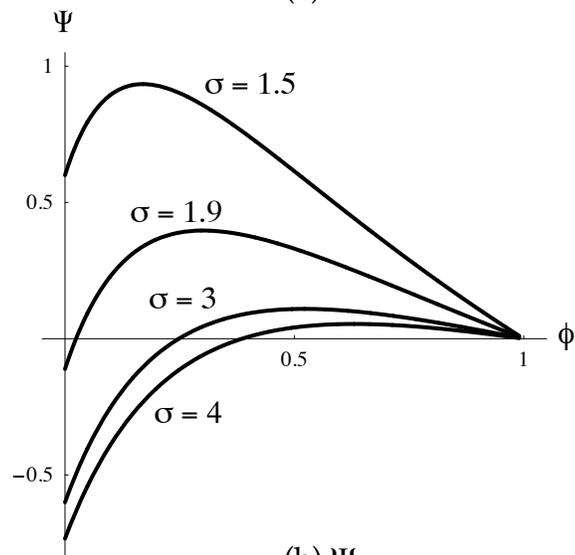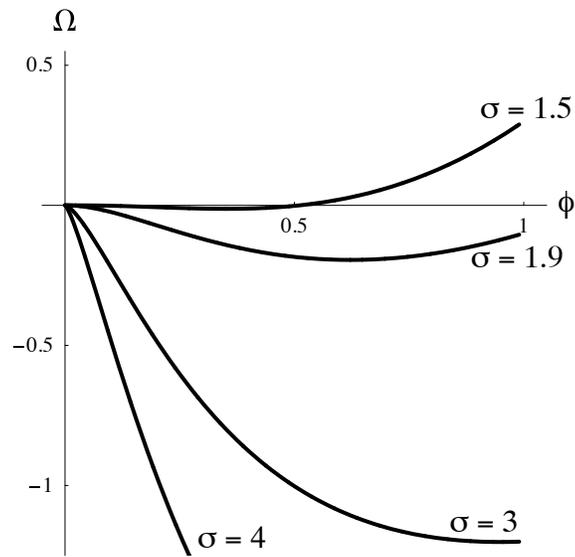
Fig. 4. $\Psi + \Omega$, Ψ and Ω as functions of ϕ when $\mu = 0.8$.
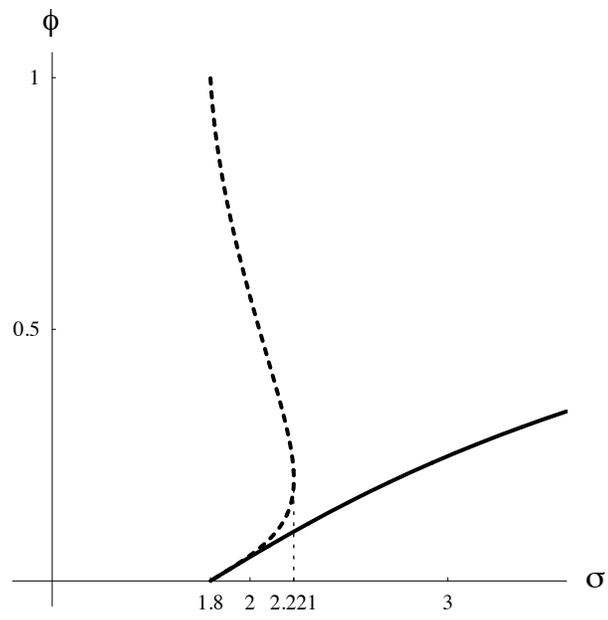
Fig. 5. The loci of break point when μ = 0.8.

(a) σ = 3.
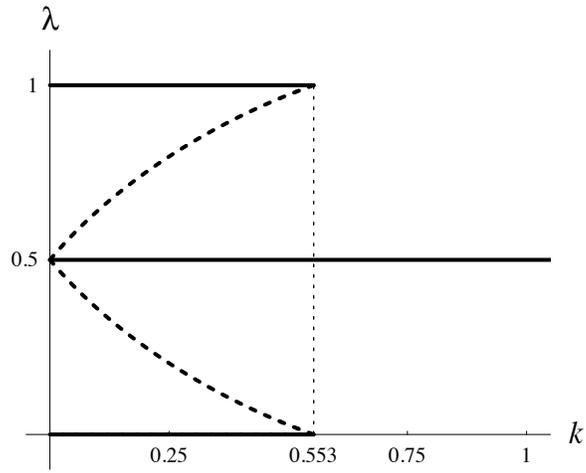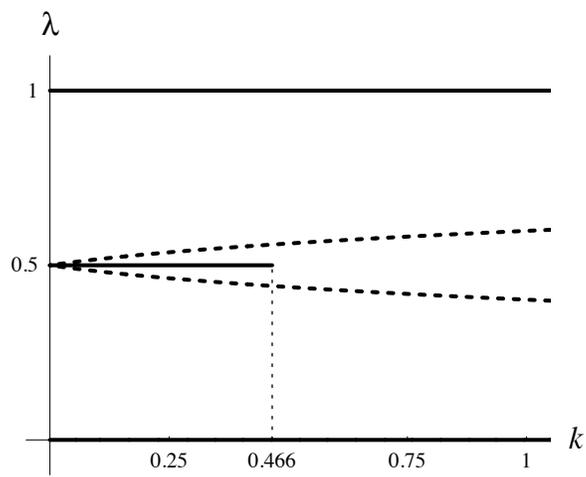

(b) σ = 1.9.

Fig. 6. Effect of k on economic geography
when $\mu = 0.8$ and $c = 0.2$.

(a) σ = 3.
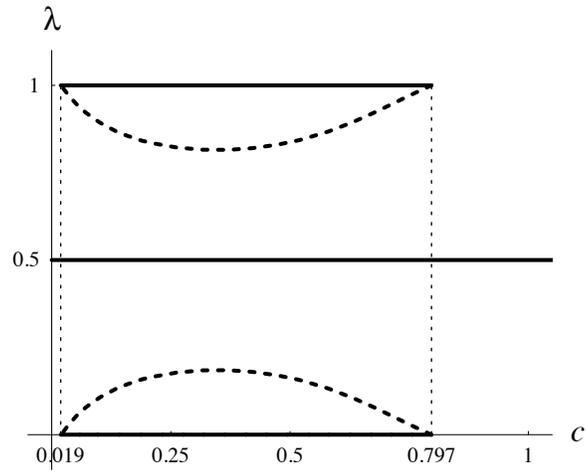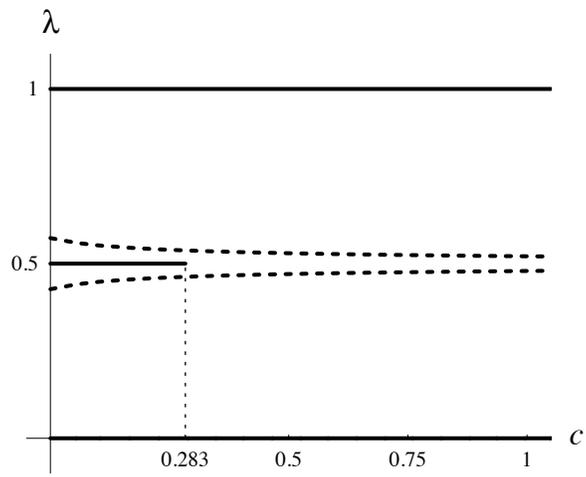
(b) σ = 1.9.

Fig. 7. Effect of *c* on economic geography
when μ = 0.8 and *k* = 0.3.