

# CSIS Discussion Paper No. 146

## On the redistribution effect of tariff integration in public transport

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November 2016

### Abstract:

This paper examines the redistribution effect of integrating tariffs of public transport operated by various institutions when congestion is present. It is shown that the consumers with medium wages benefit from the integration while those with too low wages and those with too high wages are hurt by it, as long as they face a trade-off between a fare and a travel time or the difference in travel times is not too large, before the integration.

### Keywords:

common fare; congestion; distribution of wage; travel time

### Acknowledgement:

I thank two anonymous referees for their valuable comments and suggestions, which greatly help me improve the paper. I also appreciate the comments from seminar participants at various institutions. This research is partly supported by the Grants in Aid for Research by the Ministry of Education, Science and Culture in Japan (No. 25285070)

# 1 Introduction

In many countries, especially in developed ones, the attempts to integrate public transport are becoming more and more widespread (see the report commissioned by the European Commission, NEA (2003), among others). In Tokyo, for instance, policy makers have been insisting on the merger of the two operators of its subway system in recent years. The aim of that policy is to achieve the integration of tariffs, which is one of the most important elements in the integration.<sup>1</sup>

The aim of tariff integration is to raise efficiency in the provision of transport services. One of its immediate benefits is a reduction of transaction costs (for reviews, see White (1981), Carbajo (1988), and Gilbert and Jalilian (1991)), which results in the increase in ridership, as has been recorded in a sizable body of literature.<sup>2</sup> The other benefits include the mitigation of congestion, the alleviation of a distortion in users' choices and the exploitation of economies of scale in production (for the latter two, see Takahashi (2014), for example).

In the meantime, the possibility that the tariff integration has different impacts upon consumers with varied levels of incomes has seldom attracted the interests of researchers. One of the key factors responsible for the different impacts is three effects of integration. First, the fare paid by a consumer obviously changes as a result of the tariff integration. It may decrease, resulting in a positive *fare saving effect*, or increase, a negative fare saving effect. Second, if a consumer switches over the transport service (transport route) to use, the length of time necessary to complete her trip also changes: There may be a positive or negative *time saving effect*. Third and last, the congestion in the route(s) a consumer uses may ameliorate or deteriorate, which produces a positive or negative *congestion reduction effect*, accordingly. There can be two channels. For one thing, the degree of congestion changes if she switches over the route to take. Furthermore, even if she uses the same route, the degree changes because the demands for each transport service change. Now, a consumer with a lower income cares more about a fare than a consumer with a higher income because a given amount of fare payment occupies a greater part of his income. At the same time, he places a smaller importance on a travel time because the opportunity cost of time is lower. For him, therefore, the fare

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<sup>1</sup> Three fields of integration are distinguished (see Abrate et al. (2009)): informative integration, which provides users an easier access to the information on total networks, timetables and fares; physical integration, which improves the infrastructure necessary to use the services of different operators or those by different modes; and tariff integration, which is a topic of this paper.

<sup>2</sup> As examples, see FitzRoy and Smith (1998), Matas (2004), Abrate et al. (2009), and Sharaby and Shiftan (2012).

saving effect is relatively more important compared to the time saving effect, other things being equal. In contrast, a consumer with a higher income attaches a lower value to a fare but a greater value to a travel time. Thus, the time saving effect is relatively more important compared to the fare saving effect. Because of this difference, the integration has different impacts on consumers with varied levels of incomes.

The purpose of this paper is to shed light on this redistribution effect of tariff integration through a rigorous analysis. For that purpose, we construct a simple model with two transport firms providing different transport services to consumers. Consumers decide which firm's service to use, comparing travel times, fares and degrees of congestion. We examine the effects of tariff integration, or, in other words, the introduction of a common fare.

Main finding is that consumers with medium wages benefit from tariff integration while those with too low wages and those with too high wages are hurt by it, as long as the transport service that is less expensive before the integration takes a longer travel time than the other service, or the difference in travel times is not too large. To understand this result, consider the first case where consumers, before the integration, face a trade-off between a fare and a travel time. A consumer with a low wage, before the integration, tends to choose the less expensive service, which nonetheless takes a longer time. As a result of the integration, the fare he pays rises while the travel time is shortened if he switches over the service to use, or otherwise remains unchanged. If his wage is very low, the former effect of the rise in a fare is so large as to dominate the latter effect of time saving, if any. Therefore, he is hurt by the integration. In contrast, a consumer with a high wage tends to choose the more expensive service with a shorter travel time before the integration. Consequently, the fare she pays declines by the integration, which benefits her more when her wage is lower. At the same time, the integration worsens the congestion in the service with a shorter time because it reduces the fare of that service. A consumer with a too high wage, therefore, fails to receive a sufficient amount of the benefit of fare reduction to counteract the loss from the aggravation of congestion.

Most of the studies on tariff integration are the empirical works that estimate the increase in the ridership of public transport, as has been referred to earlier. Two exceptions are worth mentioning. First, Cassone and Marchese (2005) compare the welfare impacts of tariff integration under monopoly pricing and under the benevolent regulation through Ramsey pricing to unveil the distortion caused

by a monopolistic behavior. Second, Marchese (2006) examines through a nonlinear pricing approach how consumers' surplus is extracted when tariffs are integrated. However, those studies do not discuss the redistribution effect of tariff integration.

