Introduction to Global Navigation Satellite System (GNSS) Satellite Orbits

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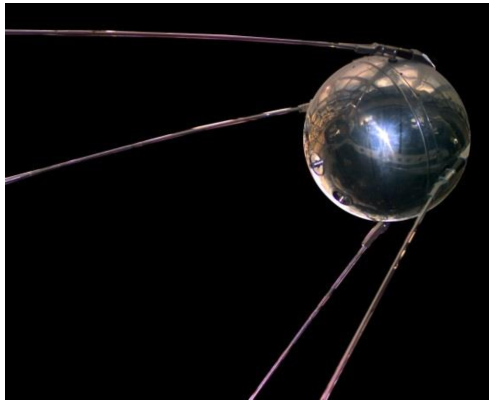
Orbital Mechanics

- **Orbital mechanics** or astrodynamics is the application of celestial mechanics to the practical problem concerning the motion of spacecraft.
  - A core discipline within space mission design, control, and operation.
- **Celestial mechanics** treats the orbital dynamics of natural astronomical bodies such as star systems, planets, and moons.

Sputnik-1
The first artificial Earth satellite launched by the Soviet Union in 1957.



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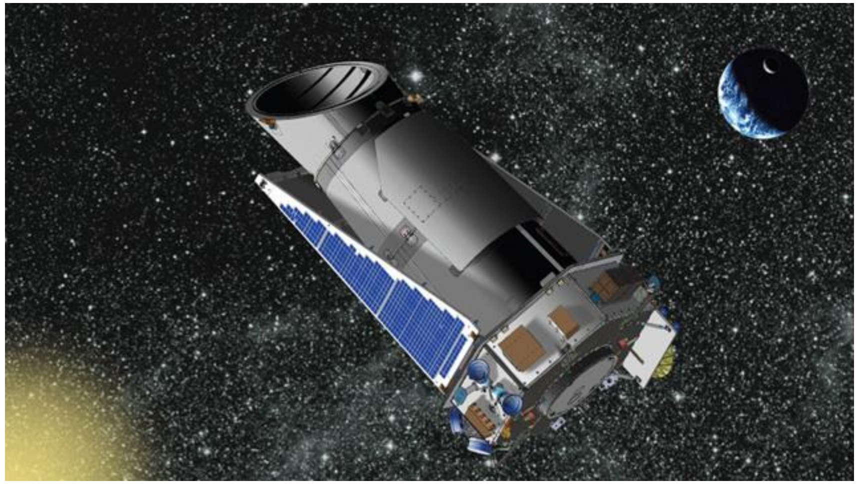
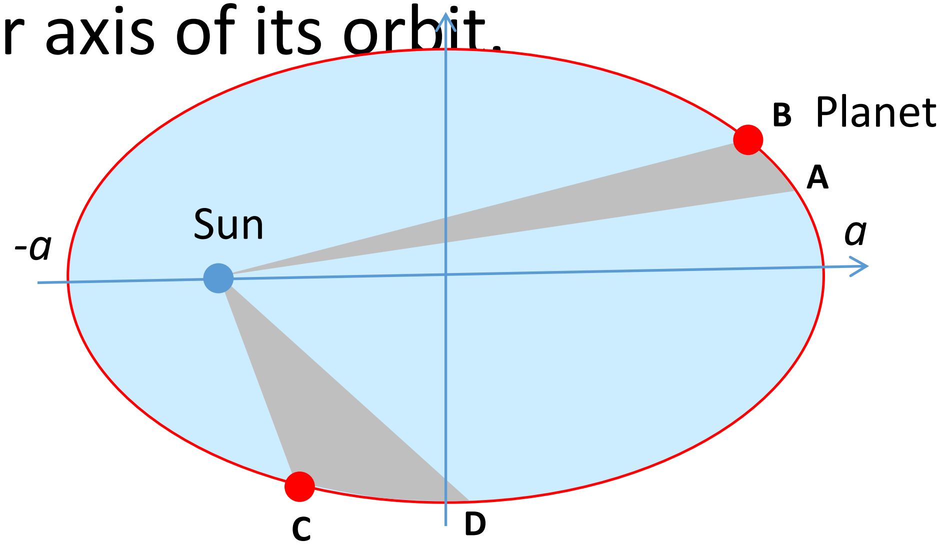
History

- There is little distinction between orbital and celestial mechanics. The fundamental techniques are the same.
- **Johannes Kepler** was the first to successfully model planetary orbits to a high degree of accuracy, publishing his laws of planetary motion in 1605.



