

# Economic analysis of tariff integration in public transport

Takaaki Takahashi

Center for Spatial Information Science, University of Tokyo, 5-1-5, Kashiwa-no-ha, Kashiwa,  
Chiba 277-8568, Japan

November 2014

## Abstract

The attempts to integrate the tariffs of public transport operated by various institutions are becoming more and more widespread. Their main purpose is a reduction of distortion in consumers' choices, which arises when a more efficient route costs more. In this paper, we examine this effect of tariff integration based on a simple model with users and operators of public transport. It is shown that under certain conditions, there exists a common fare that makes both of them better off given redistribution of incomes among users and that of revenues among operators.

**Keywords:** common fare; distortion in consumers' choices; distribution of wage; economy of scale; heterogenous consumers

**JEL Classification Numbers:** R4

---

E-mail address: [takaaki-t@csis.u-tokyo.ac.jp](mailto:takaaki-t@csis.u-tokyo.ac.jp)

This research is partly supported by the Grants in Aid for Research by the Ministry of Education, Science and Culture in Japan (No. 25285070).

# 1 Introduction

In many countries, especially in developed ones, the attempts to integrate public transport are becoming more and more widespread (see the report commissioned by the European Commission, NEA (2003), among others). One reason is the growing concerns for environmental issues: the integration is expected to improve the accessibility to public transport and thereby to induce the shift from automobile to it.

It is common to distinguish three fields of integration (see Abrate et al. (2009)): *informative integration*, which provides users an easier access to information on the total networks, timetables and fares; *physical integration*, which improves the infrastructure necessary to use the services of different operators or by different modes; and *tariff integration*, which is a topic of this paper. The most simple type of tariff integration takes the form that different operators adopt the same pricing system: several bus companies charge the same fare for a single ride, or several train companies charge the same fare for a travel of a given distance, for instance. In broader integration, this same pricing system applies even to the multi-leg services involving more than one operators. That is, the fare for a travel with a given distance is the same whether one uses only the services of one operator or mixes the services of different operators. What we observe most often is this broader type of integration combined with a flat fare. In most of European big cities, for example, they sell the tickets valid for the unlimited number of rides on subways, trams and buses, possibly run by different operators, within a predetermined time period, say 1 hour or 1 day.

One of the immediate benefits of tariff integration is the reduction of transaction costs (for reviews, see White (1981), Carbajo (1988) and Gilbert and Jalilian (1991)). In many cases, tariff integration has been realized with the help of a brand-new fare collection system often accompanied by electronic cards with or without IC chips, which greatly facilitates the usability of public transport. This improvement results in the increase in its ridership, as has been recorded in a sizable body of literature.<sup>1</sup>

However, another benefit is no less important although it has seldom attracted the interests of researchers: it is the removal or alleviation of a distortion in users' choices. Users of public transport decide their travel routes taking into account various factors such as travel time and a fare, most importantly, and the number of transfers and the degree of congestion, less importantly. If tariff is not integrated, a user may choose a less expensive route even though there are other routes that are better in terms of the other factors. If it is integrated, she will pay the same amount of money regardless of the routes to take, and consequently, choose her best route. In this way, the tariff integration, inducing users to commit themselves to take the most efficient routes, can get rid of a distortion in their choices. It is noteworthy that this distortion is much more serious in big

---

<sup>1</sup>See Taylor and Carter (1998), Lee (1999), Hirsch et al. (2000), Giuliano et al. (2000), and Ungemah et al. (2006) for the cases in the United States; Dargay and Pekkarinen (1997) in Finland; FitzRoy and Smith (1988, 1999) in Germany and Switzerland; Matas (2004) in Spain; Abrate et al. (2009) in Italy; and Sharaby and Shiftan (2012) in Israel.

cities in Japan than in the counterparts in Europe and the United States. In the latter cities, urban transit is usually run by a public sector and therefore, the problem of integration mainly concerns the one between different modes of transport such as subways and commuting trains, subway and buses, and trams and buses. In big Japanese cities, contrastingly, urban transit services are provided by a number of private companies. Mass transit in Tokyo metropolitan area is, for instance, operated by the JR East, part of the former national railway company, nine “big” private commuting rail companies, the Tokyo metropolitan government, and more. In such cities, it is not rare that users take less efficient routes to save transport costs and thus, there is a great need for tariff integration.

The first aim of this paper is to analyze this distortion in a rigorous manner. For that purpose, we construct a simple model with the consumers who use transport services and the two transport firms that provide them. The consumers are heterogenous in terms of their income levels. Each transport firm possesses its own transport route and offers the consumers a pair of travel time and a fare for its service. The consumers decide which firm’s service to use comparing the travel times and fares.

In Tokyo, what is more, even the subway lines are operated by two institutions, the Tokyo Metro, a private company, and the Toei, the sector of the Tokyo metropolitan government in charge of its transport network. To integrate the tariff schemes of the two operators, policy makers have been insisting on their merger in recent years. Although its principal aim is allegedly the reduction of the distortion in users’ choices, the merger has far-reaching effects on a local economy: it will affect the financial conditions of respective operators, alter the competitive environment surrounding the railroad companies in Tokyo, and even change the spatial structure of the city in a long run, to name a few. In order to justify the measure, therefore, one has to scrutinize its costs and benefits in a broad array of its aspects. Moreover, even if it is justifiable on economic grounds, the diversity of the effects entails conflicts among individuals and organizations concerned, which can politically hinder the adoption of the measure. For these reasons, the merger does not seem to be a realistic solution to the problem.

