

**THE BASICS OF A NETWORK ECONOMY**

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**Abstract**

Using a simple general equilibrium model with an endogenously evolving communications network, we demonstrate the basic characteristics of a network-based economy. First, we characterize the first-best solutions, compare them with the solutions under a laissez-faire economy, and show that in a laissez-faire economy, the communications network contracts and causes subsequent distortions to wages, firm size, and the number of firms. Second, we examine the effects of representative policies, such as a subsidy for the price of communications services, an income transfer, and a subsidy to firms, and evaluate these policies in terms of the total social surplus.

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## I Introduction

The Internet grows inexorably. According to the Ministry of Public Management, Home Affairs, Posts and Telecommunications [2002], the number of people worldwide to access the Internet surged from 26 million in 1995 to 544 million in 2002. A major reason for this rapid increase is that the Internet enables us to communicate with one another easily and cheaply. Suppose that you want to read the mimeographed paper of a certain professor. If you cannot access the Internet, you have to mail a letter asking the professor to send the paper. This exchange of letters would take a long time. If the professor is too busy to send the paper, either you write the letter again or you wait. Again, this process would be time-consuming. If both the professor and you can access the Internet, the situation changes dramatically. You simply have to e-mail the professor, who will send the paper to you as an attached file within a very short time. You may not even need to send an e-mail. If the professor has a homepage that includes the papers, you can download them to read at your leisure.

This paper examines the basic characteristics of an economy with a communications network and various policies that influence the development of communications networks and economic performance. To do this, we build a simple general equilibrium model with an endogenously evolving communications network. Our model has two types of consumer, three types of firm, and the government. The type of consumer that communicates with others using a communications service provided via a network is the type-IT consumer, while the type that does not is the type-NIT consumer. An increase in the number of type-IT consumers, that is, an increase in the network size, yields benefits to all type-IT consumers, because such an enlargement expands communication possibilities for each type-IT consumer. A type-NIT consumer can freely become a

type-IT consumer by receiving education, but the cost of education to become a type-IT consumer differs between individuals. The government partly subsidizes the education to become a type-IT consumer. Firms are classified into those that produce the composite consumer good, and a monopoly that provides the communications service. The former consist of type-IT firms, which hire type-IT consumers, and type-NIT firms, which hire type-NIT consumers, and produce the composite consumer good using consumers' information as an input. Knowledge useful for type-IT firm production is assumed to be increasing with network size, on the grounds that type-IT consumers accumulate information by communicating with one another. Thus, communication between type-IT consumers has positive externality on type-IT firm production, which increases the income of type-IT consumers.

First, we characterize the first-best outcome. In our model, the size of the network is determined by the price of the communications service and the government's subsidy for the cost of education required to become a type-IT consumer. In the first-best outcome, the total price of a communications service, which is the sum of the prices that a sender and receiver of information pay, equals the marginal cost. The subsidy for the cost of education is positive, that is, the private burden of the cost of education must be lower than the true cost of education, because an additional type-IT consumer generates positive externalities on the utility of other type-IT consumers and on the production of type-IT firms.

Second, as an extreme case, we characterize the solutions under the *laissez-faire* economy, in which there is no government intervention. In this case, the price of the communications service is higher than its marginal cost, due to monopoly pricing by the provider of the communications service. The private burden of the cost of education

equals its true cost, because the subsidy for the cost of education is zero. The high price of the communications service and the low subsidy for the cost of education combine to cause the communications network to contract in the laissez-faire economy. The small network size means a decrease in the number of type-IT consumers and an increase in the number of type-NIT consumers. Since the wage of type-NIT consumers and the number of employees in each type-NIT firm do not depend on the network size, the number of type-NIT firms, which is defined as the number of type-NIT consumers over the number of employees in each type-NIT firm, increases. The small network size also implies fewer externalities on type-IT firm production, which decreases the wage of type-IT consumers and increases the number of employees in each type-IT firm. A decrease in the number of type-IT consumers and an increase in the number of employees in each type-IT firm result in a decrease in the number of type-IT firms, which is defined as the number of type-IT consumers over the number of employees in each type-IT firm.

Third, we examine the effects of various policies in a situation in which the existing network is below optimal due to a higher price of the communications service or a lower subsidy for the cost of education. The major policies we focus on are: i) a subsidy for the price of the communications service; ii) an income transfer from type-IT consumers to type-NIT consumers; iii) an increase in the government's subsidy for employment in type-IT firms; and iv) an increase in the government's subsidy for employment in type-NIT firms. We consider the effects of these policies on network size, income, employment, and the number of firms, and we show whether these policies are beneficial or not in terms of the social surplus.

Our model is unique in that it can deal with various changes in economic performance brought about by an endogenous evolution of the communications network

within a consistent theoretical framework. However, our model does relate to other papers in the literature, and before proceeding, we briefly link our study to this existing literature.

First, our study is related to the literature on network externalities. In our model, an evolution of the communications network yields positive externalities in the form of increased utility and income of type-IT consumers. These positive externalities justify a subsidy for the cost of education and make the optimal private burden of the cost of education lower than its true cost. This result is an application of the studies of Squire [1973], Littlechild [1975], and Kanemoto [1990, 2001], which, in the context of telecommunications industries, show that the optimal fixed tariff in a two-part tariff mechanism is lower than the true fixed cost.

Second, this paper is related to the literature on the relationship between information technology (IT) and the labor market. For example, Krueger [1993], Doms et al. [1997], Berman et al. [1998], and Machin and van Reenan [1998] find that an increase in the demand for skilled workers coexists with an increase in the income of skilled workers in the United States and other OECD countries. Autor et al. [1998] and Bresnahan [1999] consider the possibility that IT generates wage inequality. Although our model does not focus on IT investment as a whole but on the evolution of communications networks through IT, our results are consistent with these arguments. In our model, evolution of the communications network, that is, an increase in the number of type-IT consumers increases both the number of type-IT employees and their wages. Since an expansion of the communications network does not change the wages of type-NIT consumers, an increase in the wages of type-IT consumers through an evolution of the communications network increases wage inequality. Acemogle [1998] and Galor and Moav [2000],

utilizing a different approach to explain wage inequality, use growth theory to demonstrate that technological evolution causes wage inequality. Our model shows that we can rationally explain wage inequality using simply a one-shot model, provided we take into account the evolution of communications networks.

Third, our study is related to the literature on the relationship between IT and the organization of the firm. Brynjolfsson et al. [1994] shows empirically that IT investments reduce firm employment. Brynjolfsson and Hitt [1998], Brynjolfsson and Hitt [2000], and Bresnahan et al. [2002] argue that IT, skilled labor, and work organization are complements, and suggest that the decentralization of work organization is necessary for the utilization of IT. However, these arguments lack a theoretical foundation. The only exception is Brynjolfsson [1994], which uses principal-agent theory to explain the relationship between IT and the number of employees in the firm. The problem with this approach is that it ignores effects on other aspects of firm performance, such as wages. Our general equilibrium set-up shows that evolution of the communications network, that is, an increase in the number of type-IT employees, decreases the number of type-IT employees per firm (that is, makes work organization smaller), and raises the incomes of type-IT employees. Thus, our model provides a theoretical foundation for the arguments advanced in this branch of the existing literature.

