CSIS Discussion Paper No. 60

Benefit Estimation of Transport Projects

-Three Basic Models and Their Implications-

Yukihiro Kidokoro*

Center for Spatial Information Science, University of Tokyo

Address: Faculty of Economics, University of Tokyo,

7-3-1, Hongo, Bunkyo, Tokyo, 113-0033, Japan

Tel&Fax: +81-3-5841-5641

E-mail: kidokoro@csis.u-tokyo.ac.jp

Abstract: In this paper, focusing on the forms of the utility functions, we explain the three basic models of benefit estimation for transport projects: the Mohring model, the Wardrop model, and the Logit model. The main contributions of this paper are a clarification of the relationship between the three models and a demonstration of their merits and demerits. We find that the Mohring model incorporates the Wardrop and Logit models as special cases and that the Logit model degenerates to the Wardrop model in a limiting case. Although one can rely on the Mohring model whatever method is used to estimate transport demand, the Wardrop and Logit models are practically useful in the context of new transport routes. Theoretical results are derived and illustrated by an example.

Keywords: transport, investment, benefit, appraisal, Logit model

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* I thank Tatsuo Hatta, Yoshitsugu Kanemoto, Kenneth A. Small, and Takatoshi Tabuchi for helpful suggestions and discussions. This paper was written while I was visiting the Department of Economics, at the University of California, Irvine. I thank the faculty members and staff there for their hospitality. I am solely responsible for any remaining errors and omissions.

1 Introduction

In transport economics, the approach to estimating the benefits of transport projects has been a central topic. However, transport engineers have traditionally forecast transport demand in practice, which is necessary to estimate these benefits. Since estimation of the benefits of transport projects is inextricably linked to forecasting transport demand, both should be dealt with in the context of a consistent microeconomics paradigm. This issue has not been adequately addressed by the existing literature.[1] Hence, the effects on benefit estimation of the assumptions made about modeling transport demand are unclear. The purpose of this paper is to clarify the relationship between the assumptions made about transport-demand modeling and the method used to estimate benefits, by highlighting the forms of the utility functions. To do this, we develop a benefit-estimation model for transport projects, which is fully consistent with both microeconomics and transport-demand modeling. The model is also useful for estimating benefits in complex multi-modal transport networks in practice.

This paper analyzes three basic models, which are (at least implicitly) assumed in transport-demand modeling. We focus on a situation in which Routes 1 and 2 connect Zones A and B. The three models are termed the Mohring model, the Wardrop model, and the Logit model. The Mohring model is based on Mohring (1976), in which Route 1 may be a substitute or a complement for Route 2. The Wardrop model is based on Wardrop (1952). His argument is consistent with the view that transport demand is 'derived' demand

[1] See, for example, Harrison (1974), Mohring (1976), Williams (1976, 1977), Jones (1977), Jara-Diaz and Friesz (1982), Kanemoto and Mera (1985), Jara-Diaz (1986), Williams et al. (1991, 2001a, 2001b), Button (1993), Oppenheim (1995), and Kanemoto (1996).

generated by persons wanting to travel (or move goods) between Zones A and B. In the Wardrop model, a consumer uses the route with the lowest generalized price (which incorporates both monetary and time costs) and Routes 1 and 2 are perfect substitutes. The Logit model is an application of discrete-choice models to transport-demand forecasting and benefit estimation, and has been widely used recently. In the Logit model, the routes chosen by consumers are assumed to result from their probabilistic behavior. The probabilistic term has a Gumbel distribution. The main purpose of the present paper is to identify explicitly the relationships between the three models. That is, we show that the Wardrop and Logit models are special cases of the Mohring model, and that the Logit model degenerates to the Wardrop model in a limiting case. This finding implies that one can use the benefit-estimation method corresponding to the Mohring model to estimate the benefits of transport projects, whatever method is used to model transport demand. However, our finding does not imply that the benefit-estimation methods based on the Wardrop and Logit models are useless. We identify their merits and demerits, and demonstrate that the methods based on these models are effective when applied to new transport routes.

The paper is structured as follows. In Section 2, we formulate the three basic models and explain their merits and demerits. In Section 3, the results of Section 2 are illustrated by an example. Section 4 concludes the paper.

2 The Model

Throughout this paper, the utility function is assumed to be quasi-linear in the composite consumer good, z, the price of which is normalized at unity. Given the quasi-linear utility function, there are no income effects, and consequently, the consumer's surplus equals the equivalent variation and the compensating variation.[2] If the utility function did not satisfy quasi-linearity, these three measures of the surplus would differ. However, ignoring these differences is justified in benefit estimation in practice, because, as Willig (1976) has demonstrated, the differences between the consumer's surplus, equivalent variation, and compensating variation are rather small.

Consider the simple two-zone model shown in Figure 1. Zones A and B are connected by two routes, Routes 1 and 2. Routes 1 and 2 may be the same type of transport mode or different types of transport mode. For example, Routes 1 and 2 may both be road-transport routes. Alternatively, Route 1 might be a road-transport route and Route 2 might be a rail route. Transport demand in Routes 1 and 2 is given by x^1 and x^2, respectively. Although we focus on a simple two-zone model to demonstrate the essence of the argument, our analysis can be easily extended to more complicated networks, by applying Kidokoro (2003).

We focus on three types of model, which describe the relationship between Routes 1 and 2 in different ways. In the Mohring model, which is due to Mohring (1976), the relationship between Routes 1 and 2 is unrestricted. In the Wardrop model, which is due to Wardrop (1952), Routes 1 and 2 are perfect substitutes. The Logit model corresponds to the one in which transport demand in Routes 1 and 2 is estimated by using the standard Logit model.

2-1 The Mohring Model

We begin with the Mohring model, which is due to Mohring (1976). In the Mohring model, no specific relationship between Routes 1 and 2 is assumed. That is, Routes 1 and 2 may be substitutes or complements. Consequently, the utility function of a representative consumer has the form:

[2] See, for example, Varian (1992).

$$U = z + u(x^1, x^2)\,. \tag{1}$$

The budget constraint is:

$$y = z + p^1 x^1 + p^2 x^2 \tag{2}$$

where y is consumer income and p^1 and p^2 are the generalized prices of transport services in Routes 1 and 2, which incorporate time costs. Substituting (2) into (1) to eliminate z yields:

$$U = y - p^1 x^1 - p^2 x^2 + u(x^1, x^2)\,. \tag{3}$$

Maximizing (3) with respect to x^1 and x^2, we obtain:

$$p^1 = u_1(x^1, x^2) \equiv \frac{\partial u(x^1, x^2)}{\partial x^1}\,, \tag{4}$$

$$p^2 = u_2(x^1, x^2) \equiv \frac{\partial u(x^1, x^2)}{\partial x^2}\,. \tag{5}$$

Henceforth, partial derivatives with respect to the ith argument are denoted by the subscript i. From (4) and (5), the following transport demand functions are derived:

$$x^1 = x^1(p^1, p^2)\,, \tag{6}$$

$$x^2 = x^2(p^1, p^2)\,. \tag{7}$$

The generalized prices of transport services in Routes 1 and 2, p^1 and p^2, satisfy:

$$p^1 = t^1 + TC^1(x^1, I^1)\,, \tag{8}$$

$$p^2 = t^2 + TC^2(x^2, I^2)\,, \tag{9}$$

where t^1 and t^2 represent monetary costs of x^1 and x^2, such as fuel costs and tolls for cars, and fares for rail, I^1 and I^2 denote investments in transport infrastructure in each route that are measured in monetary terms, and $TC^1(x^1, I^1)$ and $TC^2(x^2, I^2)$ are monetized time costs in each route. Both routes are subject to congestion and time costs are increasing in transport demand and decreasing in transport investments; i.e., $TC_1^1(x^1, I^1) > 0$, $TC_2^1(x^1, I^1) < 0$, $TC_1^2(x^2, I^2) > 0$, $TC_2^2(x^2, I^2) < 0$. The investments in transport infrastructure, I^1 and I^2, are assumed to be exogenously determined by the government. As we explain in more detail later, the monetary costs, t^1 and t^2, are also assumed to be exogenous.

Substituting (8) and (9) into (6) and (7), we obtain the general-equilibrium demand functions, $x^1(t^1, I^1, t^2, I^2)$ and $x^2(t^1, I^1, t^2, I^2)$, the arguments of which are the four exogenous variables, t^1, t^2, I^1, and I^2. These general-equilibrium demand functions are loci of attained equilibria. Consequently, they incorporate all the general-equilibrium changes between Routes 1 and 2, as is explained subsequently.