Kepler's Laws of Planet Motion

- The orbit of every planet is an **ellipse** with the Sun at one of the two foci (plural of focus).
- A line joining a planet and the sun sweeps out equal area during equal intervals of time.
- The square of the orbital period of planet is proportional to the cube of the semi-major axis of its orbit.



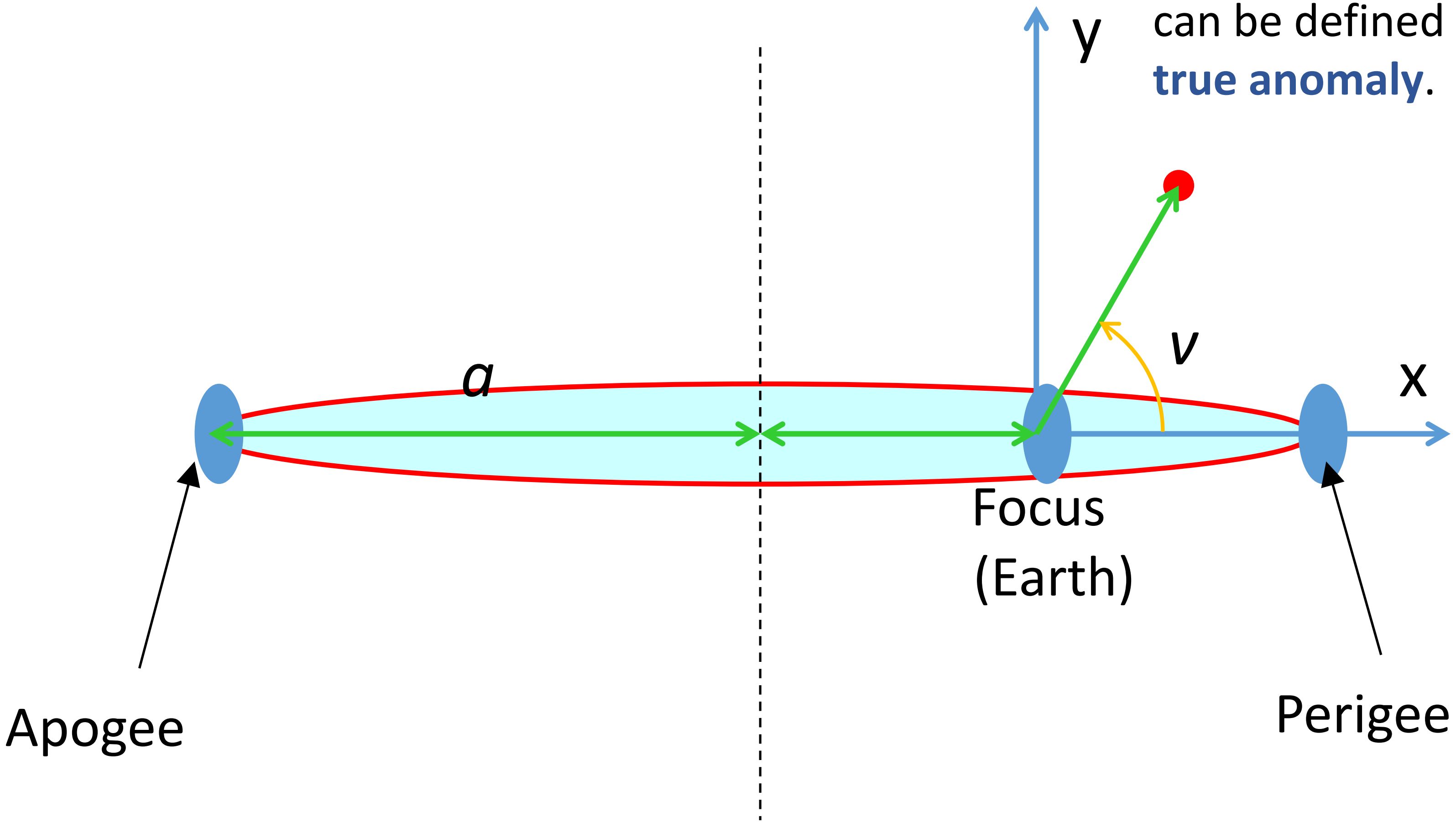
Kepler Orbit

- Kepler orbit can be uniquely defined by six parameters know as **Keplerian elements**.
  - **Semi-major axis** (*a*)
  - **Eccentricity** (*e*)
  - **Inclination** (*i*)
  - **Right ascension of the ascending node** (RAAN) (Ω)
  - **Argument of perigee** (ω)
  - **True anomaly** (ν: Greek letters *nu*)

Orbital Plane

The shape of an elliptic orbit can be defined by the **semi-major axis** and **eccentricity**.