The rest of the paper is structured as follows. In Section 2, we build the model. In Section 3, we derive the first-best outcome and compare it to the solution under the laissez-faire economy. In Section 4, we examine the effects of various policies. Section 5 concludes.

## II Model

Our model consists of two types of consumer, two types of firm that produce the composite consumer good, a provider of a communications service, and the government. Let us define these players in turn.

### 1. Consumers

Consumers are indexed by  $i \in [0,1]$ . We ignore the integer problem. Consumers are classified into two types: type-IT consumers use the communications service, say e-mail, and the composite consumer good, while type-NIT consumers only consume the composite consumer good. We assume that type-NIT consumers can freely become type-IT consumers by receiving education.

A type-IT consumer communicates with other type-IT consumers. Type-IT consumers are distributed in the range  $[0,n]$ . The utility function of type-IT consumer  $i$  is assumed to be separable in the quantity of information to be sent,  $x_{ij}^s$ , the quantity of information to be received,  $x_{ij}^r$ , and the composite consumer good,  $z$ , the price of which is normalized to be unity. The quantities of information to be sent and to be received,  $x_{ij}^s$  and  $x_{ij}^r$ , can be thought of as the frequencies of sending and receiving information. The utility function of type-IT consumer  $i$  is

$$U^{IT} = z + \int_{j=0}^n u^s(x_{ij}^s) dj + \int_{j=0}^n u^r(x_{ij}^r) dj, \quad (1)$$

where  $u^s(x_{ij}^s)$  is the utility obtained by type-IT consumer  $i$  when he or she sends information, whose volume is  $x_{ij}^s$  (or at frequency  $x_{ij}^s$ ), to consumer  $j$ , and  $u^r(x_{ij}^r)$  is  $i$ 's utility when he or she receives information, whose volume is  $x_{ij}^r$  (or at frequency  $x_{ij}^r$ ),

from consumer  $j$ . For tractability, we assume that the sub-utility function regarding sending information,  $u^s(x_{ij}^s)$ , has the same form as the sub-utility function regarding receiving information,  $u^r(x_{ij}^r)$ , that is,  $u^s(\bullet) = u^r(\bullet) = u(\bullet)$  where  $u(0) = 0$ ,  $u'(\bullet) > 0$ , and  $u''(\bullet) < 0$ . We further assume that the utility obtained by type-IT consumer  $i$  by communicating does not depend on who is communicated with; that is, for type-IT consumer  $i$ , all other type-IT consumers are symmetric. These assumptions simplify the utility function of type-IT consumer  $i$  to

$$U^{iIT} = z + n\{u(x_i^s) + u(x_i^r)\}. \quad (2)$$

The budget constraint of type-IT consumer  $i$  is

$$w^{iIT} = z + p^s \int_{j=0}^n x_i^s dj + p^r \int_{j=0}^n x_i^r dj + e(i) - t \quad (3)$$

where  $w^{iIT}$ ,  $p^s$ ,  $p^r$ ,  $e(i)$ , and  $t$  respectively denote the income of type-IT consumer  $i$ , the unit price of sending information, the unit price of receiving information,  $i$ 's cost of education incurred in becoming a type-IT consumer, and the government's education subsidy. We simplify our analysis by assuming that all type-IT consumers receive the same wage, that is,  $w^{iIT} = w^{IT}$ . Since we have assumed that all other type-IT consumers are symmetric for type-IT consumer  $i$ , the unit prices of sending and receiving information,  $p^s$  and  $p^r$ , are independent of the person communicated with, consumer  $j$ . The unit prices of sending and receiving information,  $p^s$  and  $p^r$ , can be thought of as the unit access cost to connecting to a network, such as the Internet. Equation (3) implies that if we want to send or receive a larger volume of information or if we want to send or receive information more frequently, we have to pay more. For the sake of simplicity, the unit prices of sending and receiving information are assumed to be symmetric. Thus, the

unit price of sending information is the same as that of receiving information, that is,  $p^s = p^r = p$ . Consequently, (3) reduces to

$$w^{IT} = z + pn(x_i^s + x_i^r) + e(i) - t. \quad (4)$$

Regarding  $i$ 's cost of education, we make the following assumptions: i)  $i$ 's cost of education to become a type-IT consumer,  $e(i)$ , equals its true social cost; and ii)  $i$ 's cost of education becomes higher as  $i$  approaches  $n$ , that is,

$$e_i > 0. \quad (5)$$

The first assumption implies that there are no profits in the sector that provides the education required to become a type-IT consumer, and the second assumption means that the smaller the consumer's index,  $i$ , the lower the cost of education and the greater the willingness to enter the communications network.

A type-IT consumer maximizes his or her utility, (2), with respect to  $z$  and  $x_i^s$ , subject to the budget constraint, (4). In communication by e-mail, a person normally receives all incoming e-mails by paying his or her own access costs and spending his or her own time, even if the messages include unsolicited e-mail ("spam", or "junk e-mail"). Thus, we assume that the receiver of information cannot sort out the incoming information. Consequently,  $x_i^r$  is chosen by other consumers, and hence cannot be controlled by type-IT consumers,  $i$ . The first-order condition for type-IT consumer  $i$ 's maximization problem is

$$nu'(x_i^s) - pn = 0, \quad (6)$$

from which, we obtain

$$x_i^s = x^s(p) \quad (7)$$

where

$$x_p^s \equiv \frac{\partial x^s}{\partial p} = \frac{1}{u''} < 0. \quad (8)$$

From (7), we know that the volume of information to be sent (or the frequency of sending information) is independent of  $i$ , and depends only on the unit price of the communications service. This implies

$$x_i^s = x_i^r \equiv x(p). \quad (9)$$

Type-NIT consumers, who are distributed in the range  $[n, 1]$ , have the same utility function, which includes only the quantity of the composite consumer good, that is,

$$U^{NIT} = z. \quad (10)$$

A type-NIT consumer spends all of his or her income,  $w^{NIT}$ , which is assumed to be common to all type-NIT consumers, on the composite consumer good, and consequently,

$$U^{NIT} = w^{NIT}. \quad (11)$$

The total consumer surplus, denoted by  $CS$ , can be written as

$$CS = \int_{i=0}^n U^{IT} di + \int_{i=n}^1 U^{iNIT} di \quad (12)$$

The marginal, that is,  $n$ th, subscriber to the communications network attains the same utility level whether he or she becomes a type-IT consumer or a type-NIT consumer.

That is,  $U^{IT} = U^{NIT}$ , which yields

$$w^{IT} - 2pnx(p) - e(n) + t + 2nu(x(p)) = w^{NIT}. \quad (13)$$

Denoting the marginal benefit of becoming a type-IT consumer by  $MB \equiv w^{IT} + 2n\{u(x(p)) - px(p)\} - \{e(n) - t\}$ , we can rearrange (13) as

$$MB = w^{NIT}. \quad (14)$$

To ensure a unique and stable equilibrium, the marginal benefit of becoming a type-IT consumer,  $MB$ , must be decreasing with respect to network size, that is,

$$MB_n < 0. \quad (15)$$

(See Figure 1.) If this condition is not met, there may be multiple equilibria.<sup>1</sup> However, the purpose of this paper is not to show the existence of multiple equilibria, but to show how the evolution of a communications network affects economic performance. Thus, in the following analysis, we assume  $MB_n < 0$ , for which the increasing cost of education relating to  $i$ , (5), is a necessary condition.