Alternatively, this paper discusses a more basic policy, the tariff integration combined with the redistribution of revenues among mass transit operators. The second aim of this paper is to study the effects of this policy. More specifically, we show, using the model explained earlier, that under certain conditions, there exists a fare scheme common to the two transport firms that makes both the consumers and firms better off with some redistributions. In other words, we demonstrate the existence of a fare that satisfies the following two requirements: first, it yields a sufficient amount of total revenue that *both* of the two operators continue to record a surplus after some redistribution of their revenues; and second, consumers’ surplus increases as a result of the introduction of the common fare.

Most of the studies on tariff integration are the empirical works that estimate the increase in the ridership of public transport, as has been referred to earlier. Two exceptions are worth men-

tioning. First, Cassone and Marchese (2005) compare the welfare impacts of tariff integration under monopoly pricing and under the benevolent regulation through Ramsey pricing to unveil the distortion caused by a monopolistic behavior. Second, Marchese (2006) examines through a non-linear pricing approach how consumers' surplus is extracted when tariff is integrated. However, she does not discuss the *effects* of tariff integration on individual transport firms and consumers.

The rest of the paper consists of six sections. In the next section, we briefly come back to the example of Tokyo's subway system to casually illustrate how severe the problem of the distortion can be. Section 3 presents a basic model. In Section 4, we examine the effects of tariff integration on the welfare of consumers. A case with of a specific functional form is analyzed in Section 5. In Section 6, the effects on the total welfare, namely, the welfare of consumers *and* transport firms, are discussed. The last section concludes.

## 2 An introductory case study of the Tokyo subway system

In this section, we view the subway system in Tokyo in a little more detail to get a rough image of the severity of the problem. The purpose is not to present a rigorous empirical analysis but to provide a casual observation to enhance readers' motivation.

As has been mentioned, the Tokyo's subway system is operated by two institutions, the Tokyo Metro and the Toei. Among its 13 subway lines, the Tokyo Metro operates 9, which total 195.1 km in length and carry 6.44 million passengers daily in 2012. The Toei operates 4 lines with total length and daily passengers being 109 km and 2.46 million, respectively, in 2013. The tariff is not integrated. For one thing, the fare schemes are different. Both operators set fares depending on travel distances and the Tokyo Metro charges a lower fare than the Toei for any given distance. Furthermore, if a user wants to take a route consisting of two or more than two legs, some of which are operated by the Tokyo Metro and the rest are operated by Toei, she basically needs to pay the *sum* of the fares payable to respective operators.<sup>2</sup> Therefore, such a route becomes relatively more expensive compared to the routes consisting of the legs all of which are run by the same operator. On these two accounts, consumers are often enticed to use a less expensive but less efficient route.

Now, let us take a look at travel times and fares between a pair of stations. Because there are too many subway stations in Tokyo (183 stations) to examine travel times and fares for all possible routes, we focus on a subset of the 29 stations that are served by *both* the Tokyo Metro and the Toei.<sup>3</sup> One reason for picking such stations is that there are at least two routes between these stations. For them, 812 (29 multiplied by 28) origin-destination (OD) pairs can be identified.

---

<sup>2</sup>In fact, some transfer discount is applicable provided that certain conditions are met.

<sup>3</sup>Even if a pair of stations have different names, we regard them as one station if the two operators do so in offering inter-operator transfer discounts. Furthermore, 2 stations, Meguro and Shirokane-Dai, are, although served by both operators, not included in the 29 stations, because they run trains on the same tracks between Meguro station and Shirokane-Takanawa station through Shirokane-Dai station.

Assuming that a travel starts at the noon on the September 1st in 2014, we look up the travel times and fares required for the corresponding 812 travel patterns and find the fastest route and the least expensive route for each pattern.<sup>4,5</sup>

The first four columns of Table 1 summarize the basic statistics of the travel times and fares of such routes for the 812 OD pairs. They indicate that the fastest route is faster than the least expensive route by 4.42 minutes on average, which amounts to 25.9% of the average travel time of the fastest route. The least expensive route is, furthermore, less expensive than the fastest route by 25.15 yen on average, which equals 13.4% of the average fare of the least expensive route. In an extreme case in which users always choose the least expensive routes, therefore, the travel time will decline by 25.9% (4.42 minutes) on average if tariff is integrated. However, the least expensive routes coincide the fastest routes for 353 OD pairs. For the remaining 459 OD pairs, which we call "irregular" OD pairs, the least expensive routes are not the fastest ones.<sup>6</sup> When only the irregular OD pairs concern, the time saving effect becomes much greater. The last four columns of the table show figures for the irregular OD pairs. The fastest route is now faster than the least expensive route by 7.81 minutes (43.22% of the average travel time of the fastest route) while the least expensive route is now less expensive than the fastest route by 44.25 yen (22.93% of the average fare of the least expensive route). If users always choose the least expensive routes, therefore, tariff integration will reduce the travel time by 43.22% for the irregular OD pairs.