The satellite position in the orbital plane can be defined by **true anomaly**.

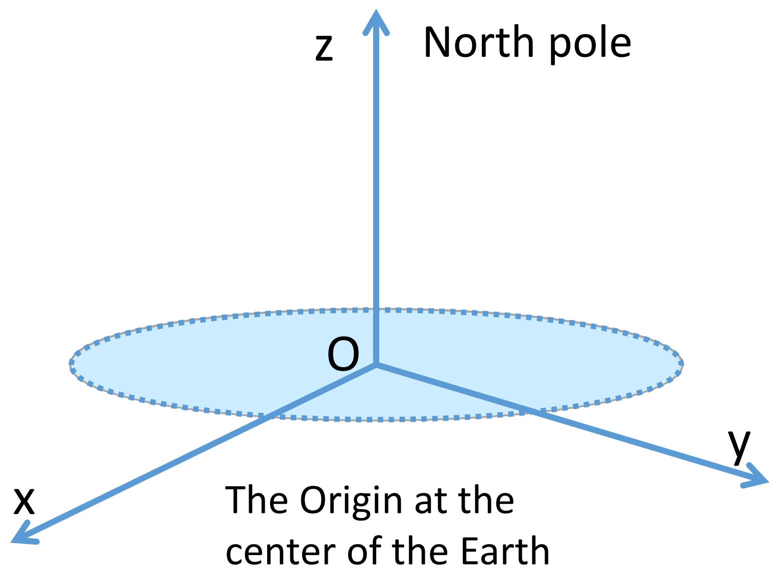


Equatorial Coordinate System

- The most common coordinate frame for describing satellite orbits is the geocentric equatorial coordinate system, which is also called an Earth-Centered Inertial (**ECI**) coordinate system.

Vernal equinox

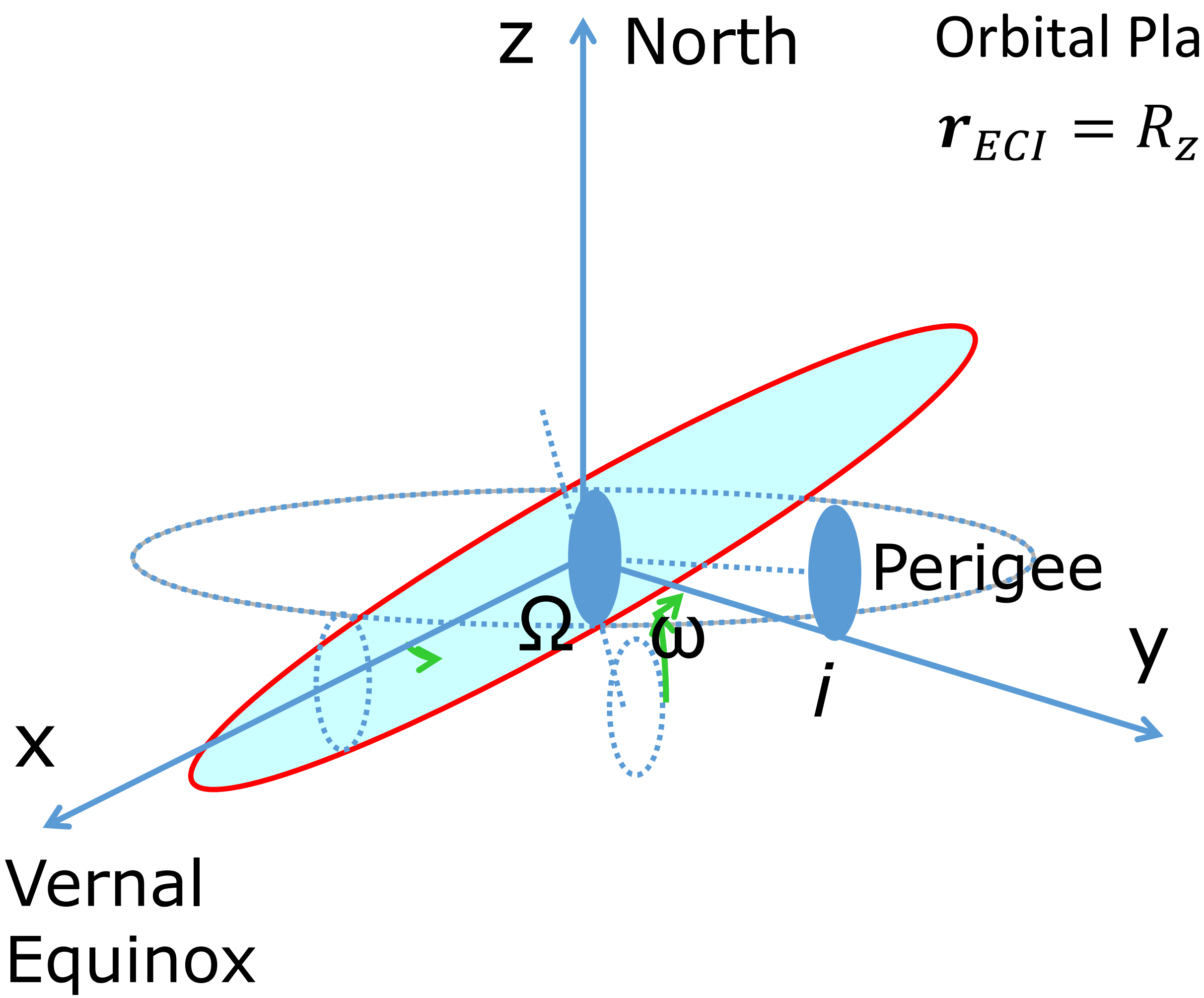
The direction of the Sun as seen from Earth at the beginning of spring time.