## 2. z-producing Firms

Two types of firm produce the composite consumer good,  $z$ . Type-IT firms are assumed to employ only type-IT consumers and produce  $z$  using their knowledge. Type-IT consumers' knowledge level is  $k(n)$ , where  $k'(n) > 0$ . This assumption implies that the exchange of information via communication extends type-IT consumers' knowledge. We assume that knowledge that is useful for type-IT firm production per type-IT employee,  $\alpha$ , is increasing with the knowledge level, that is,  $\alpha(n) \equiv l(k(n))$ , where  $l'(\bullet) > 0$  and consequently,  $\alpha'(n) > 0$ .

Type-NIT firms are assumed to hire only type-NIT consumers and produce  $z$  using their knowledge. The knowledge level of type-NIT consumers is fixed, because these consumers do not upgrade their knowledge by communicating with each other. Thus, useful knowledge for type-NIT consumers is also constant. For simplicity, useful knowledge for type-NIT consumers is normalized to be unity.

Since the price of the composite consumer good is unity, the profits of the type-IT firm can be written as

$$\pi^{IT} = f(\alpha(n)L^{IT}) - w^{IT}L^{IT}, \quad (16)$$

where  $L^{IT}$  is the number of employees that the type-IT firm hires, and consequently,

$\alpha(n)L^{IT}$  is the total amount of knowledge that the type-IT firm uses for its production.

The profits of the type-NIT firm are

$$\pi^{NIT} = g(L^{NIT}) - w^{NIT} L^{NIT}, \quad (17)$$

where  $L^{NIT}$  is the number of employees that the type-NIT firm hires. Since useful knowledge for type-NIT firms is assumed fixed at unity,  $L^{NIT}$  is also the total amount of knowledge that the type-NIT firm uses for its production.

The composite consumer good,  $z$ , can be costlessly imported from or exported to the rest of the economy, which is not modeled here, and the  $z$ -producing industry is assumed to be sufficiently competitive. Thus, in equilibrium, we obtain

$$\pi^{IT} = \pi^{NIT} = 0. \quad (18)$$

Denoting the number of type-IT firms and the number of type-NIT firms by  $m^{IT}$  and  $m^{NIT}$  respectively, we obtain

$$m^{IT} \equiv \frac{n}{L^{IT}}, \quad (19)$$

$$m^{NIT} \equiv \frac{1-n}{L^{NIT}}. \quad (20)$$

### 3. Provider of the Communications Service

For the sake of simplicity, the communications service is assumed to be supplied by a monopoly. Recall from (4) that each type-IT consumer pays  $2pnx(p)$  when connecting to the network to send or receive information. We assume that the unit marginal cost of sending information from  $i$  to  $j$  is independent of  $i$  and  $j$  and constant at  $c$ . Thus, the profits of a monopolistic provider of the communications service,  $\pi^{COM}$ , can be written as

$$\begin{aligned}\pi^{COM} &= \int_{i=0}^n 2pnx(p)di - \int_{i=0}^n cnx(p)di - F \\ &= (2p - c)n^2x(p) - F,\end{aligned}\tag{21}$$

where  $F$  is the fixed cost of providing the service.<sup>2</sup>

#### 4. Government

The government pays a subsidy for the education required to become a type-IT consumer,  $t$ . The government's surplus is

$$G = -tn.\tag{22}$$

As long as the government pays a positive subsidy, it has a budget deficit, which is assumed to be financed by non-distortional lump-sum transfers. Thus, we ignore distortions arising from collecting taxes.

#### 5. The Total Social Surplus

From (12), (18), (21), and (22), the total social surplus, denoted by  $TSS$ , can be written as

$$TSS = CS + \pi^{COM} + G.\tag{23}$$

### **III First-Best vs. Laissez-faire**

For the sake of convenience, let us begin by obtaining the relationship between the network size,  $n$ , and the performance of  $z$ -producing firms. From profit maximization by  $z$ -producing firms, we obtain the following Lemma 1.

**Lemma 1**

- i)  $w_n^{IT}(n) > 0$  and  $w^{NIT}$  is constant.
- ii)  $L_n^{IT}(n) < 0$  and  $L^{NIT}$  is constant.
- iii)  $m_n^{IT}(n) > 0$  and  $m_n^{NIT}(n) < 0$ .

The proof is shown in the Appendix.

Lemma 1-i) shows that the income of type-IT consumers is an increasing function of network size, that is, the number of type-IT consumers, and that the income of type-NIT consumers is fixed. Thus, the wage gap between type-IT and type-NIT consumers increases as the communications network develops. This results from  $\alpha'(n) > 0$ . Type-IT consumers' increased knowledge level, facilitated by evolution of the communications network, benefits type-IT firm production. Consequently, the wage of type-IT employees, that is, the income of type-IT consumers, increases as the communications network expands. Since knowledge that is useful for type-NIT firm production is fixed at unity per type-NIT employee, the income of type-NIT firms is irrelevant to network size.

Lemma 1-ii) shows that the number of employees per type-IT firm is a decreasing function of network size and the number of employees per type-NIT firm is fixed. This results from Lemma 1-i). As the network expands, the wage paid by type-IT firms rises, and consequently, the number of employees per type-IT firm decreases. Since the wage paid by type-NIT firms is irrelevant to network size, the number of employees per type-NIT firm is also independent of network size.

Lemma 1-iii) focuses on the number of firms. As the network expands, the number of type-IT consumers increases and the number of employees per type-IT firm decreases.

This implies that the number of type-IT firms increases. By contrast, since an expansion of the network reduces the number of type-NIT consumers without affecting the number of employees per type-NIT firm, the number of type-NIT firms decreases.

Next, we obtain the effects of the unit price of the communications service and the government subsidy for education on the network size as given by Lemma 2.

**Lemma 2**

- i)  $n_p \equiv \frac{\partial n(p,t)}{\partial p} < 0$
- ii)  $n_t \equiv \frac{\partial n(p,t)}{\partial t} > 0$

Lemma 2 shows that the communications network shrinks as the unit price of the communications service rises, and expands as the government subsidy for education increases. The high unit price of the communications service lowers the marginal benefit of entering the network, and consequently causes the network to contract, as shown in Figure 2. The low burden of the cost of education from a high government subsidy raises the marginal benefit of entering the network, and therefore expands the network as shown in Figure 3.

Now let us derive the first-best value of the unit price of the communications service and the government subsidy for education by maximizing the total social surplus, (23).

**Proposition 1**

In the first best, we obtain:

- i)  $p^* = \frac{1}{2}c$ ,

$$\text{ii) } t^* = 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} > 0,$$

where the superscript \* denotes the first best outcome.

In the first-best outcome, the unit price of the communications service is half of its marginal cost, that is, the sum of the prices that a sender and a receiver pay equals the marginal cost of the communications service. This result is an application of the studies of telecommunications by Littlechild [1975] and Kanemoto [1990, 2001], which show that the optimal unit price per communications service is equal to its marginal cost, although these models do not explicitly consider the benefits of communication to the receiver.