---

<sup>4</sup>There are a few travel patterns for which either the fastest route or the least expensive route, or both of them include the legs served by the other train companies like the JR East. For the sake of simplicity, we disregard such routes and concentrate on the routes along which every leg is served by either of the two subway operators.

<sup>5</sup>Travel time includes the one to wait for trains' arrivals and, if any, the one to change trains.

<sup>6</sup>We say that these pairs are "irregular" by the following reason. In the Tokyo subway system, like many others elsewhere, there is no faster train service with an additional fee (with only one exception). Furthermore, the speeds of trains are not much different among lines. If the fare schemes were the same for the two operators, therefore, a travel with a longer distance would need more money and more time. Consequently, in this "regular" situation, fare would monotonically increase with travel time.

Table 1: Travel times and fares for the selected OD pairs in the Tokyo subway system<sup>a</sup>

	all the OD pairs				only the irregular OD pairs			
	the fastest route		the least expensive route		the fastest route		the least expensive route	
	time (min.)	fare (yen)	time (min.)	fare (yen)	time (min.)	fare (yen)	time (min.)	fare (yen)
minimum	1	170	1	170	2	180	4	170
maximum	43	320	54	240	43	320	54	240
average	17.08	212.76	21.50	187.61	18.07	237.21	25.88	192.96
median	17	200	21	180	18	220	25	200
standard deviation	7.83	42.17	9.58	19.94	7.87	39.49	8.60	21.16
number of samples	812				459			

<sup>a</sup> All the figures are manually derived from an internet site, Jorudan, by the author (<http://www.jorudan.co.jp/>).

Table 2: Number of the irregular OD pairs in the Tokyo subway system<sup>a</sup>

		difference in fare (yen)												sum
		10	20	30	40	50	60	70	80	90	100	110	120	
difference in time (min.)	1-5	31	56	11	0	26	0	11	12	0	5	37	2	191
	6-10	34	38	11	0	5	4	11	23	0	2	16	0	144
	11-15	10	26	13	1	8	0	5	11	0	0	2	0	78
	16-20	13	14	3	0	0	0	0	6	0	0	1	0	37
	21-27	4	6	0	0	0	0	0	1	0	0	0	0	11
sum		92	140	38	1	39	4	27	53	0	7	56	2	459

<sup>a</sup> All the figures are computed from the data manually derived from an internet site, Jorudan, by the author (<http://www.jorudan.co.jp/>).

To investigate these points in more detail, we, focusing on the irregular OD pairs, compute the difference in travel time and fare between the fastest route and the least expensive route for each pair. The result is summarized in Table 2. It says, for instance, that for 31 OD pairs, the fastest route is faster than the least expensive route by 1 to 5 minutes but more expensive by 10 yen. We can see that the difference in travel time is large for a considerable number of pairs. Indeed, 270 pairs exhibit the time difference of more than 5 minutes, 126 pairs more than 10 minutes, and 48 pairs more than 15 minutes. Moreover, the difference in fare is also quite large for not small part of such pairs. Among the 270 pairs with more-than-5-minute time difference, for example, 95 exhibit the fare difference of 50 yen or larger. True, many consumers will probably use the fastest route if it is not much more expensive; but otherwise, they will choose a less expensive route. For the 95 pairs, thus, it is plausible that consumers do not take the fastest route to save 50 yen or larger, although it reduces travel time by more than 5 minutes.

### 3 Basic framework

#### 3.1 Geography and transport services

Consider a city with four regions,  $W$  (West),  $CW$  (Central West),  $CE$  (Central East) and  $E$  (East), each of which is considered a point. Regions  $W$  and  $E$  are suburbs and regions  $CW$  and  $CE$  are city centers. In the city, there are consumers whose places of residence and workplaces are given. The workplaces are confined to either of the two central regions, region  $CW$  or region  $CE$ . Unless consumers work at the region where they live, they commute. The commutes are inelastic: each consumer uses a constant amount, say 1 unit, of a transport service irrespective of its price (i.e., fare).

Three patterns of commutes are identified. The first is an “inner-city” commute between regions  $CW$  and  $CE$ . It occurs when consumers who live in one of the two central regions work at the other central region. Taking a unit appropriately, we normalize the mass of those inner-city commuters at 1. The second pattern is “suburban” commutes between regions  $W$  and  $CW$  and between regions  $E$  and  $CE$ : consumers who live in a suburban region commute to its nearby central region. The last pattern is “city-wide” commutes between regions  $W$  and  $CE$  and between regions  $E$  and  $CW$ . Consumers commute from a suburban region to the central region located farther away from their residence than the other central region. The masses of the suburban commuters and city-wide commuters are fixed.