The total cost functions for suppliers of transport services in each Route are denoted by $C^1(x^1)$ and $C^2(x^2)$, which are assumed to be increasing in transport demand; i.e., $C^{1\prime} > 0$ an $C^{2\prime} > 0$.[3] Since suppliers of transport services receive the prices paid by consumers for the transport services in each route, t^1 and t^2, their profits can be written as:

$$\pi^1 = t^1 x^1 - C^1(x^1), \tag{10}$$

$$\pi^2 = t^2 x^2 - C^2(x^2). \tag{11}$$

If airlines or railway companies supply transport services, π^1 and π^2 are the economic profits of these companies. In the case of private car transport, one can think of consumers deriving profits by supplying car transport services to themselves. For example, suppose that the social monetary cost of car transport, incorporating social fuel costs and road-maintenance costs, is 300 yen. Suppose also that the private price paid by consumers,

[3] For simplicity, we do not take into account the effects of investment in transport infrastructure on the cost functions of suppliers.

i.e., fuel prices including taxes, is 500 yen, and that time costs are 800 yen. Hence, the consumer pays 500+800=1300 yen for car transport, which is the generalized price of car transport. However, the generalized social cost of car transport is 300+800=1100 yen. This implies a surplus of 1300-1100=200 yen. Thus, car transport yields the consumer 200 yen in profits.[4]

In this paper, t^1 and t^2, which are the monetary prices of x^1 and x^2, are assumed to be exogenous. This assumption is reasonable if these prices are regulated by the government. In practice, the monetary prices of transport services are often controlled by government. For example, governments typically impose fuel taxes and highway tolls for road transport, which represent a considerable proportion of the total costs of road transport in many countries. In addition, air fares and rail fares are often regulated by government. However, the assumption that t^1 and t^2 are exogenous is made merely for simplicity. Alternative assumptions could be made. For instance, our analysis applies if a monopoly supplies transport services and sets price to maximize profits. That is, whether the monetary prices of transport services are exogenous or endogenous does not affect our analysis.

The total benefit, TB, is the sum of the consumer's utility and the profits of the suppliers of transport services, which from (3), (10), and (11), is:

$$\begin{aligned} TB &= U + \pi^1 + \pi^2 \\ &= y - p^1x^1 - p^2x^2 + u(x^1, x^2) + t^1x^1 - C^1(x^1) + t^2x^2 - C^2(x^2) \end{aligned}. \tag{12}$$

Suppose the government increases investment in transport infrastructure in Route 1 from I^{1WO} to I^{1W}.[5] Henceforth, the superscripts WO and W denote without and with

[4] Even if we explicitly take into account the existence of government, our argument is unaffected as long as the government surplus from the collected tax is eventually returned to consumers.

[5] Transport investment in Route 2 can be analyzed in the same way.

transport investment, respectively. In this case, the change in total benefits, ΔTB, is:

$$\Delta TB = \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right) dx^1 + \int_{p^{2W}}^{p^{2WO}} x^2 dp^2 + \int_{x^{2WO}}^{x^{2W}} \left(t^2 - C^{2\prime}\right) dx^2 , \qquad (13)$$

The derivation of (13) is given in Appendix 1. The first and second terms of the right-hand side of (13) represent the changes in the consumer and producer surpluses in Route 1, respectively. The third and fourth terms represent the corresponding changes in Route 2.

Let us interpret equation (13). We begin by focusing on changes in Route 1. The first term on the right-hand side of (13) is the change in the consumer's surplus in Route 1, measured by the general-equilibrium demand curve.[6] The standard Marshallian demand curve is drawn holding incomes and prices in other markets constant. In our model, in which the utility function is assumed to be quasi-linear, the Marshallian demand curve does not depend on income, but does depend on prices in other markets. Investment in transport infrastructure in Route 1, I^1, changes the time costs in Route 1, and hence changes the generalized price in Route 1. From (6) and (7), the change in the generalized price in Route 1 not only changes transport demand in Route 1, it also affects transport demand in Route 2. In turn, from (9), the change in transport demand in Route 2 changes the generalized price in Route 2. The result is that investment in transport infrastructure in Route 1 changes the transport demand and generalized prices in all routes. This means that the changes in Route 1's generalized price and transport demand induced by investment in transport infrastructure in Route 1 are not represented by a movement along the Marshallian demand curve for Route 1 with the generalized price in Route 2 fixed, but by a shift of the Marshallian demand curve.Figure 2-1 shows that, for Route 1, the equilibria without and with transport investment, (x^{1WO}, p^{1WO}) and (x^{1W}, p^{1W}), respectively, are on different Marshallian demand

[6] See Boadway and Bruce (1984) for details of the general-equilibrium demand curve.

curves. The general-equilibrium demand curve in Route 1 is a locus of attained equilibria, which passes through (x^{1WO}, p^{1WO}) and (x^{1W}, p^{1W}), and incorporates the associated change in Route 2. In the $x^1 - p^1$ plane in Figure 2-1, the general-equilibrium demand curve in Route 1 is shown as $x^1(p^1, p^2(p^1, I^2, t^2))$, given I^2 and t^2. The change in the consumer's surplus in Route 1 must be measured along the general-equilibrium demand curve. Consequently, this change is given by Area A.

In the standard four-step estimation procedure used in practice,[7] the forecast to be made is actual transport demand with and without a project, not hypothetical transport demand in which the transport conditions in other routes, such as the degree of congestion, remain unchanged. This implies that the conventional four-step estimation procedure is not represented by the derivation of the Marshallian demand curve with incomes and the generalized prices in other routes held constant, but is represented by the derivation of the general-equilibrium demand curve, which tracks actual transport demand with and without a project.

The second term on the right-hand side of (13) is the change in the producer's surplus in Route 1, which equals the change in profits in Route 1. We explain this using Figure 2-2.[8] In Figure 2-2, $SMC^1(I^1)$ is the social marginal cost in Route 1, which is given by $SMC^1(I^1) \equiv C^{1\prime}(x^1) + TC^1(x^1, I^1) + TC^1_1(x^1, I^1)x^1$. Since $\int SMC^1 dx^1 = C^1 + TC^1 x^1$, the area below the social marginal cost in Route 1 represents total costs in Route 1, including time costs. Including the time costs for Route 1, the consumer pays

[7] See for example, Oppenheim (1995) and Ortuzar and Willumsen (2001).

[8] Although we depict the case in which the generalized price exceeds the social marginal cost in Figure 2-2 for ease of illustration, we can analyze other cases in the same way.

$p^{1WO} x^{1WO} = \left\{ t^1 + TC^1(I^{1WO}) \right\} x^{1WO}$ in total. Thus, Area B, which is the consumer's total payment minus total costs, shows the producer's surplus in Route 1 without transport investment. Given that

$$p^{1WO} x^{1WO} - \int SMC^1 dx^1 = t^1 x^{1WO} - C^1(x^{1WO}) = \pi^{1WO}, \tag{14}$$

Area B also equals the profits in Route 1 without transport investment. Analogously, Area C represents the producer's surplus in Route 1 with transport investment, which equals the profits in Route 1 with transport investment. The result is that the change in the producer's surplus in Route 1, which equals the change in profits in Route 1, is derived as Area C minus Area B.

Consider next changes in Route 2. The third term on the right-hand side of (13) represents the change in the consumer's surplus in Route 2. As with Route 1, we can compute this change by using the general-equilibrium demand curve. As Figure 2-3 shows, the general-equilibrium demand curve in Route 2 coincides with the generalized cost curve in Route 2, which is given by (9). The reason is that equilibria must be on the line representing equation (9), regardless of the shifts of the Marshallian demand curve in Route 2 induced by investment in transport infrastructure in Route 1.[9] Thus, we can estimate the change in the consumer's surplus in Route 2 along the line that represents (9) as Area D.

The fourth term on the right-hand side of (13) represents the change in the producer's surplus in Route 2. Since the relationship given by (14) also holds for Route 2, the change in the producer's surplus in Route 2, which is equivalent to the change in profits in Route 2,

[9] This result depends on the assumption that the monetary prices in Route 2 are regulated by the government and are unaffected by investment in transport infrastructure in Route 1. If this is not the case, the general-equilibrium demand curve will be a locus of attained equilibria on a shifting generalized price curve in Route 2.

equals the change in the consumer's total payment in Route 2, minus the change in total costs in Route 2. Figure 2-4 shows that the decrease in the consumer's total payment is the sum of Areas E and F, and that the decrease in the total costs is the sum of Areas F and G. Thus, the change in the producer's surplus in Route 2 is Area G minus Area E. This is because $-\text{Area E} - \text{Area F} - (-\text{Area F} - \text{Area G}) = \text{Area G} - \text{Area E}$.