The rest of the paper consists of four sections. In the next section, we present a basic framework to discuss the effects of tariff integration. In Section 3, consumers' choices are analyzed. In Section 4, we examine the redistribution effect of tariff integration. The last section concludes.

## 2 Basic framework

In this section, we present a model with two firms (firm 1 and firm 2), e.g., railway companies, which provide transport services between two points in a space along their own routes. The "firms" may be private companies or government sectors. They are labeled so that firm 1's fare is no higher than firm 2's, i.e.,  $p_1 \leq p_2$ , without loss of generality.

There are 1 unit of consumers who make trips between the two points. To avoid unnecessary complication, we assume that the number of the trips made by each consumer is fixed: The trips are, for instance, commutes, and as long as a consumer desires to continue to be employed, she has no option to choose the number of workdays. Thus, each of them consumes a fixed amount, say 1 unit, of transport service, irrespective of its price. Consequently, the total demand for transport services is equal to 1. That is,  $D_1 + D_2 = 1$ , where  $D_1$  and  $D_2$  are the demands for the transport service provided by each firm.

Consumers have identical preference. For one thing, they obtain utility from the consumption of a composite good and leisure. It is denoted by  $u(z, l)$ , where  $z$  and  $l$  are the amounts of the composite good and leisure, respectively. To derive clear-cut results, we specify its functional form as  $u(z, l) = z^\alpha l^{1-\alpha}$  with  $\alpha \in (0, 1)$ . In addition, consumers suffer disutility from the congestion in the transport route they use. It is assumed that this disutility increases with the number of users of the route, that is to say, the demand for the transport service they consume. Thus, the disutility of a consumer who uses firm  $i$ 's service is expressed by an increasing function,  $c(D_i)$ , with  $c'(D_i) > 0$ . Furthermore, we assume that  $c(D_i)$  approaches positive infinity as  $D_i$  goes to 1, that is, the congestion would become

intolerable if the demand were concentrated in one route. The addition of this disutility reflects the numerous empirical findings that congestion has considerable negative impacts on the utilities of users (for review, see Li and Hensher (2011) and Wardman and Whelan (2011), among others). The overall utility is denoted by  $U(z, l; D_i) \equiv u(z, l) - c(D_i)$ .

Consumers are heterogeneous in that their wage rates are different. We assume that the wage rates are distributed in interval  $[\underline{w}, \bar{w}]$  ( $\underline{w} > 0$ ,  $\bar{w} > \underline{w}$ ) according to a given cumulative distribution function  $F(\cdot)$ , which is continuous. For the sake of exposition, we rather define  $F(\cdot)$  on the domain of entire non-negative real numbers. That is,  $F(w) = 0$  for  $w \in [0, \underline{w}]$  and  $F(w) = 1$  for  $w \geq \bar{w}$ .

When a consumer uses firm  $i$ 's transport service, her budget constraint is given by

$$z + wl = w(\bar{l} - t_i) - p_i \equiv y_i. \quad (1)$$

Here,  $\bar{l}$  is the available length of time and  $w$  is her wage rate, and  $t_i$  is the travel time of firm  $i$ 's service. The price of the composite good is given and taken as a numeraire. We can interpret  $w(\bar{l} - t_i)$  as a "gross" income, which is the sum of the wage income,  $w(\bar{l} - l - t_i)$ , and the opportunity cost of leisure,  $wl$ . Variable  $y_i$  represents a gross disposable income excluding the payment for transport service.

Consumers who use firm  $i$ 's service maximize  $U(z, l; D_i)$  subject to (1). Let  $v(w, y_i)$  and  $V(w, y_i; D_i)$  be the indirect utilities associated with  $u(z, l)$  and  $U(z, l; D_i)$ , respectively, that is,  $v(w, y_i) = Ky_i/w^{1-\alpha}$  and  $V(w, y_i; D_i) = v(w, y_i) - c(D_i)$ , where  $K \equiv \alpha^\alpha(1-\alpha)^{1-\alpha} > 0$  is a constant.

Consumers use the service that yields a higher value of  $V(\cdot)$ . That is, they use firm 1's service if  $V(w, y_1; D_1) > V(w, y_2; D_2)$  and firm 2's service if  $V(w, y_1; D_1) < V(w, y_2; D_2)$ . If  $V(w, y_1; D_1) = V(w, y_2; D_2)$ , furthermore, they are indifferent between the services of the two firms. Let us express the relative attractiveness of firm 1's service compared to firm 2's as  $\Psi(w; \tau, \rho)$ :  $\Psi(w; \tau, \rho) \equiv V(w, y_1; D_1) - V(w, y_2; D_2)$ . Here,  $\tau \equiv t_1 - t_2$  and  $\rho \equiv p_1 - p_2$  denote the differences in travel times and fares, respectively. By assumption,  $\rho \leq 0$ . Then, since  $V(w, y_1; D_1) \begin{cases} > \\ = \\ < \end{cases} V(w, y_2; D_2)$  if

$\Psi(w; \tau, \rho) \begin{cases} > \\ = \\ < \end{cases} 0$ , the consumers with  $\Psi(w; \tau, \rho) > 0$  use firm 1's service, those with  $\Psi(w; \tau, \rho) < 0$

use firm 2's service, and those with  $\Psi(w; \tau, \rho) = 0$  are indifferent between two firms' services.