Second, we focus on the government subsidy for education. In the first-best outcome, the government subsidy for education is positive. This is because an additional entrant to the communications network generates two positive externalities, one of which affects the utility, the other the income, of existing type-IT consumers. If consumer  $i$  becomes a type-IT consumer, existing type-IT consumers can communicate with consumer  $i$ , and consequently, existing customers obtain greater utility both as senders and receivers of information. That is, new type-IT consumers generate a positive externality in the form of higher utility for all type-IT consumers, which is given by  $2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x}$ . The other externality affects the income of type-IT consumers. As a result of accumulating information by communicating with consumer  $i$ , type-IT consumers are paid a higher wage by type-IT firms, and thus have a higher income. This means that new type-IT consumers generate a positive externality in the form of greater income for all type-IT

consumers, which is given by  $nw_n^{IT}$ . Due to these two positive externalities, the socially optimal subsidy for education is positive.

Next, we consider the laissez-faire economy, in which there is no government intervention. In this case, the total payment for the communications service is higher than its marginal cost due to monopoly pricing, and the government subsidy for education is zero. Denoting the equilibrium under laissez-faire economy by  $\hat{\cdot}$ , we derive the following proposition.

**Proposition 2**

When  $\hat{p} > p^*$  and  $\hat{t} = 0$ ,

- i)  $\hat{n} < n^*$ ,
- ii)  $\hat{w}^{IT} < w^{IT*}$ ,  $\hat{w}^{NIT} = w^{NIT*}$ ,
- iii)  $\hat{L}^{IT} > L^{IT*}$ ,  $\hat{L}^{NIT} = L^{NIT*}$ ,
- iv)  $\hat{m}^{IT} < m^{IT*}$ ,  $\hat{m}^{NIT} > m^{NIT*}$ .

(See Appendix for proof.)

Since the price of the communications service is higher and the government subsidy lower in the laissez-faire economy than in the first-best outcome, the network is below its optimal size in the laissez-faire economy. The smaller network lowers the income of type-IT consumers and increases the number of employees per type-IT firm. Since in the laissez-faire economy, the number of type-IT consumers decreases and the number of employees per type-IT firm increases, relative to the first-best outcome, the number of type-IT firms decreases. By contrast, the income of type-NIT consumers and the number

of type-NIT consumers per type-NIT firm is irrelevant to the size of the network. A constant number of employees per type-NIT firm and a larger number of type-NIT consumers in the laissez-faire economy mean that the number of type-NIT firms is larger than in the first-best outcome.

#### **IV Effects of Various Policies**

In Proposition 2, we have shown that in the laissez-faire economy, which is characterized by monopoly pricing and no education subsidy, network size decreases and consequently, distortions arise. Although Proposition 2 focuses on an extreme case, this result holds as long as either the total payment for the communications service is higher than its marginal cost or the government subsidy for education is inadequate. In reality, we have no reason to believe that the provision of communications services is competitive enough to result in marginal-cost pricing or that government subsidies for education are determined by taking into account positive externalities on type-IT consumers and type-IT firms. We then look for desirable policies when the size of the communications network is below optimal due to price exceeding marginal cost and a lower subsidy for education.

First, we consider a subsidy for the communications service. Here, we focus on a subsidy for the unit price of the communications service,  $\tau_1$ , to decrease type-IT consumers' communication costs. In this case, the unit price of the communications service for type-IT consumers becomes  $p - \tau_1$ . The results are stated as Proposition 3.

#### **Proposition 3**

A subsidy for the unit price of the communications service,  $\tau_1$ , expands  $n$ , increases

$w^{IT}$  and  $m^{IT}$ , and decreases  $L^{IT}$  and  $m^{NIT}$ . This subsidy raises the total social surplus, as long as  $\tau_1 < p - \frac{1}{2}c$ .

(See Appendix for proof.)

A subsidy for the unit price of the communications service enlarges the communications network, that is, increases the number of type-IT consumers, because it increases the benefits of entering the network. From Lemma 1, an evolution of the communications network increases the incomes of type-IT consumers and reduces the number of employees per type-IT firm, and consequently increases the number of type-IT firms. The incomes of type-NIT consumers and the number of employees per type-NIT firm remain unchanged, and consequently the number of type-NIT firms decreases.

The net social benefits of a subsidy for the unit price of the communications service are classified into two types. These are, first, the direct benefits caused by a decrease in the price of the communications service for type-IT consumers, and second, the indirect benefits that derive from an evolution of the communications network. The direct benefits are those caused by an increase in the volume of information being sent or received (or the frequencies of sending or receiving information) brought about by a decrease in the price of the communications service from  $p$  to  $p - \tau_1$ . These direct benefits are positive as long as the total payment for the communication service,  $2(p - \tau_1)$ , is higher than the marginal cost,  $c$ , that is,  $\tau_1 < p - \frac{1}{2}c$ . The indirect benefits are those due to an expansion of the network, which are positive when the size of the communications network is below optimal. Thus, when the size of the communications network is not sufficiently large, a subsidy for the price of the communications service is

welfare-improving as long as the total payment for the communications service is higher than the marginal cost.

The essence of Proposition 3 is an expansion of the network through a decrease in the price of the communications service. Obviously, the same result holds when the price of the communications service is regulated by a price-cap,  $\bar{p}$ . The results are stated as Corollary 1.

### **Corollary 1**

Price-cap regulation expands  $n$ , increases  $w^{IT}$  and  $m^{IT}$ , and decreases  $L^{IT}$  and  $m^{NIT}$ . This regulation raises the total social surplus, as long as  $\bar{p} > \frac{1}{2}c$ .

The difference between Proposition 3 and Corollary 1 is in the distribution of surplus. In Proposition 3, in which a subsidy is given to the unit price of the communications service, type-IT consumers gain benefits from a decrease in the price in exchange for the government deficit. On the contrary, in Corollary 1, in which a price-cap regulation is imposed, type-IT consumers also gain benefits, but these benefits are due to a decrease in the profits of the provider of the communications service. In our general equilibrium model, these two policies yield the same total social surplus. In reality, however, price-cap regulation would be a more desirable policy, because it would generate incentives for providers of communications services to reduce costs, and thereby additional benefits. (These cost-reducing incentives are not modeled here.)

Second, we deal with an income transfer from type-IT consumers to type-NIT consumers. For the sake of simplicity, we focus on the following mechanism: each

type-IT consumer pays a tax,  $\tau_2$ , on his or her income,  $w^{IT}$ , and the total amount of the collected tax,  $n\tau_2 w^{IT}$ , is shared equally between all type-NIT consumers, who each receive  $\frac{n\tau_2 w^{IT}}{1-n}$ . The after-tax incomes of a type-IT consumer and a type-NIT consumer are denoted by  $w^{ITA} \equiv (1-\tau_2)w^{IT}$  and  $w^{NITA} \equiv w^{NIT} + \frac{n\tau_2 w^{IT}}{1-n}$  respectively. The results are stated as Proposition 4.

#### **Proposition 4**

An income transfer from type-IT consumers to type-NIT consumers causes  $n$  to contract, increases  $L^{IT}$  and  $m^{NIT}$ , and decreases  $w^{ITA}$  and  $m^{IT}$ . Whether  $w^{NITA}$  increases or not is ambiguous. This income transfer lowers the total social surplus.

(See Appendix for proof.)