The changes in Route 2 can be understood more easily by rearranging (13) as:

$$\Delta TB = \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right) dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(p^2 - SMC^2\right) dx^2 , \qquad (15)$$

where $SMC^2 \equiv C^{2\prime} + TC^2 + TC_1^2 x^2$ is the social marginal cost in Route 2, which is the sum of the marginal monetary cost in Route 2, $C^{2\prime}$, and the marginal time cost in Route 2, $TC^2 + TC_1^2 x^2$. (See Appendix 1 for the derivation of (15).) Figures 2-3 and 2-4 show that Area D equals Area E, and that the change in Route 2, which is the sum of the change in the consumer's surplus, Area D, and the change in the producer's surplus, Area G minus Area E, is simply Area G. Area G corresponds to the third term on the right-hand side of (15), which represents the change in the deadweight loss in Route 2, which is due to the difference between the social marginal cost and the generalized price. In general, changes in the total social surplus in routes other than the route in which investments are made reduce to changes in the deadweight loss. This result represents an application of benefit estimation in an economy with distortions, as developed by Boadway and Bruce (1984).

The benefit-estimation method implied by the Mohring model is the most general and is applicable to all transport projects, and non-transport projects. In practice, the benefit-estimation method implied by the Mohring model is often implemented as the so-called ‘Rule-of-half’ method.[10] In the Rule-of-half method, changes in the consumer’s surplus in Routes 1 and 2 are given by $\frac{1}{2}(p^{1WO}-p^{1W})(x^{1WO}+x^{1W})$ and $\frac{1}{2}(p^{2WO}-p^{2W})(x^{2WO}+x^{2W})$, respectively. The Rule-of-half method can accurately calculate benefits if changes in producer’s surpluses in both routes are incorporated, except that errors arise because of the linear approximations applied by the Rule-of-half method. If the errors resulting from these linear approximations are insignificant, the Rule-of-half method based on the Mohring model computes benefits accurately, as long as the estimation of transport demand with and without a project is accurate. This result implies the following: even when transport demand is estimated by the conventional four-step method, which has no microeconomic basis, the Rule-of-half method is applicable, as long as we consider that the conventional four-step model is a reduced form of the Mohring model and as long as transport demand is estimated correctly.

However, the Mohring model has a practical disadvantage. When forecasting transport demand in practice, it is usually not possible to obtain the general-equilibrium demand function over its full range. In most cases, only transport demand with and without a transport project can be estimated, in which case, the Mohring model fails to estimate the benefits of new transport routes. Transport demand in a non-existent route is zero, and it is difficult in practice to estimate the generalized price in this case. However, benefit estimation using the Mohring model requires information on the generalized price without a project, i.e., the generalized price in a non-existent route. Thus, in this case, the Mohring model lacks practical validity. Although this disadvantage has already been noted in the literature, by for example Harberger (1972), to our knowledge, no attempt has so far been

[10] See, for example, Williams (1976).

made to resolve this problem within the framework of the Mohring model.

The merits and the demerits of the Mohring model are summarized as follows.

Merits: The Mohring model is the most general and is intrinsically applicable to all transport projects.

Demerits: To estimate the benefits of new transport routes by using the Mohring model, we need to know the generalized price of non-existent new transport routes, which is impractical.

2-2 The Wardrop Model

Transport demand is often considered a derived demand, because consumers use transport services not as an end in itself, but as a means of traveling to particular destinations. In this section, we build a model in which transport demand is explicitly assumed a derived demand.

If we treat transport demand as derived demand, a special case of the Mohring model is applicable: In such a case, the utility function is specified as

$$U = z + u(x^1 + x^2), \tag{16}$$

which shows that Routes 1 and 2 are perfect substitutes and that only total transport demand between Zones A and B, $x^1 + x^2$, is relevant in the partial utility function, u. Since this model formalizes the argument of Wardrop (1952), it is termed the Wardrop model. The Wardrop model is reasonable when there are no significant quality differences between Routes 1 and 2, except those measured by the price. Since Routes 1 and 2 are perfect substitutes in the Wardrop model, both routes are used if $p^1 = p^2$, whereas only Route 2 is used if $p^1 > p^2$, and only Route 1 is used if $p^1 < p^2$.

Defining p as $p \equiv \min\{p^1, p^2\}$, it follows that the generalized price between Zones A and B is p. Thus, the budget constraint is:

$$y = z + p(x^1 + x^2). \tag{17}$$

Substituting (17) into (16) yields:

$$U = y - p(x^1 + x^2) + u(x^1 + x^2). \tag{18}$$

Maximizing (18) with respect to x^1 and x^2, we obtain:

$$p = u'(x^1 + x^2). \tag{19}$$

As in the Mohring model, the generalized prices in Routes A and B satisfy:

$$p^1 = t^1 + TC^1(x^1, I^1), \tag{20}$$

$$p^2 = t^2 + TC^2(x^2, I^2). \tag{21}$$

If $p = p^1 = p^2$, transport demand in Routes 1 and 2, x^1 and x^2, respectively, are determined from (19), (20), and (21). If $p = p^1 < p^2$, $x^2 = 0$ and x^1 is derived from (19) and (20). If $p = p^2 < p^1$, $x^1 = 0$ and x^2 is derived from (19) and (21).

The arguments used in the Mohring model also apply to the suppliers of transport services. Thus, from (10), (11), and (18), the total benefit, TB, can be written as:

$$\begin{aligned} TB &= U + \pi^1 + \pi^2 \\ &= y - p(x^1 + x^2) + u(x^1 + x^2) + t^1 x^1 - C^1(x^1) + t^2 x^2 - C^2(x^2) \end{aligned} \tag{22}$$

In the Wardrop model, the change in total benefits when investment in transport infrastructure in Route 1, I^1, increases from I^{1WO} to I^{1W} is, from (19) and (22):

$$\Delta TB = \int_{p^W}^{p^{WO}} (x^1 + x^2) dp + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right) dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(t^2 - C^{2\prime}\right) dx^2 \tag{23}$$

(Since the derivation of (23) is essentially the same as that of (13), it is omitted.) The first term on the right-hand side of (23) is the change in the consumer's surplus between Zones A and B. The second and third terms on the right-hand side of (23) are the changes in the producer's surpluses in Routes 1 and 2, respectively.

It is apparent from the utility function in the Wardrop model, which is given by (16), that the Wardrop model is a special case of the Mohring model. Thus, we can rearrange (23) as

$$\begin{aligned}\Delta TB &= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right) dx^1 + \int_{p^W}^{p^{WO}} x^2 dp^2 + \int_{x^{2WO}}^{x^{2W}} \left(t^2 - C^{2\prime}\right) dx^2 \\ &= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right) dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(p^2 - SMC^2\right) dx^2,\end{aligned} \tag{24}$$

which has the same form as (13) and (15), except that the generalized prices in Routes 1 and 2 are $p \equiv \min\{p^1, p^2\}$, not p^1 and p^2. That is, in the Wardrop model, we can use the generalized price between Zones A and B in calculating the consumer's surplus, which is impractical in the Mohring model. This is useful in practice, because it overcomes the limitation of the Mohring model, which arises in the context of estimating the benefits of new transport routes. Suppose that Route 1 is the existing route and Route 2 is the new route for which transport investment is planned. In the absence of transport investment, the generalized price in Route 2 can be considered infinite, in which case, the generalized price in Route 1 prevails as the price between Zones A and B, since $p = p^1 < p^2 = \infty$. Thus, we can use the generalized price in Route 1 to compute the change in the consumer's surplus in Route 2. With transport investment, since Routes 1 and 2 are perfect substitutes, $p = p^1 = p^2$ if both routes are used. (If only the newly introduced Route 2 is used, then $p = p^2 < p^1$.) The result is that in the Wardrop model, we can estimate the change in the consumer's surplus by using only the generalized prices actually observed. This is particularly useful for benefit estimation in practice.

Although the Wardrop model is practically useful, the assumption of perfect substitutability between routes is strong, as has been pointed out by Arnott and Yan (2000).

For example, one can travel between Tokyo and Osaka in Japan by bullet train or by air. According to the Wardrop model, all consumers would travel by air following a slight drop in airfares *ceteris paribus*, unless there were capacity constraints in air travel. This is unrealistic. This unrealistic prediction might not eventuate were we to incorporate consumer heterogeneity. However, with homogeneous consumers, the assumption of perfect substitutability between routes might generate predictions that are not consistent with observed outcomes.