In addition, let us introduce a new function  $\gamma(x) \equiv c(x) - c(1-x)$  so that  $\gamma(D_1)$  represents a difference in the congestion disutilities.. The function is increasing since  $c(\cdot)$  is increasing; and furthermore,

$$\gamma(x) \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } x \begin{cases} > \\ = \\ < \end{cases} \frac{1}{2}. \quad (2)$$

Then, we can rewrite  $\Psi(w; \tau, \rho)$  as  $\Psi(w; \tau, \rho) \equiv \psi(w; \tau, \rho) - \gamma(D_1)$ , where

$$\psi(w; \tau, \rho) \equiv v(w, y_1) - v(w, y_2) = -\frac{K}{w^{1-\alpha}}(w\tau + \rho)$$

is a difference in the indirect utilities associated with  $u(z, l)$ . It shows three sources of the relative attractiveness of firm 1's service, namely, the saving in the time spent, the saving in the fare to pay and finally, the alleviation of the disutility from congestion. Their magnitudes are equal to  $-Kw^\alpha\tau$ ,  $-K\rho/w^{1-\alpha}$  and  $-\gamma(D_1)$ , respectively.

### 3 Consumers' choices

In this section, we study which service consumers use, considering two cases, namely, the case of  $\rho < 0$  ( $p_1 < p_2$ ) and that of  $\rho = 0$  ( $p_1 = p_2$ ), one after another.

#### 3.1 The case of $\rho < 0$

To begin with, note that

$$\frac{\partial \psi(w; \tau, \rho)}{\partial w} = \frac{1}{w^{2-\alpha}} [-\alpha w\tau + (1-\alpha)\rho]. \quad (3)$$

Three cases are distinguished.

First, suppose that  $\tau > 0$ . Then, (3) implies that  $\partial \psi(w; \tau, \rho)/\partial w < 0$ . Moreover,  $\psi(w; \tau, \rho)$  approaches positive infinity as  $w$  goes to 0 from above whereas it approaches negative infinity as  $w$  grows unboundedly. Fig. 1 shows  $\psi(w; \tau, \rho)$  and  $\gamma(D_1)$  as functions of  $w$ . As is clear from the

first panel of the figure, there exists a critical value,  $\widehat{w}(\tau, \rho)$ , for which

$$\Psi(w; \tau, \rho) \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w \begin{cases} < \\ = \\ > \end{cases} \widehat{w}(\tau, \rho). \quad (4)$$

Second, suppose that  $\tau = 0$ . It is still the case that  $\partial\psi(w; \tau, \rho)/\partial w < 0$  and that  $\psi(w; \tau, \rho)$  approaches positive infinity as  $w$  goes to 0 from above. A difference is that  $\psi(w; \tau, \rho)$  now approaches 0 not negative infinity as  $w$  grows unboundedly. At the same time,  $\gamma(D_1)$  is necessarily positive in this case. This is shown as follows. Suppose that  $\gamma(D_1)$  is nonpositive. Then, (2) implies that  $D_1 \leq D_2$ . Because  $\psi(w; \tau, \rho) > 0$  for  $\tau = 0$  and  $\rho < 0$ , however,  $\Psi(w; \tau, \rho) > 0$  and the demand for firm 2's service must be 0, which is a contradiction. Now, from these observations, we can again conclude that there exists  $\widehat{w}(\tau, \rho)$  that satisfies (4) (see Fig. 1 (b)).

Fig. 1 Consumers' demand in the case of  $\rho < 0$

Therefore, in these two cases, that is, when  $\tau \geq 0$ , there is a critical value,  $\widehat{w}(\tau, \rho)$ , such that the consumers with  $w < \widehat{w}(\tau, \rho)$  use less expensive firm 1' service, those with  $w = \widehat{w}(\tau, \rho)$  are indifferent between two firms' services, and those with  $w > \widehat{w}(\tau, \rho)$  use more expensive firm 2's service. It is straightforward to explain this result intuitively. A consumer with a lower wage does not attach as great importance to the length of a travel time as a consumer with a higher wage, because the opportunity cost of time is lower. To the contrary, the former consumer places a higher value on a fare than the latter consumer, because the payment for transport service occupies a greater part of income. Therefore, a low wage consumer is inclined to firm 1's service, which is less expensive but takes a longer time; and the opposite holds for a high wage consumer.

Note that we can safely ignore indifferent consumers because their mass is of measure zero. (In what follows, too, we will pay no attention to marginal consumers by the same reason.) Since the total mass of consumers is normalized to 1, therefore, the mass of the low wage consumers who use firm 1's service becomes equal to  $F(\widehat{w}(\tau, \rho))$ , that is,  $D_1 = F(\widehat{w}(\tau, \rho))$ . Consequently,  $\widehat{w}(\tau, \rho)$  is given as a solution to

$$\psi(w; \tau, \rho) - \gamma(F(w)) = 0. \quad (5)$$

Third and last, suppose that  $\tau < 0$ . In this case, firm 1's service is less expensive and faster than firm 2's. If there were no disutility from congestion, therefore, all the consumers would use firm 1's service. However, to the extent that they prefer firm 1's service, congestion arises in its route, which undermines its attractiveness.