Two firms, say, railway companies, provide transport services for consumers’ commutes. In parallel with commutes and commuters, the services are classified into three groups according to the origins and destinations of carriages; the services to carry consumers between the two central regions, those between a suburban region and the central region closer to it, and those between a suburban region and the central region farther away from it. These groups of services are referred to as inner-city services, suburban services and city-wide services, respectively. Two assumptions are imposed. First, city-wide commuters do not jointly consume an inner-city service and a suburban service.<sup>7</sup> It is because the city-wide services are, probably with the operation of direct trains, speedy and convenient enough to attract all of them. Second, without loss of generality, we assume that firm 1’s inner-city service takes longer time than firm 2’s. Let  $t_i$  be the travel time involved when an inner-city commuter uses firm  $i$ ’s service ( $i \in \{1, 2\}$ ). Then, our assumption is that  $t_1 > t_2$ .

Firm 1 owns a transport route from region  $W$  to region  $CE$  via region  $CW$  while firm 2 does a route from region  $E$  to region  $CW$  via region  $CE$  (see Figure 1). Thus, firm 1 supplies the inner-city transport service between region  $CW$  and  $CE$ , the suburban service between regions  $W$  and  $CW$ , and the city-wide service between regions  $W$  and  $CE$ , whereas firm 2 supplies the inner-city transport service between region  $E$  and  $CE$ , the suburban service between regions  $E$  and  $W$ , and the city-wide service between regions  $E$  and  $CW$ .

Figure 1: Transport routes

By construction, inner-city commuters have an access to both firms’ inner-city services. They determine which firm’s service to use comparing the travel times and fares of the services of the two firms. We denote the mass of the inner-city commuters who use firm  $i$ ’s service by  $D_i$  with  $D_1 + D_2 = 1$  ( $i \in \{1, 2\}$ ). In contrast, suburban commuters and city-wide commuters use only the service of either firm. First, suburban services are provided by only one firm. Therefore, the suburban commuters who commute between regions  $W$  and  $CW$  (regions  $E$  and  $CE$ , resp.)

---

<sup>7</sup>In addition, there is a theoretical possibility that an inner-city commuter or a suburban commuter buys a city-wide transport service and uses only part of it. To avoid this possibility, it is assumed that the fares of city-wide services are sufficiently higher than those of inner-city services and suburban services.

can use only firm 1's (firm 2's, resp.) service. Second, as has mentioned earlier, it is supposed that city-wide commuters do not jointly consume an inner-city service and a suburban service. Consequently, the city-wide commuter who commutes between regions  $W$  and  $CE$  (regions  $E$  and  $CW$ , resp.) are to use firm 1's (firm 2's, resp.) service. We denote the masses of the city-wide commuters who use firm 1's service (or, commute between regions  $W$  and  $CE$ ) by  $\bar{L}_1$  and those who use firm 2's service (or, commute between regions  $E$  and  $CW$ ) by  $\bar{L}_2$ .

### 3.2 Consumers

The preferences of consumers are identical and described by a utility function,  $u(z, l)$ , where  $z$  and  $l$  are the amounts of a composite good and leisure. Consumers are heterogenous in that their wages are different. We assume that the wages are distributed in the interval  $[\underline{w}, \bar{w}]$  according to a given distribution function  $G(\cdot)$ , which is continuous. When a consumer uses for her commute a transport service that takes  $t$  units of time, her budget constraint is given by

$$z + wl = w(\bar{l} - t) - p \equiv y. \quad (1)$$

Here,  $\bar{l}$  is the available length of time and  $w$  and  $p$  are her wage rate and a fare of the transport service. The price of the composite good is given and it is taken as a numeraire. Furthermore,  $y$  represents consumer's "gross" income, which is a wage income,  $w(\bar{l} - t - l)$ , plus the opportunity cost of leisure,  $wl$ . Consumers maximize  $u(z, l)$  subject to (1). The resulting indirect utility is denoted by  $v(w, y)$ . Because the prices of composite good and leisure are fixed, travel time and a fare affect the indirect utility only through the gross income. Therefore, consumer surplus can be measured by the gross income.

### 3.3 Transport firms

To concentrate on the effects of tariff integration, we do not discuss the determination of the fares of the three types of transport services. They are considered fixed.

Let us look at the revenue of each firm. To begin with, the fares paid by inner-city commuters to firm  $i$  amount to  $p_i D_i$  ( $i \in \{1, 2\}$ ), where  $p_i$  is the fare of an inner-city service charged by firm  $i$  ( $i \in \{1, 2\}$ ). In contrast, the sum of the fares paid by suburban commuters and city-wide commuters becomes a constant, which we denote by  $\bar{R}_i$ . This is because their demands for transport services and the fares of a suburban service and a city-wide service are fixed. Thus, we can write firm  $i$ 's revenue as  $p_i D_i + \bar{R}_i$ .

Production cost of each transport firm is given by

$$C_i(D_i) = \gamma(D_i + \bar{L}_i) + F_i, \quad i \in \{1, 2\}. \quad (2)$$

$\gamma(\cdot)$  gives the variable cost to supply a transport service between the two central regions as a function of the mass of its users. Here, we are implicitly assuming that the *productions* of inner-city services and city-wide services are conducted jointly although their *consumptions* are done

separately. That is, the trains connecting the two central regions and those connecting a suburb and the central region farther away from it are different. However, both use the same tracks between the two central regions, and therefore, the incurred costs associated with that inner-city part depend on the total number of the users that pass it. For firm  $i$ , they are  $D_i$  inner-city commuters and  $\bar{L}_i$  city-wide commuters. It is assumed that  $\gamma(\cdot)$  is identical for the two firms and increasing. Furthermore,  $F_i$  is the sum of fixed costs and the variable costs that are exogenous. The exogenous variable costs are the variable costs of operating trains between a suburban region and the central region closer to it. Firm 1, for instance, runs trains between regions  $W$  and  $CW$  to provide a suburban service between regions  $W$  and  $CW$  and a city-wide service between regions  $W$  and  $CE$ . The variable costs incurred in this operation depends on the mass of their users, which is fixed. Therefore, the variable costs are also fixed.