The merits and the demerits of the Wardrop model can be summarized as follows.

Merits: The Wardrop model facilitates benefit estimation in practice, particularly in relation to the introduction of new transport routes.

Demerits: The strong assumption of perfect substitutability between routes suggests that the Wardrop model might not explain observed outcomes.

2-3 The Logit Model

The Logit model applies discrete-choice modeling to forecasting transport demand and benefit estimation. Suppose that a consumer demands one trip between Zones A and B. We assume that the utilities in Routes 1 and 2, respectively are V^1 and V^2, which are $V^1 = v^1 + \varepsilon^1$ and $V^2 = v^2 + \varepsilon^2$, where v^1 and v^2 are deterministic utility terms and ε^1 and ε^2 are residual terms, which are assumed to have a probability distribution. For instance, ε^1 and ε^2 may correspond to unobservable components of utility that represent the variety of choices made by observationally identical consumers. Route 1 is selected if $V^1 > V^2$, while Route 2 is selected if $V^1 < V^2$. Whether $V^1 > V^2$ or $V^1 < V^2$ depends on the probabilistic terms, ε^1 and ε^2. The probability with which Route 1 is selected, $P1$, is:

$$P1 = \text{Prob}(V^1 > V^2) = \text{Prob}(v^1 - v^2 > \varepsilon^2 - \varepsilon^1). \tag{25}$$

Under the assumption that the probabilistic terms, ε^1 and ε^2, are independently subject to the Gumbel distribution function, $F(\varepsilon) = \exp\left\{-\exp\left(-\frac{\varepsilon}{\mu}\right)\right\}$ (in which $\mu > 0$), we obtain:

$$P1 = \frac{\exp(v^1/\mu)}{\exp(v^1/\mu) + \exp(v^2/\mu)}. \tag{26}$$

(The probability with which Route 2 is selected, $P2$, is $P2 = 1 - P1$.) Transport demand in Routes 1 and 2, x^1 and x^2 is:

$$x^1 = X \times P1 = X \frac{\exp(v^1/\mu)}{\exp(v^1/\mu) + \exp(v^2/\mu)}, \tag{27}$$

$$x^2 = X \times P2 = X \frac{\exp(v^2/\mu)}{\exp(v^1/\mu) + \exp(v^2/\mu)}, \tag{28}$$

where X denotes the total number of consumers.

Suppose that the deterministic utilities, v^1 and v^2, are estimated econometrically as $v^1 \equiv \alpha - \beta p^1$ and $v^2 \equiv \alpha - \beta p^2$ ($\alpha \geq 0$, $\beta > 0$). Following Small and Rosen (1981), the change in the consumer's surplus when there is an increase in transport investment in Route 1 is given by:

$$\Delta U = \frac{\mu X}{\lambda}\left\{\ln\sum_{i=1}^{2}\exp(v^{iW}/\mu) - \ln\sum_{i=1}^{2}\exp(v^{iWO}/\mu)\right\} \tag{29}$$

where λ is the marginal utility of income. The marginal utility of income, λ, is derived from Roy's Identity:

$$\lambda = \frac{\partial v^1}{\partial y} = -\frac{\frac{\partial v^1}{\partial p^1}}{x^{1c}} \tag{30}$$

where x^{1c} is 'conditional' transport demand in Route 1, given that a consumer demands transport services. For instance, consider daily transport demand for a commuter service. If a consumer makes one daily round-trip, $x^{1c} = 1$.

The Logit model has microeconomic foundations because it can be derived from the utility-maximizing behavior of the representative consumer. We investigate this further by applying the approach of Anderson et al. (1992). Consider the following utility-maximizing problem of the representative consumer (M):

$$\text{(M)}\quad \begin{aligned} &\max_{\{z,x^1,x^2\}} U = z + \frac{\alpha}{\beta}(x^1 + x^2) - \frac{\mu}{\beta}\left(x^1 \ln\frac{x^1}{X} + x^2 \ln\frac{x^2}{X} \right) \\ &s.t. \quad y = z + p^1x^1 + p^2x^2 \\ &\qquad\quad x^1 + x^2 = X \end{aligned} .$$

Solving the utility-maximizing problem, we obtain the demand functions in Routes 1 and 2:

$$x^1 = X \frac{\exp\left(\dfrac{\alpha - \beta p^1}{\mu}\right)}{\exp\left(\dfrac{\alpha - \beta p^1}{\mu}\right) + \exp\left(\dfrac{\alpha - \beta p^2}{\mu}\right)}, \tag{31}$$

$$x^2 = X \frac{\exp\left(\dfrac{\alpha - \beta p^2}{\mu}\right)}{\exp\left(\dfrac{\alpha - \beta p^1}{\mu}\right) + \exp\left(\dfrac{\alpha - \beta p^2}{\mu}\right)}. \tag{32}$$

Recall that $v^1 \equiv \alpha - \beta p^1$ and $v^2 \equiv \alpha - \beta p^2$. The result is that (31) and (32) correspond to the demand functions in the Logit model, given by (27) and (28), respectively. Substituting (31) and (32) into the utility function in (M), we obtain the following expenditure function:

$$e(v^1, v^2, u) = u - \frac{\mu X}{\beta} \ln \sum_{i=1}^{2} \exp(v^i / \mu), \tag{33}$$

where u is the utility level. Since, from (30),

$$\lambda = \frac{\partial v^1}{\partial y} = -\frac{\dfrac{\partial v^1}{\partial p^1}}{x^{1c}} = -\frac{-\beta}{1} = \beta, \tag{34}$$

the change in the consumer's surplus when transport investment increases in Route 1 is

$$\begin{aligned}\Delta U &= e(v^{1WO}, v^{2WO}, u) - e(v^{1W}, v^{2W}, u) \\ &= \frac{\mu X}{\beta}\left\{\ln\sum_{i=1}^{2}\exp(v^{iW}/\mu) - \ln\sum_{i=1}^{2}\exp(v^{iWO}/\mu)\right\}, \\ &= \frac{\mu X}{\lambda}\left\{\ln\sum_{i=1}^{2}\exp(v^{iW}/\mu) - \ln\sum_{i=1}^{2}\exp(v^{iWO}/\mu)\right\}\end{aligned} \tag{35}$$

which corresponds to (29).

It is clear from the utility-maximization problem (M) that the Logit model is a special case of the Mohring model. Hence, the benefit-estimation methods derived in (13) and (15) are valid even in the Logit model. In the case of the Logit model, an alternative form for the change in the consumer's surplus when transport investment increases in Route 1 is derived as:

$$\begin{aligned}\Delta U &= \int_{p^{1W}}^{p^{1WO}} \frac{\partial e(v^1, v^2, u)}{\partial p^1} dp^1 \\ &= \int_{p^{1W}}^{p^{1WO}} \left(\frac{\partial e}{\partial v^1}\frac{\partial v^1}{\partial p^1} + \frac{\partial e}{\partial v^2}\frac{\partial v^2}{\partial p^2}\frac{\partial p^2}{\partial p^1}\right) dp^1 \\ &= -\frac{\mu X}{\beta}\int_{p^{1W}}^{p^{1WO}} \left(\frac{-\frac{\beta}{\mu}\exp(v^1/\mu)}{\sum_{i=1}^{2}\exp(v^i/\mu)} + \frac{-\frac{\beta}{\mu}\exp(v^2/\mu)}{\sum_{i=1}^{2}\exp(v^i/\mu)}\frac{\partial p^2}{\partial p^1}\right) dp^1 \\ &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{p^{2W}}^{p^{2WO}} x^2 dp^2,\end{aligned} \tag{36}$$

where $\int_{p^{1W}}^{p^{1WO}} x^1 dp^1$ corresponds to the first term on the right-hand side of (13), which is the change in the consumer's surplus in Route 1, and $\int_{p^{2W}}^{p^{2WO}} x^2 dp^2$ corresponds to the third term on the right-hand side of (13), which is the change in the consumer's surplus in Route 2.[11]

[11] Equation (36) could also be derived directly from the properties of the expenditure function. See, for example, Varian (1992).