Three observations follow. First,  $\psi(w; \tau, \rho)$  approaches positive infinity as  $w$  goes to 0 from above, and as  $w$  grows unboundedly. Second, (3) implies that

$$\frac{\partial \psi(w; \tau, \rho)}{\partial w} \begin{cases} < \\ = \\ > \end{cases} 0 \text{ if } w \begin{cases} < \\ = \\ > \end{cases} w^* \equiv \frac{(1-\alpha)\rho}{\alpha\tau}.$$

Therefore,  $\psi(w; \tau, \rho)$  has a single trough point at  $w^*$  as is shown in the last panel of Fig. 1. Third and last,  $\gamma(D_1) > \psi(w^*; \tau, \rho)$ , that is, the horizontal  $\gamma(D_1)$  line lies above the  $\psi(w; \tau, \rho)$  curve at  $w^*$  in Fig. 1 (c). This is shown as follows: Suppose that  $\gamma(D_1) \leq \psi(w^*; \tau, \rho)$ . Then,  $\Psi(w; \tau, \rho) > 0$  for any  $w$  except for  $w = w^*$ . Therefore, (almost) all the consumers use firm 1's service.<sup>3</sup> However, we are assuming that  $c(D_1)$  approaches positive infinity in that case, which gives a contradiction. These three observations lead us to the conclusion that the  $\gamma(D_1)$  line intersects the  $\psi(w; \tau, \rho)$  curve twice. We denote the values of  $w$  at such intersections by  $w'(\tau, \rho)$  and  $w''(\tau, \rho)$  with  $w'(\tau, \rho) < w''(\tau, \rho)$ .

Since  $\Psi(w; \tau, \rho) \begin{cases} > \\ = \\ < \end{cases} 0$  if  $\begin{cases} w < w'(\tau, \rho) \text{ or } w > w''(\tau, \rho) \\ w = w'(\tau, \rho) \text{ or } w = w''(\tau, \rho) \\ w \in (w'(\tau, \rho), w''(\tau, \rho)) \end{cases}$ , consumers with  $w < w'(\tau, \rho)$  or  $w >$

$w''(\tau, \rho)$  use firm 1's service while those with  $w \in (w'(\tau, \rho), w''(\tau, \rho))$  use firm 2's service.

This non-monotonic consequence is attributed to the interplay among the three elements that determine consumers' utility, namely, the fare, the travel time and the congestion. As has been mentioned earlier, a consumer with a lower wage attaches a higher importance to a low fare. When his wage is extremely low, this factor becomes dominating and he chooses the service with a lower fare, namely, firm 1's service. In contrast, a consumer with a higher wage places a greater value on a short travel time. When her wage is extremely high, it becomes such a decisive factor that she resorts to the service with a shorter travel time, namely, firm 1's service. In this way, the reasons to choose firm 1's service are different between the consumers with  $w < w'(\tau, \rho)$  and those with  $w > w''(\tau, \rho)$ .

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<sup>3</sup> We can ignore the consumers with  $w = w^*$  since their measure is 0.



The two variables  $w'(\tau, \rho)$  and  $w''(\tau, \rho)$  are determined by the system of the following two equations (their arguments are omitted for the sake of clarity):

$$\psi(w'; \tau, \rho) - \gamma(F(w') + 1 - F(w'')) = 0 \quad \text{and}$$

$$\psi(w''; \tau, \rho) - \gamma(F(w') + 1 - F(w'')) = 0.$$

For a subsequent analysis, it is worth noting that when  $w''(\tau, \rho)$  is no smaller than  $\bar{w}$  (the uppermost wage), the non-monotonicity disappears and this case degenerates to the first two cases. That is, all the consumers whose wage exceeds  $w'(\tau, \rho)$  use firm 2's service while those with  $w < w'(\tau, \rho)$  use firm 1's. Since  $w'(\tau, \rho)$  is to solve (5) in this case,  $w'(\tau, \rho)$  coincides with  $\hat{w}(\tau, \rho)$ . A sufficient condition for  $w''(\tau, \rho)$  being no smaller than  $\bar{w}$  is that the  $\psi(w; \tau, \rho)$  curve hits the bottom at  $w \geq \bar{w}$ , or equivalently,  $w^*$  is greater than or equal to  $\bar{w}$ . This is more likely when the difference in fares,  $p_2 - p_1$ , is larger and the difference in travel times,  $t_2 - t_1$ , is smaller. Consequently, when the fare differential is sufficiently large and/or the time differential is sufficiently small, the consumers with  $w < \hat{w}(\tau, \rho)$  use firm 1's service while those with  $w > \hat{w}(\tau, \rho)$  use firm 2's service even in the case of  $\tau < 0$ . The interpretation is straightforward. When the price differential is extremely large, low wage consumers' bias in favor of the less expensive service swells to the extent that it crowds out the high wage consumers with  $w > w''(\tau, \rho)$ , who are willing to use that service. When the time differential is very small, in contrast, even the high wage consumers with  $w > w''(\tau, \rho)$ , who highly appreciate a shorter travel time, do not consider firm 1's service attractive enough.

### 3.2 The case of $\rho = 0$

Next, we turn to the case where the two firms charge an equal fare.