These discussions establish the profit of each firm:

$$\pi_i = \pi_i(p_i, D_i) \equiv p_i D_i + \bar{R}_i - C_i(D_i), \quad i \in \{1, 2\}. \quad (3)$$

## 4 Effect of a common fare on consumers' welfare

The two transport firms charge different fares,  $p_1^0$  and  $p_2^0$ , for inner-city transport services initially, and then a common fare,  $p^c$ , after the integration of tariff. We compare the welfare levels of consumers before and after the tariff integration. In what follows, superscripts 0 and  $c$  (for "common fare") refer to the variables before and after the tariff integration, respectively.

Let us begin with the economy before the tariff integration. Since  $t_1 > t_2$ , all the inner-city commuters use firm 2's service if  $p_1^0 \geq p_2^0$ . This is not an interesting case and therefore, we focus on the other case with  $p_1^0 < p_2^0$ . Furthermore, the gross income of an inner-city commuter who uses firm  $i$ 's service is denoted by  $y_i^0 \equiv w(\bar{L} - t_i) - p_i^0$ .

Inner-city commuters use the service that yields a higher indirect utility. That is, they use the service of firm  $\left\{ \frac{1}{2} \right\}$  if  $v(w, y_1^0) \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} v(w, y_2^0)$ . If  $v(w, y_1^0) = v(w, y_2^0)$ , they are indifferent between the services of the two firms. It is obvious that  $v(w, y_1^0) \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} v(w, y_2^0)$  if  $y_1^0 \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} y_2^0$ , since the indirect utility is increasing in consumer's income. Therefore, the consumers use the service of firm  $\left\{ \frac{1}{2} \right\}$  if  $y_1^0 \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} y_2^0$ , or equivalently, if  $w \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} \hat{w} \equiv (p_2^0 - p_1^0) / (t_1 - t_2)$ , where  $\hat{w} > 0$ .<sup>8</sup> This reflects the fact that a consumer with a lower wage attaches relatively smaller importance to the time costs of travel compared to a consumer with a higher wage. We call the set of the inner-city commuters with  $w < \hat{w}$  a *low wage group* and those with  $w > \hat{w}$  a *high wage group*.

Since the mass of inner-city commuters is normalized at 1, the size of the low wage group is  $G(\hat{w})$ , which is equal to the demand for firm 1's inner-city service. Similarly, the demand for firm

<sup>8</sup>Since the measure of the consumers whose incomes satisfy  $y_1^0 = y_2^0$  is zero, we can safely disregard them.

2's inner-city service is equal to  $1 - G(\hat{w})$ :

$$D_1^0 = G(\hat{w}) \quad \text{and} \quad D_2^0 = 1 - G(\hat{w}). \quad (4)$$

Suppose that a common fare is introduced for inner-city services. We concentrate on the common fare in between the fares of the two firms before the tariff integration, that is,  $p^c \in [p_1^0, p_2^0]$ . It is obvious that inner-city commuters receive higher gross incomes by using firm 2's transport service than firm 1's since  $w(\bar{l} - t_2) - p^c > w(\bar{l} - t_1) - p^c$ . Therefore, all of them use firm 2's service, and consequently, the sizes of demands are given by

$$D_1^c = 0 \quad \text{and} \quad D_2^c = 1. \quad (5)$$

We denote the resulting levels of their gross income by  $y^c \equiv w(\bar{l} - t_2) - p^c$ .

As a result of the introduction of a common fare, a consumer in the low wage group switches the service to use from firm 1's to firm 2's. Thus, her gross income increases by

$$y^c - y_1^0 = w(t_1 - t_2) - (p^c - p_1^0).$$

This also represents the increase in her surplus,  $\Delta s_L(p^c)$  (subscript  $L$  refers to the low wage group) since consumer surplus is measured by the level of a gross income. For given  $w$ ,  $\Delta s_L(p^c)$  increases with  $t_1 - t_2$ , the reduction in travel time, and decreases with  $p^c - p_1^0$ , the rise in the payment for a transport service. The sign of  $\Delta s_L(p^c)$  depends on  $w$ : a consumer in the low wage group may or may not benefit from the introduction of a common fare. More specifically,  $\Delta s_L(p^c) \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} 0$  if  $w \left\{ \begin{array}{l} \geq \\ < \end{array} \right\} \tilde{w} \equiv (p^c - p_1^0) / (t_1 - t_2)$ , where  $\tilde{w} \in [0, \hat{w}]$ . Therefore, no consumer in the low wage group becomes worse off by the tariff integration if