Consequently, equations (35) and (36) show that the well-known method of calculating the consumer's surplus using log-sum terms (or inclusive values), as shown in (35) and originally formulated by Williams (1977) and Small and Rosen (1981), is the method that gives the sum of the changes in consumers' surpluses in all routes. The change in total benefits according to the Logit model, which takes into account both consumer's and producer's surpluses, can be written as follows:

$$\begin{aligned}
\Delta TB &= \frac{\mu X}{\lambda}\left\{\ln\sum_{i=1}^{2}\exp(v^{iW}/\mu) - \ln\sum_{i=1}^{2}\exp(v^{iWO}/\mu)\right\} + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}}\left(t^2 - C^{2\prime}\right)dx^2 \\
&= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{p^W}^{p^{WO}} x^2 dp^2 + \int_{x^{2WO}}^{x^{2W}}\left(t^2 - C^{2\prime}\right)dx^2 \\
&= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}}\left(p^2 - SMC^2\right)dx^2.
\end{aligned} \tag{37}$$

Equation (37) confirms that the Logit model is a special case of the Mohring model.

The Gumbel distribution used in the Logit model implies that in the converted utility-maximization problem (M), the utility function of the representative consumer has a particular form, which is useful for estimating benefits when a new transport route is introduced. For example, suppose that Route 2 is being introduced by a transport project. Without the transport project, the required travel time in Route 2 can be considered infinite, in which case, $v^{2WO} = -\infty$, because the generalized price in Route 2 is also infinite; i.e., $p^2 = \infty$. When $v^{2WO} = -\infty$, it follows that $\exp(v^{2WO}/\mu) = 0$, which means that the generalized price in Route 2 without the transport project has no influence on the estimation of the consumer's surplus in (35). That is, the issue of how to estimate the generalized price of a non-existent route is of no importance when using a Logit model that specifies the form of the utility function.

The parameter μ in the Gumbel distribution, $F(\varepsilon)=\exp\left\{-\exp\left(-\frac{\varepsilon}{\mu}\right)\right\}$, is typically termed a scale parameter. Since the variance of the Gumbel distribution is $\frac{(\mu\pi)^2}{6}$, the greater is μ, the wider the distribution. As Small (1992) notes, $\mu=1$ is usually assumed. As μ approaches zero, the variance of the Gumbel distribution also approaches zero, and the effects of the probabilistic terms, ε^1 and ε^2, diminish. In the limit, the Logit model reduces to a special case of the Wardrop model. This is apparent from the utility-maximization problem (M) when $\mu \to 0$.

The Logit model is useful for consistent transport-demand forecasting and benefit estimation. However, it is apparent from the utility-maximization problem (M) that the results are based on the assumption that total transport demand is fixed. Thus, we must modify our analysis to deal with the effect of transport projects on total transport demand.

To model changing total transport demand, we need to incorporate the consumer choice of 'do not travel'. One approach is to incorporate the choice of 'do not travel' into the original Logit model. An alternative method is to construct a 'nested' model, in which consumers choose between 'do not travel' and 'travel' in the first stage, and in the second stage, consumers who selected 'travel' in the first stage choose between 'travel in Route 1' and 'travel in Route 2'. As we show subsequently, even when the choice of 'do not travel' is incorporated to represent a change in total transport demand, our results from the Logit model remain valid.

Initially, consider incorporating the choice of 'do not travel' into the original Logit model. Suppose that demand for 'do not travel' is x^0 and that the utility derived from this choice is given by $V^0=0+\varepsilon^0$, where deterministic utility is zero and ε^0 is independently subject to the Gumbel distribution. The corresponding utility-maximization problem of the representative consumer (M1) can be written as:

$$
\text{(M1)}\quad
\begin{aligned}
&\max_{\{z,x^0,x^1,x^2\}} U = z + \frac{\alpha}{\beta}(x^1 + x^2) - \frac{\mu}{\beta}\left(x^0 \ln\frac{x^0}{X} + x^1 \ln\frac{x^1}{X} + x^2 \ln\frac{x^2}{X} \right) \\
&s.t. \quad y = z + p^1 x^1 + p^2 x^2 \\
&\qquad\quad x^0 + x^1 + x^2 = X .
\end{aligned}
$$

As before, the Logit model reduces to a special case of the Wardrop model as $\mu \to 0$. Solving the utility-maximization problem (M1), we obtain the following demand functions for x^0, x^1, and x^2:

$$
x^0 = X \frac{1}{\exp(v^1/\mu) + \exp(v^2/\mu) + 1}, \tag{38}
$$

$$
x^1 = X \frac{\exp(v^1/\mu)}{\exp(v^1/\mu) + \exp(v^2/\mu) + 1}, \tag{39}
$$

$$
x^2 = X \frac{\exp(v^2/\mu)}{\exp(v^1/\mu) + \exp(v^2/\mu) + 1}. \tag{40}
$$

Substituting (38), (39), and (40) into the utility function in (M1) yields the following expenditure function:

$$
e(v^1, v^2, u) = u - \frac{\mu X}{\beta} \ln\left\{ \sum_{i=1}^{2} \exp(v^i/\mu) + 1 \right\}. \tag{41}
$$

From (41), the change in the consumer's surplus when transport investment increases in Route 1 is:

$$
\Delta U = \frac{\mu X}{\beta}\left[\ln\left\{ \sum_{i=1}^{2} \exp(v^{iW}/\mu) + 1 \right\} - \ln\left\{ \sum_{i=1}^{2} \exp(v^{iWO}/\mu) + 1 \right\} \right]. \tag{42}
$$

Given that $1 = \exp(0) = \exp(v^0/\mu)$, equation (42) extends (35) to include the choice of 'do not travel'. The issue of how to estimate the generalized price of a non-existent route is also of no importance in this context, because $p^{iWO} = \infty$ implies $\exp(v^{iWO}/\mu) = 0$, which shows that the generalized price of a non-existent route has no effect on (42).

By following the procedure used to derive (36) and (37), we can derive the following expression for the change in total benefits:

$$\begin{aligned}\Delta TB &= \frac{\mu X}{\beta}\left[\ln\left\{\sum_{i=1}^{2}\exp(v^{iW}/\mu)+1\right\}-\ln\left\{\sum_{i=1}^{2}\exp(v^{iWO}/\mu)+1\right\}\right]+\int_{x^{1WO}}^{x^{1W}}\left(t^1-C^{1\prime}\right)dx^1+\int_{x^{2WO}}^{x^{2W}}\left(t^2-C^{2\prime}\right)dx^2\\ &=\int_{p^W}^{p^{WO}}x^1dp^1+\int_{x^{1WO}}^{x^{1W}}\left(t^1-C^{1\prime}\right)dx^1+\int_{p^W}^{p^{WO}}x^2dp^2+\int_{x^{2WO}}^{x^{2W}}\left(t^2-C^{2\prime}\right)dx^2\\ &=\int_{p^W}^{p^{WO}}x^1dp^1+\int_{x^{1WO}}^{x^{1W}}\left(t^1-C^{1\prime}\right)dx^1+\int_{x^{2WO}}^{x^{2W}}\left(p^2-SMC^2\right)dx^2.\end{aligned} \tag{43}$$

In the above equation, there is no term for a change in the consumer's surplus relating to x^0 because deterministic utility of zero for the choice of 'do not travel' implies that the price of x^0 is invariant at zero, which yields no change in the consumer's surplus relating to x^0. Equation (43) confirms that the benefit-estimation method based on the Mohring model is applicable in this case.

As an alternative approach, we develop the 'nested' Logit model to incorporate the choice of 'do not travel' in the first stage. This nested Logit model is based on the assumption that ε^0, ε^1, and ε^2 have the following multivariate cumulative distribution function,

$$F(\varepsilon^0,\varepsilon^1,\varepsilon^2)=\exp\left[-\left\{\exp\left(-\frac{\varepsilon^0}{\mu^0}\right)\right\}^{\frac{\mu^0}{\mu}}-\left\{\exp\left(-\frac{\varepsilon^1}{\mu^1}\right)+\exp\left(-\frac{\varepsilon^2}{\mu^1}\right)\right\}^{\frac{\mu^1}{\mu}}\right], \tag{44}$$

in which $0\le\mu^0\le\mu$ and $0\le\mu^1\le\mu$. Following Verboven (1996), the utility-maximization problem of the representative consumer corresponding to the nested Logit model (M2) can be formulated as:

(M2)

$$\max_{\{z,x^0,x^1,x^2\}} \; U = z + \frac{\alpha}{\beta}(x^1 + x^2)$$

$$-\frac{1}{\beta}\left\{ x^0 \ln\left(\frac{x^0}{X}\right)^{\mu} + x^1 \ln\left(\frac{x^1}{x^1+x^2}\right)^{\mu^1}\left(\frac{x^1+x^2}{X}\right)^{\mu} + x^2 \ln\left(\frac{x^2}{x^1+x^2}\right)^{\mu^1}\left(\frac{x^1+x^2}{X}\right)^{\mu} \right\}$$

$$s.t. \quad y = z + p^1 x^1 + p^2 x^2$$

$$x^0 + x^1 + x^2 = X.$$

When $\mu = \mu^0 = \mu^1$, the utility-maximization problem (M2) coincides with the utility-maximization problem (M1). Hence, the nested Logit model includes the Logit model as a special case. This is also apparent from the fact that the multivariate cumulative distribution function, (44), reduces to

$$F(\varepsilon^0, \varepsilon^1, \varepsilon^2) = \exp\left\{-\exp\left(-\frac{\varepsilon^0}{\mu}\right)\right\}\exp\left\{-\exp\left(-\frac{\varepsilon^1}{\mu}\right)\right\}\exp\left\{-\exp\left(-\frac{\varepsilon^2}{\mu}\right)\right\}, \qquad (45)$$

which yields the Logit model when $\mu = \mu^0 = \mu^1$. This relationship between the nested Logit model and the Logit model implies that the nested Logit model reduces to a special case of the Wardrop model when $\mu = \mu^0 = \mu^1$ and $\mu \to 0$. Moreover, the utility-maximization problem (M2) implies that the nested Logit model is also a special case of the Mohring model.

Solving the utility-maximization problem (M2), we obtain the following demand functions for x^0, x^1, and x^2:

$$x^0 = X \frac{1}{A(v^1, v^2) + 1}, \qquad (46)$$

$$x^1 = X \frac{\exp(v^1/\mu^1)}{\exp(v^1/\mu^1) + \exp(v^2/\mu^1)} \frac{A(v^1, v^2)}{A(v^1, v^2) + 1}, \qquad (47)$$

$$x^2 = X\,\frac{\exp(v^2/\mu^2)}{\exp(v^1/\mu^1)+\exp(v^2/\mu^1)}\,\frac{A(v^1,v^2)}{A(v^1,v^2)+1}\,. \tag{48}$$

where $A(v^1,v^2) \equiv \exp\left\{\left(\frac{\mu^1}{\mu}\right)\ln\sum_{i=1}^{2}\exp(v^i/\mu^i)\right\}$. Substituting (46), (47), and (48) into the utility function in (M2), we obtain the following expenditure function:

$$e(v^1,v^2,u) = u - \frac{\mu X}{\beta}\ln\left\{A(v^1,v^2)+1\right\}. \tag{49}$$

From (49), the change in the consumer's surplus when transport investment increases in Route 1 is:

$$\Delta U = \frac{\mu X}{\beta}\left[\ln\left\{A(v^{1W},v^{2W})+1\right\} - \ln\left\{A(v^{1WO},v^{2WO})+1\right\}\right]. \tag{50}$$

The generalized price of a non-existent route has no effect on $A(v^{1WO},v^{2WO})$, and therefore has no effect on (50), because $\exp(v^{iWO}/\mu^i)=0$ when $p^{iWO}=\infty$. Thus, in the nested Logit model also, the issue of how to estimate the generalized price of a non-existent route is of no importance.

By again following the procedure used to obtain (36) and (37), we can derive the following expression for the change in total benefits in this case:

$$\begin{aligned}
\Delta TB &= \frac{\mu X}{\beta}\left[\ln\left\{A(v^{1W},v^{2W})+1\right\} - \ln\left\{A(v^{1WO},v^{2WO})+1\right\}\right] + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}}\left(t^2 - C^{2\prime}\right)dx^2 \\
&= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{p^W}^{p^{WO}} x^2 dp^2 + \int_{x^{2WO}}^{x^{2W}}\left(t^2 - C^{2\prime}\right)dx^2 \\
&= \int_{p^W}^{p^{WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}}\left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}}\left(p^2 - SMC^2\right)dx^2.
\end{aligned} \tag{51}$$

In the above equation, there is again no term for a change in the consumer's surplus relating to x^0. This is because the price of x^0 is invariant at zero, which implies no change in the consumer's surplus. Equation (51) confirms that the benefit-estimation method based on the Mohring model is applicable to the nested Logit model.

Although the Logit and nested Logit models are convenient for consistent transport-demand forecasting and benefit estimation, they are special cases of the Mohring model in which specific utility functions are assumed. Consequently, if transport demand is forecast by using the Logit (or nested Logit) model, we can accurately estimate benefits by using the method based on the Mohring model. Our ability to estimate benefits easily using the Logit (or nested Logit) model when a new transport route is introduced relies on having specific forms for the utility functions. That is, tractable benefit estimation in the context of new transport routes is achieved at the expense of making specific assumptions about the forms of the utility functions.

The merits and the demerits of the Logit model can be summarized as follows.

Merits: The Logit model enables consistent transport-demand forecasting and benefit estimation. The Logit model, as well as the Wardrop model, also facilitates benefit estimation when new transport routes are introduced.

Demerits: The utility function underlying the Logit (and nested Logit) model is specific. Hence, the Logit (and nested Logit) model could be criticized for unjustifiably assuming such specific forms for the utility functions.

3 An Example

In this section, we illustrate how to estimate a change in total benefits by using an example in which transport demand is forecast by the Logit model. As we showed in Section 2, the Logit model is a special case of the Mohring model. Thus, the change in total benefits can be computed by using the benefit-estimation method based on the Mohring model, or the one based on the Logit model, in which the change in the consumer's surplus is the difference in the log-sum terms, as shown in (35).[12]

The example in Table 1 describes the following situation. Consider a rail project. This project reduces rail-travel time from 40 (minutes) to 20 (minutes), but increases its average cost from 500 (yen) to 650 (yen). Without the project, rail travel breaks even. With the project, the average cost of rail travel increases by 150 (yen). This increase is financed by a rise in the rail fare of 100 (yen) and an increased government subsidy of 50 (yen) per rail user. The project increases rail travel and reduces car travel, which slightly relieves car congestion. Reduced car congestion decreases car-travel time from 45 (minutes) to 44 (minutes). The cost of car travel, i.e., the gross fuel price, which includes fuel tax, also falls, from 700 (yen) to 680 (yen). The fuel-tax rate is 100%, so the gross fuel price is twice as high as the net fuel price. Implementation of the project causes total transport demand to increase from 0.278+0.252=0.530 (million trips per day) to 0.313+0.242=0.555 (million trips per day). Transport demand is assumed to be forecast by the Logit model, in which the utility of each route is given by:

$$\begin{aligned} \mathrm{V} &= -0.000250 \times (\mathrm{Time} \times 40 + \mathrm{Monetary\ Price}) \\ &= -0.000250 \times (\mathrm{Generalized\ Price}), \end{aligned} \tag{52}$$

where the time cost is 40 (yen per minute).

We begin by applying the benefit-estimation method based on the Logit model with variable total demand, given by (42). The scale parameter of the Gumbel distribution is assumed to be unity; i.e., $\mu = 1$. Without the project, the log-sum term,

[12] We cannot apply the benefit-estimation method based on the Wardrop model, (23), to the following example, because both rail and car are used but their generalized prices are unequal. If the model were based on the Wardrop model, the generalized prices of rail and car that are actually used, would be equal.

$\ln\left\{\sum_i \exp(v^{iWO}/\mu)+1\right\}$, where i = rail and car in this example, is calculated as:

$$\ln\left\{\sum_i \exp(v^{iWO}/\mu)+1\right\} = \ln\left\{\exp(-0.00025\times 2100)+\exp(-0.00025\times 2500)+1\right\} = 0.755.$$

(53)

With the project, the log-sum term, $\ln\left\{\sum_i \exp(v^{iW}/\mu)+1\right\}$, is:

$$\ln\left\{\sum_i \exp(v^{iW}/\mu)+1\right\} = \ln\left\{\exp(-0.00025\times 1400)+\exp(-0.00025\times 2440)+1\right\} = 0.810.$$

(54)

By applying (35), from (53) and (54), we obtain the following change in the consumer's surplus:

$$\Delta U = \frac{1}{0.00025}(0.810-0.755) = 222 \text{ (million yen).} \tag{55}$$

Next, we calculate the producer's surplus. The change in the producer's surplus in rail travel is:

$$(600-650)\times 0.313-(500-500)\times 0.278 = -15.7 \text{ (million yen).} \tag{56}$$

The change in the producer's surplus in car travel is:

$$(680-340)\times 0.242-(700-350)\times 0.252 = -5.91 \text{ (million yen).} \tag{57}$$

Adding (55) to (57) yields a change in the total benefit of 200 (million yen).