Equatorial plane

The fundamental plane is consisting of the projection of the Earth's equator

Orientation of the Orbital Plane



Orbital Plane to ECI

$$\boldsymbol{r}_{ECI} = R_z(-\Omega)R_x(-i)R_z(-\omega)\boldsymbol{r}$$

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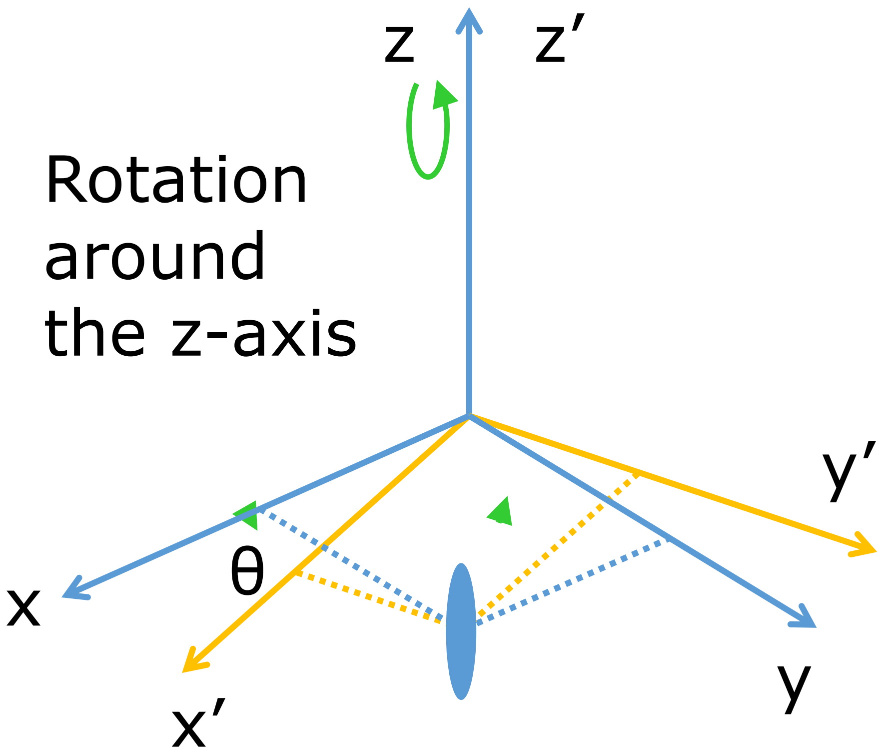
Rotation Matrices



$$R_z(\theta)=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Around the x-axis

$$R_x(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix}$$

Around the y-axis

$$R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix}$$

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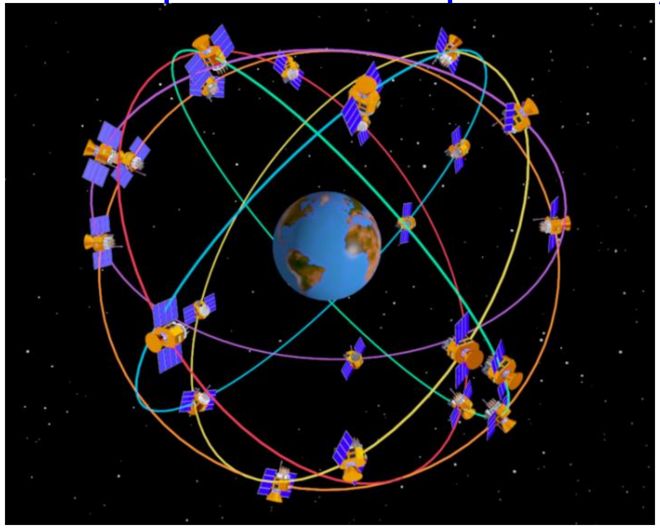
Typical GPS Orbit