$$\underline{w} \geq \tilde{w}, \quad (6)$$

that is, if income inequality is sufficiently small. This condition for weak Pareto-improvement is more likely to be satisfied when  $t_1 - t_2$  is larger and  $p^c - p_1^0$  is smaller. This is because larger  $t_1 - t_2$  and smaller  $p^c - p_1^0$  are associated with larger  $\Delta s_L(p^c)$ , as we have seen. In contrast, a consumer in the high wage group uses firm 2's service not only after but also before the tariff integration; and therefore, the increase in his surplus is equal to  $\Delta s_H(p^c) \equiv y^c - y_2^0 = p_2^0 - p^c$  (subscript  $H$  denotes a consumer in the high wage group).<sup>9</sup> As long as  $p^c \in [p_1^0, p_2^0]$ ,  $\Delta s_H(p^c)$  becomes nonnegative: no consumer in the high wage group becomes worse off by the introduction of a common fare.

Note that the surplus of a suburban commuter and that of a city-wide commuter do not change. Therefore, the total effect of the introduction of a common fare on the consumer surplus, denoted by  $\Delta S(p^c)$ , is equal to the sum of  $\Delta s_L(p^c)$ 's and  $\Delta s_H(p^c)$ 's for all the inner-city

<sup>9</sup>The increase in a gross income is also equal to the equivalent variation (EV) and compensating variation (CV), which become identical in this simple setting. For the low wage group, they are given by  $e(w, v(w, y^c)) - e(w, v(w, y_1^0)) = \Delta s_L(p^c)$ ; and for the high wage group, they are given by  $e(w, v(w, y^c)) - e(w, v(w, y_2^0)) = \Delta s_H(p^c)$ . Here,  $e(\cdot, \cdot)$  denotes an expenditure function.

commuters, that is,  $\Delta S(p^c) = \int_{\underline{w}}^{\hat{w}} \Delta s_L(p^c) dG(w) + \int_{\hat{w}}^{\bar{w}} \Delta s_H(p^c) dG(w)$ . It is straightforward to obtain

$$\Delta S(p^c) = (t_1 - t_2) \int_{\underline{w}}^{\hat{w}} w dG(w) + (p_1^0 - p^c)G(\hat{w}) + (p_2^0 - p^c)[1 - G(\hat{w})]. \quad (7)$$

The first term in the right hand side represents the time cost saving effect of a common fare. The remaining two terms indicate the monetary cost saving effects for the low wage group and for the high wage group, respectively. The effect is negative for the former group and positive for the latter.

As a benchmark, we examine the common fare for which the sum of the two firms' revenues does not change, that is,  $p_1^0 D_1^0 + p_2^0 D_2^0 = p^c D_1^c + p^c D_2^c$ . (4) and (5) imply that such a fare is given by  $p^{c*} \equiv p_1^0 G(\hat{w}) + p_2^0 [1 - G(\hat{w})]$ . This is the average of the two firms' fares weighted by the respective masses of the users of their inner-city services.

For this common fare,  $\tilde{w}$ , the lowest value of  $\underline{w}$  for which no inner-city commuter becomes worse off, is given by

$$\tilde{w} = \hat{w} [1 - G(\hat{w})]. \quad (8)$$

To get a further intuition, it is helpful to use Figure 2, which describes function  $G(w)$ . Using (8), we can rewrite the weak Pareto-improvement condition (6) as

$$\frac{G(\hat{w})}{\hat{w} - \underline{w}} \geq \frac{1}{\hat{w}'},$$

where the left hand and right hand sides represent the slopes of lines  $AB$  and  $OC$ , respectively: the condition is that line  $AB$  is steeper than line  $OC$ .

Figure 2: The weak Pareto-improvement condition

The directions of the effects of changes in parameters upon  $\tilde{w}$  are ambiguous.<sup>10</sup> This is because they affect  $\tilde{w}$  not only directly but also indirectly through a change in the common fare. As  $t_1 - t_2$  rises (as travel time is reduced by the tariff integration more), for instance,  $\Delta s_L(p^c)$  increases, other things being equal. However, other things are not equal:  $\hat{w}$  declines and, as a result, the common fare  $p^{c*}$  rises, which gives a negative impact on  $\Delta s_L(p^c)$ .

Furthermore, it immediately follows from (7) that  $\Delta S(p^{c*}) = (t_1 - t_2) \int_{\underline{w}}^{\hat{w}} w dG(w) > 0$ : consumers *as a whole* benefit from the introduction of this common fare.

### Proposition 1

*Consumers as a whole benefit from the tariff integration when the common fare is determined so as to keep the total revenue of transport firms unchanged.*

<sup>10</sup>Indeed, we obtain the following results:

$$\frac{d\tilde{w}}{d(p_2^0 - p_1^0)} = \frac{1 - G(\hat{w}) - \hat{w}G'(\hat{w})}{t_1 - t_2} \quad \text{and} \quad \frac{d\tilde{w}}{d(t_1 - t_2)} = -\hat{w} \frac{d\tilde{w}}{d(p_2^0 - p_1^0)}.$$

The signs of the two derivatives are indeterminate.

## 5 A case with a specific distribution of wages

Before proceeding to the analysis of total welfare, we examine a special case with a specific distribution of wages, which will help us intuitively understand some of the above findings.