Now, we calculate the change in the total benefit by applying the benefit-estimation method based on the Mohring model, disregarding errors due to linear approximations. First, on the basis of (13), we compute the total benefit as the sum of the changes in the consumer's and producer's surpluses. The change in the consumer's surplus in rail travel is:

$$0.5\times(2100-1400)\times(0.278+0.313) = 207 \text{ (million yen).} \tag{58}$$

The change in the consumer's surplus in car travel is:

$$0.5 \times (2500 - 2440) \times (0.252 + 0.242) = 14.8 \text{ (million yen).} \tag{59}$$

Equations (58) and (59) confirm that the change in the consumer's surplus calculated by using the method based on the Logit model, given by (55), is approximately equal to the sum of the changes in the consumer's surpluses in rail travel and car travel, given by (58) and (59), respectively. Adding (56) and (57) to the sum of (58) and (59), we again obtain a change in the total benefit of 200 (million yen).

Second, on the basis of (15), we calculate the change in the total benefit by adding the change in the consumer's surplus in rail travel, the change in the producer's surplus in rail travel, and the change in the deadweight loss in car travel. The change in the deadweight loss in car travel corresponds to Area G in Figure 2-4. Although Area G arises because the social marginal cost curve differs from the general-equilibrium demand curve, we can compute Area G without knowing the social marginal cost curve itself, which is difficult to estimate in practice. Note that the social total cost, including time costs, is given by the area below the social marginal cost curve. Thus, the sum of Areas G and F in Figure 2-4 equals the change in the social total cost. Subtracting Areas G and F from Area F, which, being the area below the general-equilibrium demand curve, equals the private total cost, yields Area G. That is, since

$$\text{(Area F)} \qquad 0.5 \times (0.242 - 0.252) \times (2440 + 2500) = -24.7 \text{ (million yen) and} \tag{60}$$

$$\text{(Area G+ Area F)} \qquad (44 \times 40 + 340) \times 0.242 - (45 \times 40 + 350) \times 0.252 = -33.6 \text{ (million yen),} \tag{61}$$

the change in the deadweight loss in car travel is:

$$-24.7 - (-33.6) = 8.89 \text{ (million yen).} \tag{62}$$

This figure equals the sum of the change in the consumer's surplus in car travel, given by (59), and the change in the producer's surplus in car travel, given by (57). Adding the change in the consumer's surplus in rail travel, given by (58), to the change in the producer's surplus in rail travel, given by (56), yields a change in the total benefit of 200 (million yen), as before.

4 Conclusion

In this paper, we have explained three basic benefit-estimation models and discussed their merits and demerits. The relationships between the three models are shown in Figure 3. The most general model is the Mohring model, which includes the Wardrop model and the Logit model as special cases. The Logit model reduces to the Wardrop model when the scale parameter of the Gumbel distribution, μ, is zero. (Considering the Logit model as a special case of the nested Logit model, the area corresponding to the nested Logit model is shown by the dotted line in Figure 3.) These results imply that the properties of the Mohring model apply to the Wardrop and Logit models, but not *vice versa*.

The analysis of this paper suggests that it is appropriate to use the Mohring model to estimate the benefits of transport projects. However, the Morhing model lacks practical validity when applied to new transport routes. In these cases, we need to use the Wardrop model, assuming that all routes are perfectly substitutable, or use the Logit (or nested Logit) model.

In the Wardrop model, in which the generalized prices of all routes are equal, we may consider the generalized price between zones. In the Logit (or nested Logit) model, the log-sum terms can be considered the generalized price between zones. However, it must be noted that the specific properties of the Wardrop and Logit (or nested Logit) models enable us to define the generalized prices between zones. In the Mohring model, we cannot define the generalized prices between zones for benefit estimation. These points are misunderstood by many researchers and practitioners, but should be carefully considered. For example, some manuals on benefit estimation in Japan, such as that of the Institute for Transport Policy Studies (1999a, 1999b) define the generalized price between zones as the weighted average of the generalized prices in each route. This price is then applied to transport demand between zones, although the manuals implicitly rely on the Mohring model. Such an inconsistent approach yields inaccurate benefit calculations, as discussed in Kidokoro (2003).

Appendix 1

Derivation of (13)

Totally differentiating (12) with respect to I^1, from (4) and (5), we obtain:

$$\begin{aligned}\frac{dSW}{dI^1} &= -\left(\frac{dp^1}{dI^1}\right)x^1 - p^1\frac{dx^1}{dI^1} - \left(\frac{dp^2}{dI^1}\right)x^2 - p^2\frac{dx^2}{dI^1} \\ &\quad + u_1\frac{dx^1}{dI^1} + u_2\frac{dx^2}{dI^1} + \left(t^1 - C^{1\prime}\right)\frac{dx^1}{dI^1} + \left(t^2 - C^{2\prime}\right)\frac{dx^2}{dI^1} \\ &= -\left(\frac{dp^1}{dI^1}\right)x^1 - \left(\frac{dp^2}{dI^1}\right)x^2 + \left(t^1 - C^{1\prime}\right)\frac{dx^1}{dI^1} + \left(t^2 - C^{2\prime}\right)\frac{dx^2}{dI^1}.\end{aligned} \tag{A1}$$

Integrating (A1) yields (13).

Q.E.D

Derivation of (14)

Rearranging the third and the fourth terms of the right-hand side of (13) using (9), we obtain (15):

$$\begin{aligned}\Delta SW &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right)dx^1 + \int_{p^{2W}}^{p^{2WO}} x^2 dp^2 + \int_{x^{2WO}}^{x^{2W}} \left(t^2 - C^{2\prime}\right)dx^2 \\ &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2W}}^{x^{2WO}} x^2 \frac{dp^2}{dx^2} dx^2 + \int_{x^{2WO}}^{x^{2W}} \left(p^2 - TC^2(x^2, I^2) - C^{2\prime}\right)dx^2 \\ &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(-TC_1^2 x^2 + p^2 - TC^2(x^2, I^2) - C^{2\prime}\right)dx^2 \\ &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(p^2 - (C^{2\prime} + TC^2 + TC_1^2 x^2)\right)dx^2 \\ &= \int_{p^{1W}}^{p^{1WO}} x^1 dp^1 + \int_{x^{1WO}}^{x^{1W}} \left(t^1 - C^{1\prime}\right)dx^1 + \int_{x^{2WO}}^{x^{2W}} \left(p^2 - SMC^2\right)dx^2 \quad .\end{aligned} \tag{A2}$$

Q.E.D

Table

Table 1: A Numerical Example

Without a Project

Without	Time (minutes)	Monetary price (yen)	Rail's Average Cost (yen)	Fuel Tax (yen)	Generalized Price (yen)	Demand (million trips per day)
Do not travel	N/A					0.470
Rail	40	500	500	N/A	2100	0.278
Car	45	700	N/A	350	2500	0.252
Total demand						1.00

With a Project

With	Time (minutes)	Monetary price (yen)	Rail's Average Cost (yen)	Fuel Tax (yen)	Generalized Price (yen)	Demand (million trips per day)
Do not travel	N/A					0.445
Rail	20	600	650	N/A	1400	0.313
Car	44	680	N/A	340	2440	0.242
Total demand						1.00