First, suppose that  $\tau > 0$ . Four observations follow. First, a reasoning similar to the one used above drives us to the conclusion that  $\gamma(D_1)$  is necessarily negative.<sup>4</sup> Second,  $\psi(w; \tau, \rho)$  is decreasing in  $w$ . Third,  $\psi(0; \tau, \rho) = 0$ . Fourth,  $\psi(w; \tau, \rho)$  approaches negative infinity as  $w$  grows unboundedly. These observations imply that there exists  $\hat{w}(\tau, \rho)$  that satisfies (4). The consumers with  $w < \hat{w}(\tau, \rho)$

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<sup>4</sup> Suppose that  $\gamma(D_1) \geq 0$ . Then, (4) implies that  $D_1 \geq D_2$ . At the same time,  $\psi(w; \tau, \rho) < 0$  implies that  $\Psi(w; \tau, \rho) < 0$  and therefore,  $D_1 = 0$ , which is a contradiction.

use firm 1's service while those with  $w > \hat{w}(\tau, \rho)$  use firm 2's. This result is quite natural. Since there is no difference in fares, what matters is a difference in travel times. Consumers with higher wages, who attach a greater importance to a travel time, end up using the faster service, i.e., firm 1's service.

Second, suppose that  $\tau = 0$ . Then, it can be easily shown that the only possibility is  $\gamma(D_1) = 0$ .<sup>5</sup> Because  $\Psi(w; \tau, \rho)$  becomes 0, consumers are indifferent between two firms' services. In this case, it might be natural to suppose that half the consumers who are randomly chosen use firm 1's service and the remaining half use firm 2's. For the sake of notational convenience, however, we rather consider that the consumers whose wage rates are lower than the average use firm 1's service while those whose wage rates exceed it use firm 2's. This contrivance enables us to state that the consumers with  $w < \hat{w}(0, 0)$  use firm 1's service while those with  $w > \hat{w}(0, 0)$  use firm 2's, because  $\psi(\hat{w}(0, 0); 0, 0) = 0$  and therefore,  $F(\hat{w}(0, 0)) = 1/2$ .

Thus, we can again combine these two cases. When  $\tau \geq 0$ , the consumers with  $w < \hat{w}(\tau, \rho)$  use firm 1's service whereas those with  $w > \hat{w}(\tau, \rho)$  use firm 2's.

Third and last, suppose that  $\tau < 0$ . In this case,  $\gamma(D_1)$  is necessarily positive.<sup>6</sup> Moreover,  $\psi(w; \tau, \rho)$  is increasing in  $w$ , becomes 0 at  $w = 0$ , and approaches positive infinity as  $w$  grows unboundedly. Therefore, a critical value,  $\tilde{w}(\tau)$ , exists so that the consumers with  $w < \tilde{w}(\tau)$  use firm 2's service and those with  $w > \tilde{w}(\tau)$  use firm 1's. The variable  $\tilde{w}(\tau)$  is obtained as a solution to

$$\psi(w; \tau, 0) - \gamma(1 - F(w)) = 0. \quad (6)$$

In this last case, the faster service is firm 1's not firm 2's. Therefore, firm 1's service is used by higher wage consumers. Finally, note that  $\psi(w; \tau, 0) > 0$  when  $\tau < 0$ . Consequently,  $\gamma(1 - F(\tilde{w}(\tau))) > 0$  by (6), which implies that  $F(\tilde{w}(\tau)) < 1/2$  (see (2)).

To close the section, it is convenient to sum up the above findings as follows.

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<sup>5</sup> Suppose that  $\gamma(D_1) > 0$ . Then,  $D_1 > D_2$ . At the same time,  $\psi(w; \tau, \rho) = 0$  implies that  $\Psi(w; \tau, \rho) < 0$  and therefore,  $D_1 = 0$ , which is a contradiction. Similarly, we can show that  $\gamma(D_1) < 0$  leads to a contradiction.

<sup>6</sup> The reasoning is similar to the one used to show that  $\gamma(D_1)$  is negative when  $\tau > 0$ . We can simply reverse all the inequality signs in footnote 4.

### Lemma 1

- i) If  $\rho \leq 0$  and  $\tau \geq 0$ , the consumers with  $w < \hat{w}(\tau, \rho)$  use firm 1's service while those with  $w > \hat{w}(\tau, \rho)$  use firm 2's.
- ii) If  $\rho < 0$  and  $\tau < 0$ , the consumers with  $w < w'(\tau, \rho)$  and those with  $w > w''(\tau, \rho)$  use firm 1's service while those with  $w \in (w'(\tau, \rho), w''(\tau, \rho))$  use firm 2's.
- iii) If  $\rho = 0$  and  $\tau < 0$ , the consumers with  $w > \tilde{w}(\tau)$  use firm 1's service while those with  $w < \tilde{w}(\tau)$  use firm 2's.

## 4 Redistribution effect of tariff integration

We consider the following tariff integration. Before the integration, two firms' fares,  $p_1^0$  and  $p_2^0$ , are different from each other. We assume without loss of generality that the fare of firm 1 is lower than that of its competitor, that is,  $p_1^0 < p_2^0$ . After the integration, a common fare equal to  $p'$  is introduced. We concentrate on the common fare in between two firms' fares before the tariff integration, i.e.,

$$p_1^0 < p' < p_2^0.$$

Let us examine a representative consumer who uses firm  $i$ 's service before the integration and firm  $j$ 's service after it. Her benefit from the integration is equal to the change in the comprehensive indirect utility, that is,  $b_{ij}(D_i^0, D_j') \equiv V(w, y_j'; D_j') - V(w, y_i^0; D_i^0)$  where  $y_i^0$  and  $y_j'$  are her gross disposable incomes before and after the integration, respectively; and  $D_i^0$  and  $D_j'$  are the demand for firm  $i$ 's service before the integration and that for firm  $j$ 's service after it, respectively. It is straightforward to see that

$$b_{ij}(D_i^0, D_j') = \frac{K}{w^{1-\alpha}} [w(t_i - t_j) + p_i^0 - p'] + c(D_i^0) - c(D_j').$$

The term with  $w(t_i - t_j)$ , that with  $p_i^0 - p'$  and that of  $c(D_i^0) - c(D_j')$  represent the time saving effect, the fare saving effect and the congestion reduction effect, respectively, of the tariff integration.