Suppose that wages are uniformly distributed over  $[\underline{w}, \bar{w}]$ , that is,  $G(w) = (w - \underline{w}) / (\bar{w} - \underline{w})$ .

To begin with, it is straightforward to obtain  $\tilde{w} = \hat{w}(\bar{w} - \hat{w}) / (\bar{w} - \underline{w})$ , which enables us to rewrite (6) as

$$\underline{w} - \tilde{w} = \frac{(\hat{w} - \underline{w}) [\hat{w} - (\bar{w} - \underline{w})]}{\bar{w} - \underline{w}} \geq 0. \quad (9)$$

If  $\bar{w} \leq 2\underline{w}$ , then  $\hat{w} - (\bar{w} - \underline{w}) \geq 0$  given that  $\hat{w} \geq \underline{w}$ . In this case, therefore, (9) is always satisfied. If  $\bar{w} > 2\underline{w}$ , instead, whether (9) is satisfied or not depends on the sign of  $\hat{w} - (\bar{w} - \underline{w})$ : it is satisfied if and only if

$$\hat{w} \geq \bar{w} - \underline{w}. \quad (10)$$

The inverse of the left hand side of (10) is the slope of the line connecting the origin and  $(\hat{w}, 1)$ , namely, line  $OC$  in Figure 2. The inverse of its right hand side is, moreover, the slope of the distribution function, which corresponds to the slope of line  $AB$  in that figure. Consequently, the weak Pareto-improvement condition is that line  $AB$  is steeper than line  $OC$ , which is what we have demonstrated earlier.

Furthermore, the directions of the effects of changes in parameters upon the condition are unambiguously determined in this specific example. In the relevant case with  $\bar{w} > 2\underline{w}$ , it is more likely that no inner-city commuter becomes worse off by the introduction of the common fare  $p^{c*}$  when  $p_2^0 - p_1^0$  is larger,  $t_1 - t_2$  is smaller, and  $\bar{w} - \underline{w}$  is smaller. This result immediately follows from (10).

In Figure 3, the gross income,  $y_i^0$ , of an inner-city commuter who uses firm  $i$ 's service before the tariff integration is described by line  $l_i^0$  ( $i \in \{1, 2\}$ ). Lines  $l_1^0$  and  $l_2^0$  intersect each other at  $w = \hat{w}$ . Note that the former line lies above the latter for  $w \in [\underline{w}, \hat{w})$ . Thus, the consumers with  $w < \hat{w}$  obtain a higher gross income by using firm 1's service than firm 2's. The opposite holds for  $w \in (\hat{w}, \bar{w}]$ : the consumers with  $w > \hat{w}$  get a higher gross income by using firm 2's service. Correspondingly, firms' revenues before the tariff integration are given by the areas of the two shaded regions, respectively, i.e., the area of rectangular  $ABCD$  for firm 1 and that of rectangular  $BEFG$  for firm 2.

Figure 3: The case of uniform distribution

The gross income of each consumer after the tariff integration is represented by line  $l^c$ , which is parallel to line  $l_2^0$  and lies above it. Its increase is measured by the vertical distance between lines  $l^c$  and  $l_1^0$  for the low wage group and that between lines  $l^c$  and  $l_2^0$  for the high wage group. The figure describes the case where some consumers in the low wage group (the consumers with the wage lower than  $\hat{w}$ ) become worse off, that is, the case where the weak Pareto-improvement

condition (9) is violated. The total increase of the gross income of consumers in each group is equal to the area of triangle  $RST$  minus the area of triangle  $PQR$  for the low wage group and the area of parallelogram  $STUV$  for the high wage group. Firm 2's revenue after the tariff integration is equal to the area of rectangular  $AEJH$  while firm 1 receives no revenue.

In general, we cannot determine if the overall increase in gross income, namely, the area of quadrilateral  $RTUV$  minus that of triangle  $PQR$ , exceeds the loss of total revenue, namely, the area of rectangular  $FGIJ$  subtracted by that of rectangular  $DCIH$ . However, Proposition 1 says that it is indeed the case for  $p^c = p^{c*}$ . In that case, the total revenue remains constant and therefore, the loss is zero. In other words, the area of rectangular  $FGIJ$  is equal to that of rectangular  $DCIH$ . The proposition guarantees that quadrilateral  $RTUV$  is larger than triangle  $PQR$ .

## 6 Effect of a common fare on total welfare

In this section, we examine the change in total welfare. First of all, it is assumed that both firms earn nonnegative profits before the tariff integration, that is,  $\pi_i(p_i^0, D_i^0) \geq 0$  for  $i \in \{1, 2\}$ . Next, the change in the sum of the profits of the two firms is equal to

$$\Delta\Pi(p^c) \equiv \sum_{i \in \{1, 2\}} \left[ \pi_i(p^c, D_i^c) - \pi_i(p_i^0, D_i^0) \right] = p^c - \left( p_1^0 D_1^0 + p_2^0 D_2^0 \right) - (\Delta C_1 + \Delta C_2), \quad (11)$$

where  $\Delta C_1 \equiv \gamma(\bar{L}_1) - \gamma(D_1^0 + \bar{L}_1) < 0$  and  $\Delta C_2 \equiv \gamma(1 + \bar{L}_2) - \gamma(D_2^0 + \bar{L}_2) > 0$  are the changes in the costs for firm 1 and firm 2, respectively (see (2) and (3)). Note that firm 1's cost decreases while firm 2's increases.