Figures

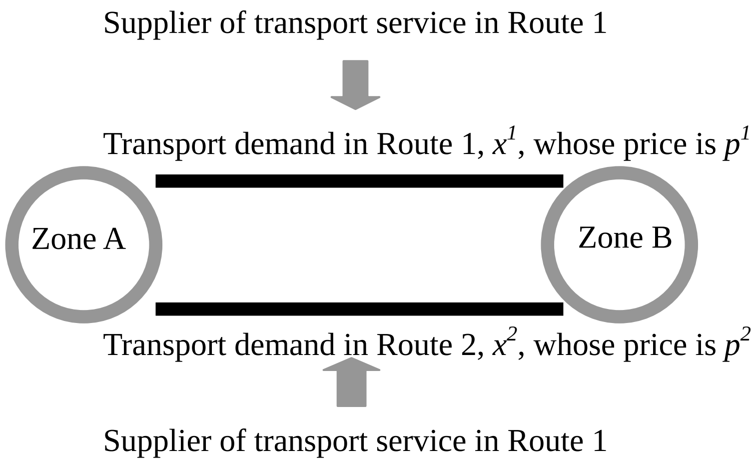

Figure 1: Visual Representation of the Model

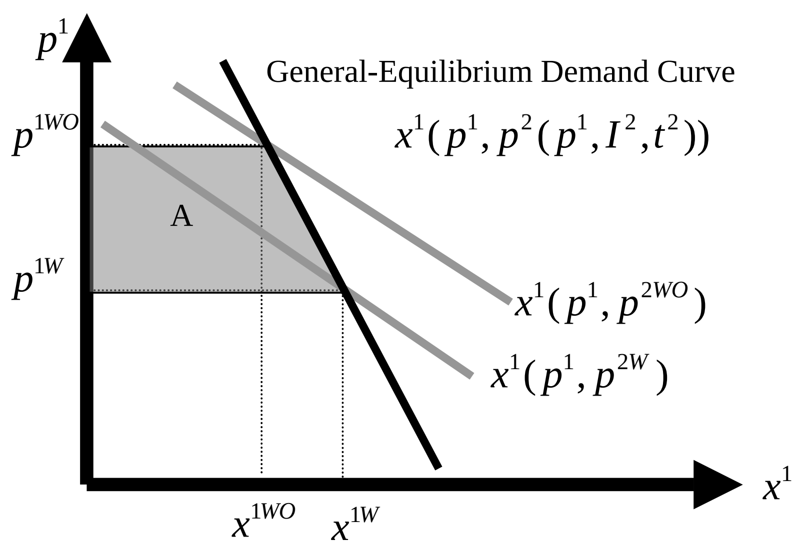

Figure 2-1：Consumer’s Surplus in Route 1

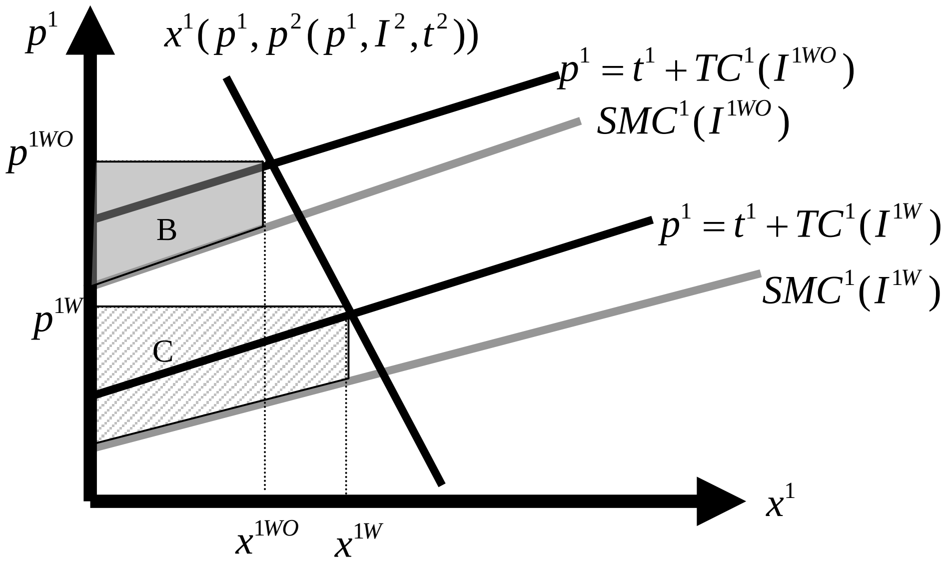

Figure 2-2：Producer’s Surplus in Route 1

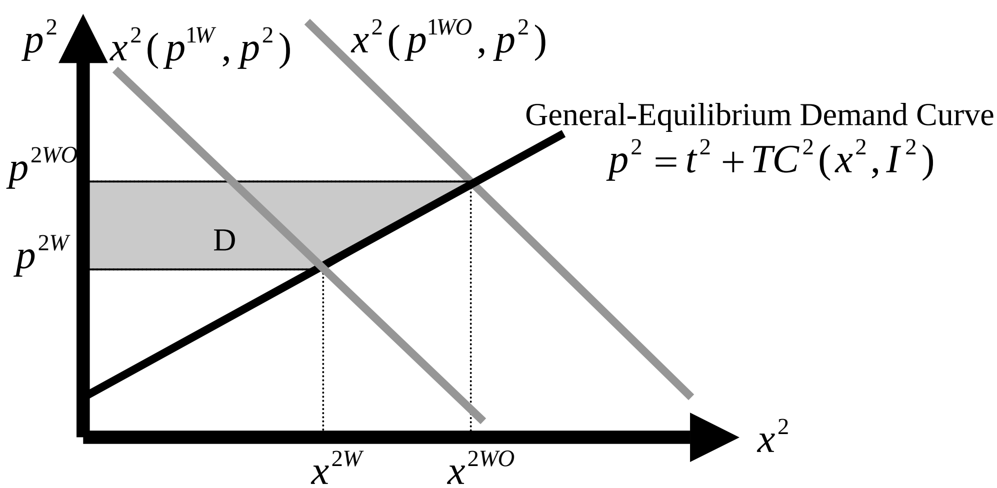

Figure 2-3：Consumer’s Surplus in Route 2

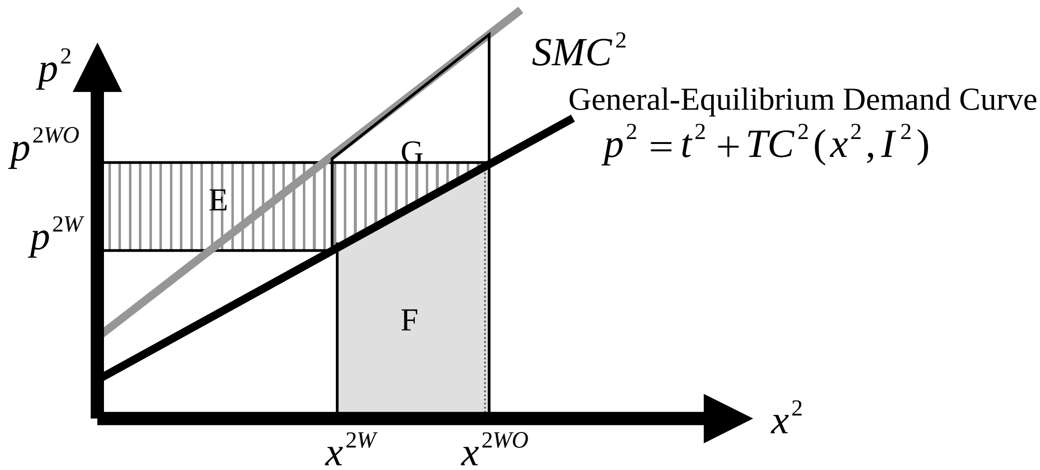

Figure 2-4：Producer’s Surplus in Route 2

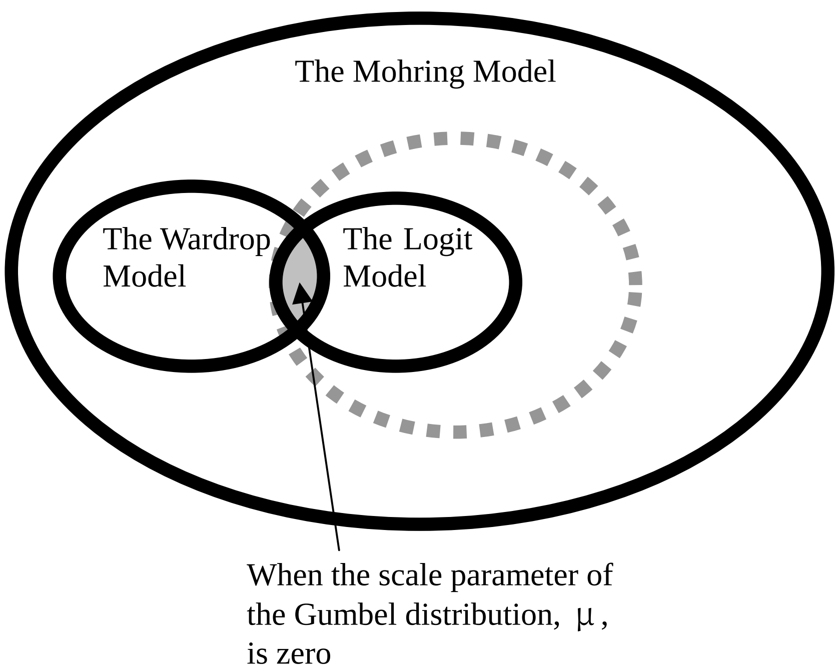

Figure 3：The Relationships between the Mohring Model, the Wardrop Model, and the Logit Model

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