- 26,560 km semi-major axis (20,200 km altitude)
  - The orbital period is approximately 12 hours
- Less than 0.01 eccentricity (near circular)
- 55 degree inclination
- 6 orbital planes with at least 4 satellites in each plane
  - The ascending nodes of the orbital planes are separated by 60 degree



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	GPS YUMA ALMANAC FILE
ID	PRN ID of SV
Health	000 = usable
Eccentricity	This shows the amount of the orbit deviation from circular (orbit). It is the distance between the foci divided by the length of the semi-major axis (our orbits are very circular).
Time of applicability	The number of seconds in the orbit when the almanac was generated. Kind of a time tag.
Orbital Inclination	The angle to which the SV orbit meets the equator (GPS is at approximately 55 degrees). Roughly, the SV's orbit will not rise above approximately 55 degrees latitude. The number is part of an equation: # = π/180 = the true inclination.
Rate of Right Ascension	Rate of change in the measurement of the angle of right ascension as defined in the Right Ascension mnemonic.
SQRT(A) Square Root of Semi-Major Axis	This is defined as the measurement from the center of the orbit to either the point of apogee or the point of perigee.
Right Ascension at Time of Almanac (TOA)	Right Ascension is an angular measurement from the vernal equinox ($(OMEGA)_0$).
Argument of Perigee	An angular measurement along the orbital path measured from the ascending node to the point of perigee, measured in the direction of the SV's motion.
Mean Anomaly	Angle (arc) traveled past the longitude of ascending node (value = 0±180 degrees). If the value exceeds 180 degrees, subtract 360 degrees to find the mean anomaly. When the SV has passed perigee and heading towards apogee, the mean anomaly is positive. After the point of apogee, the mean anomaly value will be negative to the point of perigee.
Af0	SV clock bias in seconds.
Af1	SV clock drift in seconds per seconds.
Af2	GPS week (0000–1023), every 7 days since 1999 August 22.
GPS Week	

Example of Yuma Almanac File for GPS

- ******** Week 887 almanac for PRN-01 ********
- ID : 01
- Health : 000
- Eccentricity : 0.5854606628E-002
- Time of Applicability(s) : 589824.0000
- Orbital Inclination(rad) : 0.9652777840
- Rate of Right Ascen(r/s) : -0.7714607059E-008
- SQRT(A) (m 1/2) : 5153.593750
- Right Ascen at Week(rad) : 0.2492756606E+001
- Argument of Perigee(rad) : 0.531310874
- Mean Anom(rad) : 0.3110215331E+001
- Af0(s) : 0.3147125244E-004
- Af1(s/s) : 0.0000000000E+000
- Week : 887

http://qz-vision.jaxa.jp/USE/en/almanac https://celestrak.com/GPS/almanac/Yuma/definition.asp

Perturbation Forces

- Satellite orbit will be an ellipse only if treating each of satellite and Earth as a point mass.
- In reality, Earth's gravitational field is not a point mass.
- Main force acting on GNSS satellites is Earth's central gravitational force, but there are many other significant perturbations.
  - Non sphericity of the Earth's gravitational potential
  - Third body effect
    - Direct attraction of Moon and Sun
  - Solar radiation pressure
    - Impact on the satellite surfaces of photons emitted by the Sun

Accelerations Acting on GNSS Satellites

Term	Acceleration [m/s^2]
Earth's central gravity	0.56
Flatness of the Earth (J2)	5×10^{-5}
Other gravity	3×10^{-7}
Moon and Sun	5×10^{-6}
Solar Radiation Pressure	10^{-7}

Effects of SRP on GNSS satellite position: 5~10 m

Satellite orbit in Navigation Message

- **Broadcast ephemeris**
  - Kepler orbit parameters and satellite clock corrections
  - 9 orbit perturbation corrections parameters
  - 2 m satellite position accuracy for 2 hours
  - Each GNSS satellite broadcasts only its own ephemeris data
- **Almanac**
  - Kepler orbit parameters and satellite clock corrections
  - Less accurate but valid for up to several months
  - Each GNSS satellite broadcasts almanac data for all satellites in the constellation

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