How does a wage rate affect the benefit from the integration? To answer this question, note that

$$\frac{db_{ij}(D_i^0, D_j')}{dw} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w(t_i - t_j) \begin{cases} > \\ = \\ < \end{cases} \frac{1-\alpha}{\alpha} (p_i^0 - p'). \quad (7)$$

The benefit increases with a wage rate when the time saving effect exceeds an adjusted value of the price saving effect, whereas it decreases when the opposite holds. This is again explained by the now familiar notion that a higher wage consumer values a travel time more than a lower wage consumer. To obtain further result, we need to distinguish two cases according to the relative lengths of the travel times, namely, the case of  $\tau \geq 0$  and that of  $\tau < 0$ .

#### 4.1 The case of $\tau \geq 0$

In this case, the transport service provided at a lower fare before the integration (i.e., firm 1's service) takes a longer or equal travel time.

As is shown in Lemma 1, the consumers whose wage rates are lower than a critical value use firm 1's service before the integration. Although the same remark applies to the post-integration phase, the critical values are different, namely,  $\hat{w}(\tau, \rho^0)$  before the integration and  $\hat{w}(\tau, 0)$  after the integration, where  $\rho^0 \equiv p_1^0 - p_2^0 < 0$ . Unfortunately, we cannot determine which is larger, and therefore, we need to consider two cases.

##### 4.1.1 The case of $\hat{w}(\tau, \rho^0) \geq \hat{w}(\tau, 0)$

For the first case of  $\hat{w}(\tau, \rho^0) \geq \hat{w}(\tau, 0)$ , three types of consumers are identified. (The marginal consumers who are indifferent between two firms' services are not considered because they are of measure 0.)

First, the consumers with  $w < \hat{w}(\tau, 0)$  use firm 1's service before the integration since  $w < \hat{w}(\tau, \rho^0)$ , and continue to use it after the integration. For them, therefore,  $i = j = 1$ ,  $D_i^0 = F(\hat{w}(\tau, \rho^0))$  and  $D_j^1 = F(\hat{w}(\tau, 0))$ . It follows from Lemma 2 and  $\hat{w}(\tau, 0) \leq \hat{w}(\tau, \rho^0)$  that  $c(F(\hat{w}(\tau, \rho^0))) - c(F(\hat{w}(\tau, 0))) \geq 0$ . Thus, the nonnegative congestion reduction effect works against the negative fare saving effect; and therefore, the sign of the total effect is ambiguous. Furthermore, (7) implies

that the benefit is greater for a consumer with a higher wage.<sup>7</sup>

Second, the consumers with  $w \in (\widehat{w}(\tau, 0), \widehat{w}(\tau, \rho^0))$  use firm 1's service before the integration but switches over to firm 2's after the integration. (If  $\widehat{w}(\tau, 0) = \widehat{w}(\tau, \rho^0)$ , the mass of such consumers becomes measure 0.) Since  $i = 1$ ,  $j = 2$ ,  $D_i^0 = F(\widehat{w}(\tau, \rho^0))$  and  $D_j' = 1 - F(\widehat{w}(\tau, 0))$ , the fare saving effect is negative while the sign of the congestion reduction effect is ambiguous. For this type of consumers, furthermore, one additional effect, namely, the time saving effect, appears. It is positive. Again the sign of the total effect is ambiguous. Moreover, we can obtain the same conclusion that the benefit is greater for a consumer with a higher wage.

Third and last, the consumers with  $w > \widehat{w}(\tau, \rho^0)$  always use firm 2's service:  $i = j = 2$ ,  $D_i^0 = 1 - F(\widehat{w}(\tau, \rho^0))$  and  $D_j' = 1 - F(\widehat{w}(\tau, 0))$ . Their benefits become mirror images of those of the first type, and consist of the positive fare saving effect and the negative congestion reduction effect. The benefit is greater for a consumer with a lower wage.<sup>8</sup>

We can sum up these findings as follows. Suppose that a wage rate gradually rises from  $\underline{w}$ . Then, the benefit from the integration first grows; but it shrinks after the wage rate reaches  $\widehat{w}(\tau, \rho^0)$ .

#### 4.1.2 The case of $\widehat{w}(\tau, \rho^0) < \widehat{w}(\tau, 0)$

The case of  $\widehat{w}(\tau, \rho^0) < \widehat{w}(\tau, 0)$  is analyzed likewise. First, for the consumers with  $w < \widehat{w}(\tau, \rho^0)$ , who always use firm 1's service, the fare saving effect and the congestion reduction effect are negative, and therefore, the benefit of tariff integration is negative. The benefit increases with  $w$ . Second, the consumers with  $w \in (\widehat{w}(\tau, \rho^0), \widehat{w}(\tau, 0))$  use firm 2's service before the integration and firm 1's after it.