An income transfer from type-IT consumers to type-NIT consumers decreases incomes of type-IT consumers and increases those of type-NIT consumers at first. This income transfer implies that the benefits of becoming a type-IT consumer fall and those of remaining a type-NIT consumer rise. As a result, the communications network shrinks. Subsequently, a contraction of the communications network further reduces the incomes of type-IT consumers, which results in a lower income transfer per type-NIT consumer,  $\frac{n\tau_2 w^{IT}}{1-n}$ . In sum, an income transfer from type-IT consumers to type-NIT consumers always reduces incomes of type-IT consumers, but its effect on the incomes of type-NIT

consumers is ambiguous. A decrease in the incomes of type-IT consumers results in an increase in the number of employees per type-IT firm, which together with a decrease in the number of type-IT consumers, implies a decrease in the number of type-IT firms. Since the number of employees per type-NIT firm remains unchanged, a contraction of the communications network means an increase in the number of type-NIT firms. An income transfer from type-IT consumers to type-NIT consumers decreases the total social surplus through a contraction of the network when network size is below optimal.

Proposition 4 suggests that such an income equalization policy is harmful for the total social surplus, because it doubly distorts the incentives to become a type-IT consumer: a tax for type-IT consumers decreases the benefits of entering the network and a subsidy for type-NIT consumers increases the opportunity costs of entering the network. This policy may even harm type-NIT consumers, who are supposed to receive benefits if the income transfer is large. A contraction of the network reduces the incomes of type-IT consumers, and consequently reduces the income transfer to a type-NIT consumer,  $\frac{n\tau_2 w^{IT}}{1-n}$ , given  $\tau_2$ . When  $\tau_2$  is high enough, a decrease in  $\frac{n\tau_2 w^{IT}}{1-n}$  by a contraction of the network is likely to upset an increase in it by an increase in  $\tau_2$ , and in this case, an additional income transfer to type-NIT consumers lowers their incomes and utility. The upshot is that this policy further magnifies the distortions stemming from a small network size, without guaranteed benefits even to type-NIT consumers.

Third, we consider policies for the labor market, which are aimed at improving the total social surplus. We here focus on a subsidy,  $\tau_3$ , per employee in type-IT firms. The

results are stated as Proposition 5.

**Proposition 5**

An increase in the government's subsidy per employee in type-IT firms,  $\tau_3$ , expands  $n$ , increases  $w^{IT}$  and  $m^{IT}$ , and decreases  $L^{IT}$  and  $m^{NIT}$ . This subsidy increases the total social surplus.

(See Appendix for proof.)

The government's subsidy per employee in type-IT firms,  $\tau_3$ , increases the wage paid by type-IT firms, that is, the incomes of type-IT consumers, by the full amount,  $\tau_3$ . Initially, the net wage paid by type-IT firms is unchanged, and hence so too is the number of employees per type-IT firm. Subsequently, however, the increase in the wage of type-IT consumers increases the incentive to enter the communications network, which thus expands. The development in the communications network further raises the wage of type-IT consumers and decreases the number of employees per type-IT firm. Since the total number of type-IT consumers increases while the number of employees per type-IT firm decreases, the number of type-IT firms increases. A decrease in the total number of type-NIT consumers reduces the number of type-NIT firms, because the incomes of type-NIT consumers and the number of employees per type-NIT firm remain unchanged.

The essential point in Proposition 5 is that an increase in the incomes of type-IT consumers through this subsidy in effect lowers the private burden of the cost of education from  $e(i) - t$  to  $e(i) - t - \tau_3$ . Thus, this subsidy is equivalent to a subsidy for education. This subsidy increases the total social surplus when the size of the network is below optimal. Proposition 5 suggests that subsidies for IT-related workers are justified

as a substitute when the subsidies for the cost of education are inadequate.

On the contrary, for comparison, let us examine the case where the government gives the subsidy,  $\tau_4$ , per employee in type-NIT firms. The results are stated as Proposition 6.

**Proposition 6**

An increase in the government's subsidy per employee in type-NIT firms,  $\tau_4$ , contracts  $n$ , increases  $w^{NIT}$ ,  $L^{IT}$ , and  $m^{NIT}$ , and decreases  $w^{IT}$  and  $m^{IT}$ . This subsidy decreases the total social surplus.

(See Appendix for proof.)

The government subsidy of employment in type-NIT firms,  $\tau_4$ , increases the wage paid by type-NIT firms, that is, the incomes of type-NIT consumers, by the full amount,  $\tau_4$ . Since the net wage paid by type-NIT firms is unchanged, the number of employees per type-NIT firm remains the same. Since an increase in the wage of type-NIT consumers means an increase in the opportunity costs of becoming a type-IT consumer, the communications network shrinks. Applying Lemma 1, we know that with regard to the wage of type-IT consumers, the number of employees per type-IT firm, the number of type-IT firms, and the number of type-NIT firms, the government subsidy per employee in type-NIT firms yields opposite effects to those of Proposition 5.

An increase in the incomes of type-NIT consumers through this subsidy in effect raises the private burden of the cost of the education required to enter the network from  $e(i) - t$  to  $e(i) - t + \tau_4$ . Accordingly, this subsidy adversely affects the size of the communications network and decreases the total social surplus when the network size is below optimal.

## V Concluding Remarks

In this paper, we built a general equilibrium model in which evolution of the communications network is explicitly considered. We also compared the solutions under laissez-faire with the first-best outcomes. In addition, we examined the effects of various policies on network size, income, the number of employees, and the number of firms. These policies included a subsidy for the price of the communications service, an income transfer from type-IT consumers to type-NIT consumers, and subsidies to type-IT and type-NIT firms. Here, we relate our results to actual policies and conclude our analysis.

First, policies in telecommunications markets are very important. As we showed in Proposition 3 and Corollary 1, a decrease in the price of communications services increases the total social surplus, as long as price exceeds marginal cost. Price-cap regulation for a monopolistic provider would be more desirable due to additional potential benefits stemming from cost-reducing efforts, but even in the case of a subsidy for the price of the communications service, the total social surplus increases. By providing this subsidy, the government runs a budget deficit. However, in terms of the total social surplus, this deficit is completely offset by the reduced payments made by consumers to the provider of the communications service. The remaining effect is an expansion of the communications network, which increases the total social surplus. Thus, lowering the price of communications services is beneficial. Although our model does not explicitly correspond to the fixed (or lump-sum) payment system that has become more common in broadband communications, the mechanisms described in this paper would be applicable even in that case, at least approximately. Thus, we could suggest desirable policy directions for broadband communications. If broadband networks

prevail, the total social surplus remains low, as long as the price of communications services via broadband networks is high. To increase the total social surplus, the access price to broadband networks should be kept low, at the marginal cost if possible, through competition policies and subsidies.

Second, our results provide a perspective on the so-called digital divide problem.<sup>3</sup> As we showed in Proposition 2, in the laissez-faire economy, in which the price of the communications service is higher than its marginal cost and in which consumers have to pay the entire cost of education to become type-IT consumers, the size of the communications network is below its optimal. Since evolution of the network has positive externalities on the utility and income of all type-IT consumers, policies to evolve communications networks are needed for a higher social surplus. This result suggests that preventing the digital divide is justified on the grounds of efficiency, rather than on the grounds of equity.