<sup>7</sup> Indeed, we can derive that

$$b_{11} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w \begin{cases} > \\ = \\ < \end{cases} \omega \equiv \left[ \frac{K(p' - p_1^0)}{c(F(\widehat{w}(\tau, \rho^0))) - c(F(\widehat{w}(\tau, 0)))} \right]^{\frac{1}{1-\alpha}};$$

The consumers whose wages are higher (lower, resp.) than a critical value,  $\omega$ , benefit (suffer, resp.) from the integration.

<sup>8</sup> Indeed, since

$$b_{22} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w \begin{cases} < \\ = \\ > \end{cases} \omega' \equiv \left[ \frac{K(p_2^0 - p^c)}{c(1 - F(\widehat{w}(\tau, 0))) - c(1 - F(\widehat{w}(\tau, \rho^0)))} \right]^{\frac{1}{1-\alpha}},$$

the consumers whose wages are lower (higher, resp.) than a critical value,  $\omega'$ , benefit (suffer, resp.) from the integration.

The time saving effect is nonpositive, the fare saving effect is positive and the sign of the congestion reduction effect is ambiguous. The benefit decreases with  $w$ . Third and last, the consumers with  $w > \hat{w}(\tau, \rho^0)$ , who always use firm 2's service, the fare saving effect and the congestion reduction effect are positive, and therefore, the benefit is positive. The benefit decreases with  $w$ . Consequently, we can obtain the same conclusion as for the case of  $\hat{w}(\tau, \rho^0) \geq \hat{w}(\tau, 0)$ , which is recapitulated as follows:

### Proposition 1

Suppose that a wage rate gradually rises from  $\underline{w}$  when  $\tau \geq 0$ . Then, the benefit from the integration first grows, but shrinks after the wage rate reaches  $\hat{w}(\tau, \rho^0)$ .

Thus, the consumers whose wage rates are close to  $\hat{w}(\tau, \rho^0)$  are more likely to benefit. It must be straightforward to interpret this result. The consumers with low wages, that is, those with  $w < \hat{w}(\tau, \rho^0)$ , use less expensive firm 1's service before the integration. Thus, they suffer a loss from the tariff integration because of the rise in a fare. The higher their wage rates are, the less intensively they care about this loss, and therefore, the greater the benefit of integration is. For the consumers who switch the services to use from firm 1's to firm 2's, in particular, there enters an additional effect of time saving. Because those with higher wages place more importance on a travel time compared to a fare, this effect strengthens the tendency that higher wage consumers benefit more. In contrast, the consumer with high wages, namely, those with  $w > \hat{w}(\tau, \rho^0)$ , use more expensive firm 2's service before the integration. Tariff integration benefits them through the decline in a fare. This benefit is greater for those with lower wages. Thus, the lower their wage rates are, the more they benefit from the integration.

## **4.2 The case with $\tau < 0$**

If  $\tau < 0$ , many possibilities arise on account of the non-monotonicity in consumers' choices before the tariff integration, as is summarized in ii) of Lemma 1. Because exhausting all the cases does not give much insight worth the effort, we rather focus on a special case, the case where the travel times of two firms' services are not too divergent. We have seen in the previous section that when  $t_2 - t_1$  is small

enough, the non-monotonicity does not arise even if  $\rho < 0$ . In that case, the consumers with  $w < \hat{w}(\tau, \rho)$  use firm 1's service while those with  $w > \hat{w}(\tau, \rho)$  use firm 2's service. This is the case we consider in this section.

Let us begin with the following observation.

Lemma 2

$\tilde{w}(\tau) < \hat{w}(\tau, \rho^0)$ .

Proof of Lemma 2

When  $\tau < 0$ ,  $\psi(w; \tau, \rho^0) > 0$ . Since  $\hat{w}(\tau, \rho^0)$  solves (5), however,  $\gamma(F(\hat{w}(\tau, \rho^0))) > 0$ , which implies that  $\hat{w}(\tau, \rho^0) > 1/2$ . This, along with the fact that  $\tilde{w}(\tau) < 1/2$ , establishes the desired result. ■

The results summarized in Lemma 1 prescribe three types of consumers to consider.

First, consider the consumers with  $w < \tilde{w}(\tau)$ , who use firm 1's service before the integration and firm 2's after it. Although the time saving effect and fare saving effect are both negative, the sign of the congestion reduction effect is ambiguous. Furthermore, note that (7) is reduced to

$$\frac{db_{12}(D_1^0, D_2^0)}{dw} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w \begin{cases} < \\ = \\ > \end{cases} w^{**} \equiv -\frac{(1-\alpha)(p' - p_1^0)}{\alpha\tau} > 0.$$

Since  $\tau < 0$  and  $p' - p_1^0 > 0$ ,  $w^{**}$  approaches positive infinity as  $\tau$  goes to 0 from below. Therefore, the derivative is positive when the travel time differential is sufficiently small.

Second, the consumers with  $w \in (\tilde{w}(\tau), \hat{w}(\tau, \rho^0))$  always use firm 1's service. The fare saving effect is negative but the sign of the congestion reduction effect is ambiguous. The benefit necessarily increases with  $w$ .

Third and last, the consumers with  $w > \hat{w}(\tau, \rho^0)$  use firm 2' service before the integration but switch to firm 1's after the integration. Although both the time and fare saving effects are positive, the congestion reduction effect is negative (Lemma 2 implies that  $1 - F(\hat{w}(\tau, \rho^0)) < 1 - F(\tilde{w}(\tau))$ ).