Third, our results suggest desirable policy directions for labor markets and education. To relieve the distortions stemming from an underdeveloped network, not only is a low price of communications services needed, but so too is a subsidy for the cost of education in order to reduce the private burden of the cost of education. Proposition 5 demonstrates that a subsidy for employees in type-IT firms increases the total social surplus and can be used as a substitute for a subsidy for the cost of education, because it raises the benefits of becoming a type-IT consumer. On the contrary, Proposition 6 shows that this mechanism is completely reversed in the case of a subsidy for employees in type-NIT firms. An income equalization policy in Proposition 4 produces a worse outcome, because it doubly lowers the benefits of becoming a type-IT consumer by transferring income from type-IT consumers to type-NIT consumers. To develop communications networks and increase

the total social surplus, government policy must be designed to increase the benefits of being a type-IT consumer and/or decrease those of being a type-NIT consumer.

## Appendix

### Proof of Lemma 1

From profit maximization by the type-IT firm, we obtain

$$\alpha(n)f'(\alpha(n)L^{IT}) - w^{IT} = 0. \quad (\text{A1})$$

The zero-profit condition, (18), gives us

$$f(\alpha(n)L^{IT}) - w^{IT}L^{IT} = 0. \quad (\text{A2})$$

Totally differentiating (A1) and (A2) yields

$$w_n^{IT}(n) = f'(\alpha(n)L^{IT})\alpha'(n) > 0, \quad (\text{A3})$$

$$L_n^{IT}(n) = -\frac{\alpha'(n)L^{IT}}{\alpha} < 0. \quad (\text{A4})$$

Profit maximization by the type-NIT firm yields

$$g'(L^{NIT}) - w^{NIT} = 0. \quad (\text{A5})$$

From the zero-profit condition, (18), we obtain

$$g(L^{NIT}) - w^{NIT}L^{NIT} = 0. \quad (\text{A6})$$

Solving (A5) and (A6), we obtain the fixed value of  $w^{NIT}$  and  $L^{NIT}$ .

The number of type-IT firms,  $m^{IT}$ , and the number of type-NIT firms,  $m^{NIT}$ , respectively satisfies

$$m_n^{IT}(n) = \frac{L^{IT} - nL_n^{IT}}{L^{IT^2}} > 0, \quad (\text{A7})$$

$$m_n^{NIT}(n) = -\frac{1}{L^{NIT}} < 0. \quad (\text{A8})$$

Q.E.D.

### Proof of Lemma 2

Totally differentiating (13), we obtain

$$n_p(p, t) = \frac{2nx}{MB_n} < 0, \quad (\text{A9})$$

$$n_t(p, t) = -\frac{1}{MB_n} > 0. \quad (\text{A10})$$

Q.E.D.

### Proof of Proposition 1

The total social surplus, which is defined in (23), can be reduced to

$$TSS(n(p, t), p) = nw^{IT} + (1-n)w^{NT} + n^2 \{2u(x(p)) - cx(p)\} - \int_{i=0}^n e(i)di - F. \quad (\text{A11})$$

from (2), (4), (9), (11), (12), (21), and (22). The first-order conditions are

$$TSS_p = TSS_1 n_p + TSS_2 = 0, \quad (\text{A12})$$

$$TSS_t = TSS_1 n_t = 0, \quad (\text{A13})$$

where  $TSS_i$  is the derivative of  $TSS$  with respect to the  $i$ th argument. From (A10) and (A13), we obtain

$$TSS_1 = 0. \quad (\text{A14})$$

Eqs. (A12) and (A14) yield

$$TSS_2 = 0, \quad (\text{A15})$$

from which we have

$$p^* = \frac{1}{2}c. \quad (\text{A16})$$

Rearranging (A14), we obtain

$$t^* = 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} > 0, \quad (\text{A17})$$

where  $u_{\hat{x}} - \frac{1}{2}c \geq 0$  from  $u_{\hat{x}} \geq p^* = \frac{1}{2}c$  and  $nw_n^{IT} > 0$  from (A3).

Q.E.D.

### Proof of Proposition 2

When  $\hat{p} > p^*$  and  $\hat{t} = 0$ , from (A9) and (A10), we have

$$\hat{n} < n^*. \quad (\text{A18})$$

From Lemma 1 and (A18), we obtain

$$\widehat{w}^{IT} < w^{IT*}, \quad (\text{A19})$$

$$\widehat{w}^{NIT} = w^{NIT*}, \quad (\text{A20})$$

$$\widehat{L}^{IT} > L^{IT*}, \quad (\text{A21})$$

$$\widehat{L}^{NIT} = L^{NIT*} \quad (\text{A22})$$

$$\widehat{m}^{IT} < m^{IT*}, \quad (\text{A23})$$

$$\widehat{m}^{NIT} > m^{NIT*}. \quad (\text{A24})$$

Q.E.D.

### Proof of Proposition 3

Defining the unit price of the communications service after a subsidy,  $\tilde{p}$ , by

$\tilde{p} \equiv p - \tau_1$ , (A9) now holds in terms of  $\tilde{p}$ , that is,

$$n_{\tilde{p}}(\tilde{p}, t) < 0. \quad (\text{A25})$$

From (A3), (A4), (A7), (A8), and (A25), we have

$$n_{\tau_1} = n_{\tilde{p}}(\tilde{p}, t)\tilde{p}_{\tau_1} = -n_{\tilde{p}}(\tilde{p}, t) > 0, \quad (\text{A26})$$

$$w_{\tau_1}^{IT}(n(\tilde{p}(p, \tau_1), t)) = w_n^{IT} n_{\tilde{p}} \tilde{p}_{\tau_1} = -w_n^{IT} n_{\tilde{p}} > 0, \quad (\text{A27})$$

$$L_{\tau_1}^{IT}(n(\tilde{p}(p, \tau_1), t)) = L_n^{IT} n_{\tilde{p}} \tilde{p}_{\tau_1} = -L_n^{IT} n_{\tilde{p}} < 0, \quad (\text{A28})$$

$$m_{\tau_1}^{IT}(n(\tilde{p}(p, \tau_1), t)) = m_n^{IT} n_{\tilde{p}} \tilde{p}_{\tau_1} = -m_n^{IT} n_{\tilde{p}} > 0, \quad (\text{A29})$$

$$m_{\tau_1}^{NIT}(n(\tilde{p}(p, \tau_1), t)) = m_n^{NIT} n_{\tilde{p}} \tilde{p}_{\tau_1} = -m_n^{NIT} n_{\tilde{p}} < 0. \quad (\text{A30})$$

The total social surplus in this case,  $TSS^1$ , can be written as

$$\begin{aligned} TSS^1 &= nw^{IT} + (1-n)w^{NIT} + 2n^2u(x(\tilde{p})) - 2\tilde{p}n^2x(\tilde{p}) + (2p-c)n^2x(\tilde{p}) - \int_{i=0}^n e(i)di - F - 2\tau_1n^2x(\tilde{p}) \\ &= nw^{IT} + (1-n)w^{NIT} + n^2 \{2u(x(\tilde{p})) - cx(\tilde{p})\} - \int_{i=0}^n e(i)di - F. \end{aligned} \quad (\text{A31})$$

From (A31), we obtain

$$\begin{aligned} \frac{dTSS^1(n(\tilde{p}, t), \tilde{p})}{d\tau_1} &= TSS_1^1 n_{\tilde{p}} \tilde{p}_{\tau_1} + TSS_2^1 \tilde{p}_{\tau_1} \\ &= - \left[ 2nx \left( \tilde{p} - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^{\tilde{x}} \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tilde{p}} - 2n^2 \left( \tilde{p} - \frac{1}{2}c \right) x_{\tilde{p}}. \end{aligned} \quad (\text{A32})$$

Since  $TSS_1^1 > 0$  when  $n < n^*$ , the term inside the square brackets is positive. Thus, from

(A25), we obtain

$$\frac{dTSS^1}{d\tau_1} > 0 \quad (\text{A33})$$

as long as  $\tilde{p} - \frac{1}{2}c > 0$ , that is,  $\tau_1 < p - \frac{1}{2}c$ .

Q.E.D.

### Proof of Corollary 1

We focus on the case in which price-cap regulation is binding, that is,  $p = \bar{p}$ . In this case, from (A3), (A4), (A7), (A8), and (A9), we have

$$n_{\bar{p}} = n_{\bar{p}}(\bar{p}, t) < 0, \quad (\text{A34})$$

$$w_{\bar{p}}^{IT}(n(\bar{p}, t)) = w_n^{IT} n_{\bar{p}} < 0, \quad (\text{A35})$$

$$L_{\bar{p}}^{IT}(n(\bar{p}, t)) = L_n^{IT} n_{\bar{p}} > 0, \quad (\text{A36})$$

$$m_{\bar{p}}^{IT}(n(\bar{p}, t)) = m_n^{IT} n_{\bar{p}} < 0, \quad (\text{A37})$$

$$m_{\bar{p}}^{NIT}(n(\bar{p}, t)) = m_n^{NIT} n_{\bar{p}} > 0. \quad (\text{A38})$$

From (A11), we obtain

$$\begin{aligned} \frac{dTSS(n(\bar{p}, t), \bar{p})}{d\bar{p}} &= TSS_1 n_{\bar{p}} + TSS_2 \\ &= \left[ 2nx \left( \bar{p} - \frac{1}{2}c \right) + 2n \int_{\bar{x}=0}^x \left( u_{\bar{x}} - \frac{1}{2}c \right) d\bar{x} + nw_n^{IT} - t \right] n_{\bar{p}} + 2n^2 \left( \bar{p} - \frac{1}{2}c \right) x_{\bar{p}}. \end{aligned} \quad (\text{A39})$$

Since  $TSS_1 > 0$  when  $n < n^*$ , the term inside the square brackets is positive. Thus, from

(A34), we obtain

$$\frac{dTSS}{d\bar{p}} < 0 \quad (\text{A40})$$

as long as  $\bar{p} > \frac{1}{2}c$ .

Q.E.D.

#### Proof of Proposition 4

An income transfer,  $\tau_2$ , changes (13) to

$$w^{ITA} - 2pnx(p) - e(n) + t + 2nu(x(p)) = w^{NITA}. \quad (\text{A41})$$

Totally differentiating (A41) yields

$$n_{\tau_2}(p, t, \tau_2) = \frac{w^{IT}}{MB_n + \Delta} < 0, \quad (\text{A42})$$

$$\text{where } \Delta = -\frac{\tau_2 \{w_n^{IT}(1-n) + w^{IT}\}}{(1-n)^2} < 0.$$

From (A3), (A4), (A7), (A8), and (A42), we have

$$w_{\tau_2}^{ITA}(n(p, t, \tau_2), \tau_2) = -w^{IT} + (1 - \tau_2)w_n^{IT}n_{\tau_2} < 0, \quad (\text{A43})$$

$$w_{\tau_2}^{NITA}(n(p, t, \tau_2), \tau_2) = \frac{nw^{IT}}{1-n} + \frac{n_{\tau_2}\tau_2 \{w^{IT} + (1-n)nw_n^{IT}\}}{(1-n)^2}, \quad (\text{A44})$$

$$L_{\tau_2}^{IT}(n(p, t, \tau_2)) = L_n^{IT}n_{\tau_2} > 0, \quad (\text{A45})$$

$$m_{\tau_2}^{IT}(n(p, t, \tau_2)) = m_n^{IT}n_{\tau_2} < 0, \quad (\text{A46})$$

$$m_{\tau_2}^{NIT}(n(p, t, \tau_2)) = m_n^{NIT}n_{\tau_2} > 0, \quad (\text{A47})$$

where the sign of  $w_{\tau_2}^{NITA}$  is ambiguous, because the first term is positive but the second term is negative.

The total social surplus in this case,  $TSS^2$ , can be written as

$$\begin{aligned} TSS^2 &= nw^{ITA} + (1-n)w^{NITA} + n^2 \{2u(x(p)) - cx(p)\} - \int_{i=0}^n e(i)di - F \\ &= nw^{IT} + (1-n)w^{NIT} + n^2 \{2u(x(p)) - cx(p)\} - \int_{i=0}^n e(i)di - F \end{aligned} \quad (\text{A48})$$

From (A48), we obtain

$$\begin{aligned} \frac{dTSS^2(n(p, t, \tau_2))}{d\tau_2} &= TSS_n^2 n_{\tau_2} \\ &= \left[ 2nx \left( p - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tau_2}. \end{aligned} \quad (\text{A49})$$

Since  $TSS_n^2 > 0$  when  $n < n^*$ , the term inside the square brackets is positive. Thus, from

(A42), we obtain

$$\frac{dTSS^2}{d\tau_2} < 0. \quad (\text{A50})$$

Q.E.D.

### Proof of Proposition 5

A subsidy per employee in type-IT firms changes the profits of a type-IT firm to

$$\pi^{IT} = f(\alpha(n)L^{IT}) - (w^{IT} - \tau_3)L^{IT}. \quad (\text{A51})$$

From profit maximization by the type-IT firm, we obtain

$$\alpha(n)f'(\alpha(n)L^{IT}) - (w^{IT} - \tau_3) = 0. \quad (\text{A52})$$

The zero-profit condition, (18), yields

$$f(\alpha(n)L^{IT}) - (w^{IT} - \tau_3)L^{IT} = 0. \quad (\text{A53})$$

Totally differentiating (A52) and (A53) gives us

$$w_1^{IT}(n, \tau_3) = f'(\alpha(n)L^{IT})\alpha'(n) > 0, \quad (\text{A54})$$

$$w_2^{IT}(n, \tau_3) = 1 > 0, \quad (\text{A55})$$

$$L_n^{IT}(n) = -\frac{\alpha'(n)L^{IT}}{\alpha} < 0, \quad (\text{A56})$$

where  $w_i^{IT}$  is the derivative of  $w^{IT}$  with respect to the  $i$  th argument. Totally differentiating (13), we have

$$n_{\tau_3}(p, t, \tau_3) = -\frac{1}{MB_n} > 0 \quad (\text{A57})$$

from (A54) and (A55). Thus, from (A54) to (A57), we obtain

$$w_{\tau_3}^{IT}(n(p, t, \tau_3), \tau_3) = w_1^{IT}n_{\tau_3} + w_2^{IT} > 0, \quad (\text{A58})$$

$$L_{\tau_3}^{IT}(n(p, t, \tau_3)) = L_n^{IT}n_{\tau_3} < 0. \quad (\text{A59})$$

From profit maximization by the type-NIT firm and the zero-profit condition, (18), we obtain (A5) and (A6), which gives us the fixed value of  $w^{NIT}$  and  $L^{NIT}$ .