Moreover, (7) is reduced to

$$\frac{db_{21}(D_2^0, D_1^1)}{dw} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } w \begin{cases} > \\ = \\ < \end{cases} w^{***} \equiv -\frac{(1-\alpha)(p_2^0 - p')}{\alpha\tau} > 0.$$

Again, as  $\tau$  goes to 0 from below,  $w^{***}$  approaches positive infinity. Therefore, when the time differential is sufficiently small, the benefit decreases with  $w$ .

Summing up, we obtain the result similar to that for the case of  $\tau \geq 0$ .

### Proposition 2

Suppose that a wage rate gradually rises from  $\underline{w}$  when  $\tau < 0$ . As long as the travel time differential of two firms' services is sufficiently small, the benefit from the integration first grows, but shrinks after the wage rate reaches  $\hat{w}(\tau, \rho^0)$ .

## 5. Concluding remarks

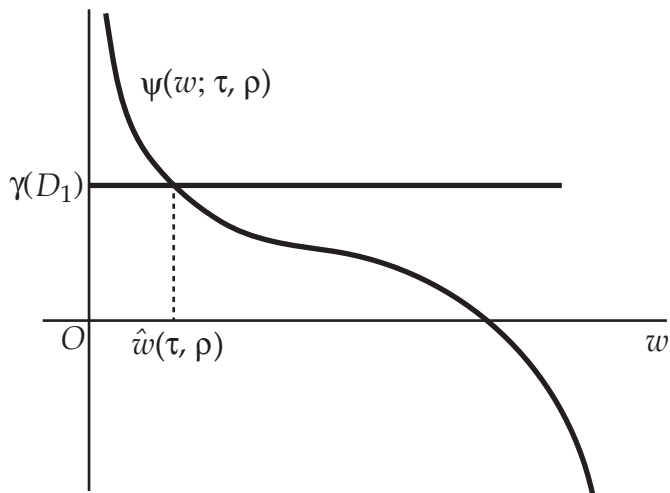
In this paper, we have examined the redistribution effect of integrating tariffs of public transport operated by various institutions. It has been shown that the consumers with medium wages benefit from the integration while those with too low wages and those with too high wages are hurt by it, as long as they face a trade-off between a fare and a travel time or the difference in travel times is not too large, before the integration.

We conclude the paper with two remarks about its limitations. First, we have not discussed the optimal level of a common fare when tariffs are integrated. To examine it, it will be necessary to fully analyze the behaviors of transport firms concerning production, which is beyond the scope of this paper. To assess the policy, however, the study of the optimal tariff would be important. Second, our findings need to be verified through empirical analyses. One could evaluate the redistribution effect of tariff integration using detailed data on the fares of various routes and actual consumers' trip behaviors, probably with a help of a discrete choice model. It might be another important research agenda, although accomplishing it requires considerable efforts, particularly when a big city is concerned.

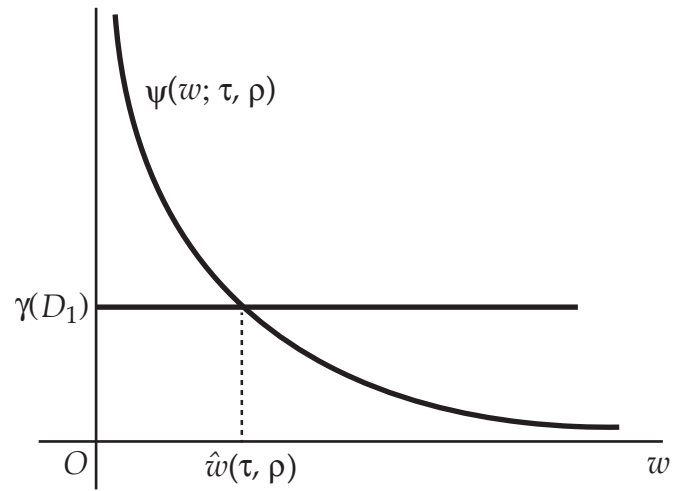


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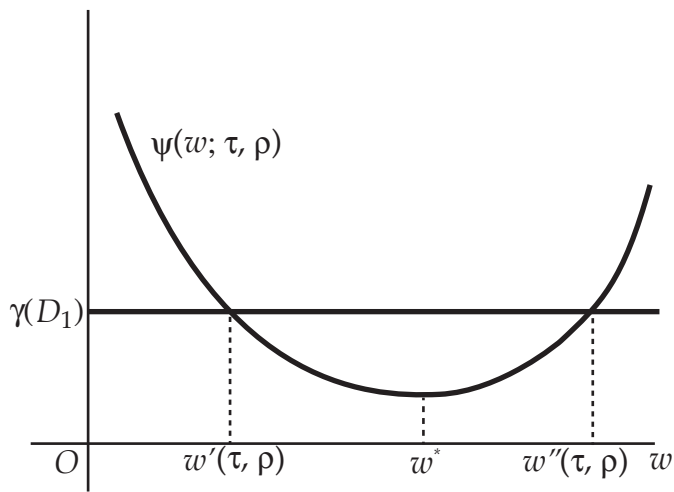
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(a) The case of  $\tau > 0$



(b) The case of  $\tau = 0$



(c) The case of  $\tau < 0$

Fig. 1 Consumers' demand in the case of  $\rho < 0$