From (A7), (A8), and (A57), we obtain

$$m_{\tau_3}^{IT}(n(p, t, \tau_3)) = m_n^{IT} n_{\tau_3} > 0 \quad (\text{A60})$$

$$m_{\tau_3}^{NIT}(n(p, t, \tau_3)) = m_n^{NIT} n_{\tau_3} < 0 \quad (\text{A61})$$

The total social surplus in this case,  $TSS^3$ , can be written as

$$TSS^3 = nw^{IT} + (1-n)w^{NIT} + n^2 \{2u(x(p)) - cx(p)\} - \int_{i=0}^n e(i)di - F - \tau_3 n \quad (\text{A62})$$

From (A55) and (A62), we obtain

$$\begin{aligned} \frac{dTSS^3(n(p, t, \tau_3), p, \tau_3)}{d\tau_3} &= TSS_1^3 n_{\tau_3} + TSS_3^3 \\ &= \left[ 2nx \left( p - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tau_3} + nw_2^{IT} - n \\ &= \left[ 2nx \left( p - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tau_3}. \end{aligned} \quad (\text{A63})$$

Since  $TSS_1^3 > 0$  when  $n < n^*$ , the term inside the square brackets is positive. Thus, from (A57), we obtain

$$\frac{dTSS^3}{d\tau_3} > 0. \quad (\text{A64})$$

Q.E.D.

### Proof of Proposition 6

A subsidy per employee in type-NIT firms changes the profits of a type-NIT firm to

$$\pi^{NIT} = g(L^{NIT}) - (w^{NIT} - \tau_4)L^{NIT}. \quad (\text{A65})$$

From profit maximization by the type-NIT firm, we obtain

$$g'(L^{NIT}) - (w^{NIT} - \tau_4) = 0. \quad (\text{A66})$$

The zero-profit condition, (18), yields

$$g(L^{NIT}) - (w^{NIT} - \tau_4)L^{NIT} = 0. \quad (\text{A67})$$

Totally differentiating (A66) and (A67) gives us the fixed value of  $L^{NIT}$  and

$$w_{\tau_4}^{NIT}(\tau_4) = 1 > 0 \quad (\text{A68})$$

Totally differentiating (13), from (A3) and (A68), we have

$$n_{\tau_4}(p, t, \tau_4) = \frac{1}{MB_n} < 0 \quad (\text{A69})$$

From (A3), (A4), and (A69), we obtain

$$w_{\tau_4}^{IT}(n(p, t, \tau_4)) = w_n^{IT} n_{\tau_4} < 0, \quad (\text{A70})$$

$$L_{\tau_4}^{IT}(n(p, t, \tau_4)) = L_n^{IT} n_{\tau_4} > 0. \quad (\text{A71})$$

From (A7), (A8), and (A69), we have

$$m_{\tau_4}^{IT}(n(p, t, \tau_4)) = m_n^{IT} n_{\tau_4} < 0, \quad (\text{A72})$$

$$m_{\tau_4}^{NIT}(n(p, t, \tau_4)) = m_n^{NIT} n_{\tau_4} > 0. \quad (\text{A73})$$

The total social surplus in this case,  $TSS^4$ , can be written as

$$TSS^4 = nw^{IT} + (1-n)w^{NIT} + n^2 \{2u(x(p)) - cx(p)\} - \int_{i=0}^n e(i)di - F - \tau_4(1-n) \quad (\text{A74})$$

From (A68) and (A74), we obtain

$$\begin{aligned}
\frac{dTSS^4(n(p, t, \tau_4), p, \tau_4)}{d\tau_4} &= TSS_1^4 n_{\tau_4} + TSS_3^4 \\
&= \left[ 2nx \left( p - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tau_4} + (1-n)w_{\tau_4}^{NIT} - (1-n) \\
&= \left[ 2nx \left( p - \frac{1}{2}c \right) + 2n \int_{\tilde{x}=0}^x \left( u_{\tilde{x}} - \frac{1}{2}c \right) d\tilde{x} + nw_n^{IT} - t \right] n_{\tau_4}.
\end{aligned} \tag{A75}$$

Since  $TSS_1^4 > 0$  when  $n < n^*$ , the term inside the square brackets is positive. Thus, from

(A69), we obtain

$$\frac{dTSS^4}{d\tau_4} < 0. \tag{A76}$$

Q.E.D.

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## Notes

1. See Rohlfs [1974], Oren and Smith [1981], and Economides and Himmerlberg [1995] for analyses that focus on the existence of multiple equilibria.

2. For the sake of simplicity, we implicitly assume that a provider of communications services does not hire either type-IT consumers or type-NIT consumers. That is, we disregard employment by the provider of communications services.

3. According to the OECD [2001, p. 5], the term “digital divide” is defined as “the gap between individuals, households, businesses, and geographic areas at different socio-economic levels with regard both to their opportunities to access information and communication technologies and to their use of the Internet for a variety of activities.”

Figure 1

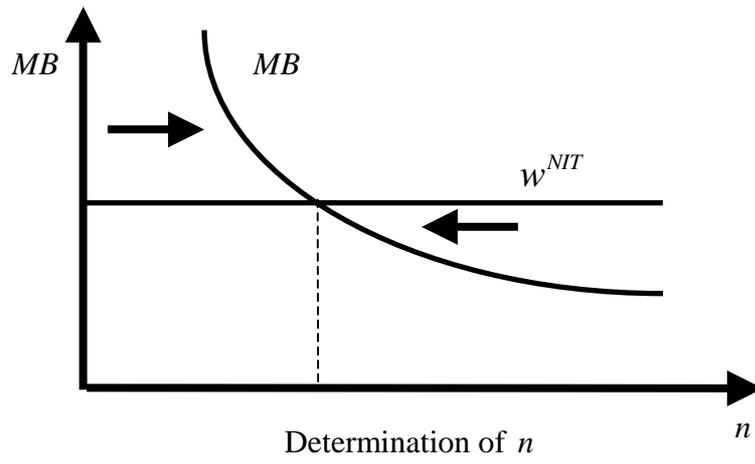


Figure 2

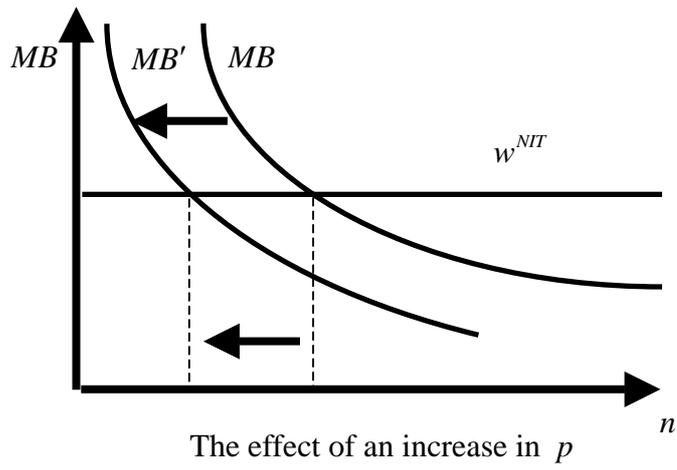


Figure 3