In a special case where the marginal cost is constant, the total cost remains unchanged after the tariff integration:  $\Delta C_1 + \Delta C_2 = 0$ . Therefore, (11) implies that  $\Delta\Pi(p^{c*}) = 0$ . We have established the following result.

### Proposition 2

*When the marginal cost is constant, there is a common fare for which consumers as a whole benefit from the tariff integration and the sum of the profits of the two transport firms is nonnegative after the integration.*

In other words, we can always find a common fare that makes consumers better off and *neither* of the two firms worse off with an appropriate redistribution of incomes among consumers and that of revenues between the firms.

To study a more general case where the marginal cost is not constant, we define  $\phi(x) \equiv \gamma(x + D_1^0) - \gamma(x)$ . Then,  $D_1^0 + D_2^0 = 1$  implies that

$$\Delta C_1 + \Delta C_2 = \phi(D_2^0 + \bar{L}_2) - \phi(\bar{L}_1). \quad (12)$$

First, suppose that function  $\gamma(\cdot)$  is concave, that is, the marginal cost is nonincreasing, in the relevant range of output. In this case,  $\phi(\cdot)$  is nonincreasing. When  $D_2^0 + \bar{L}_2 \geq \bar{L}_1$ , therefore,  $\phi(D_2^0 + \bar{L}_2) \leq \phi(\bar{L}_1)$  and  $\Delta C_1 + \Delta C_2 \leq 0$  by (12). In other words, when firm 2 gets relatively *larger* exogenous demand compared to firm 1, the total cost does not increase as a result of the tariff integration. Instead, if function  $\gamma(\cdot)$  is convex,  $\phi(\cdot)$  is nondecreasing. In this case,  $\Delta C_1 + \Delta C_2 \leq 0$  when  $D_2^0 + \bar{L}_2 \leq \bar{L}_1$ : the total cost does not increase when firm 2 gets relatively *smaller* exogenous demand. In addition, (11) implies that  $\Delta\Pi(p^{c*}) \geq 0$  when  $\Delta C_1 + \Delta C_2 \leq 0$ . This leads to the following result:

**Proposition 3**

*If the marginal cost is nonincreasing (nondecreasing, resp.) and firm 2 gets relatively larger (smaller, resp.) exogenous demand compared to firm 1, there is a common fare for which consumers as a whole benefit from the tariff integration and the sum of the profits of the two transport firms is nonnegative after the integration.*

This result is not surprising. When tariff is integrated, all the inner-city commuters use firm 2's service, which is more efficient than firm 1's in terms of travel time. Therefore, if there is an economy of scale (in the sense of a decreasing marginal cost), the total cost after the tariff integration is lower when firm 2 gets a *larger* exogenous demand. To the contrary, if there is a diseconomy of scale, the total cost is lower when that firm gets a *smaller* exogenous demand.

## 7 Concluding remarks

One of the most important benefits of tariff integration is the removal or alleviation of the distortion in consumers' choices, which can arise when a more efficient route costs more. The distortion is particularly serious in Japanese big cities, where urban transit services are provided by a number of different institutions and tariff is not integrated. In this paper, we have examined how tariff integration reduces this distortion based on a simple but rigorous model. Furthermore, we have shown that under certain conditions, there exists a common fare that makes both consumers and transport firms better off given a redistribution of incomes among consumers and that of revenues among firms.

To conclude the paper, we briefly discuss the limitations of this paper. First, our results are based on a model with a quite specific setting. It is assumed, for instance, that each consumer's demand for a transport service is inelastic. To give another example, the fares of a suburban service and a city-wide service are assumed to be independent of the fare of an inner-city service. Second, we have not discussed the determination of fares. In particular, the fares prevailing before tariff integration are considered given. In the reality, however, they are determined by firms themselves or authorities. For example, if the production of a transport service exhibits

the property of decreasing cost, as widely alleged, the fares are probably regulated at average cost. Furthermore, the analysis is focused on a special common fare, i.e., the average of respective firms' fares weighted by the masses of their users before the integration. The question of what common fare is socially desirable is left unanswered. Third and finally, we have not conducted a full empirical analysis that estimates the effects of tariff integration, but have presented only an introductory case study. One could evaluate the size of the distortion in consumers' choices attributed to unintegrated tariff, using detailed data on the fares of various routes and the actual consumers' trip behaviors, probably with a help of a discrete choice model. It might be one of the most important agenda, although accomplishing it requires considerable efforts, particularly when a big city like Tokyo is concerned.

## References

- [1] Abrate, G., M. Piacenza, and D. Vannoni. (2009) The impact of integrated tariff systems on public transport demand: Evidence from Italy, *Regional Science and Urban Economics*, 39, 120-127.
- [2] Carbajo, J. (1988) The economics of travel passes, *Journal of Transport Economic Policy*, 22, 153-173.
- [3] Cassone, A., and C. Marchese. (2005) Welfare effects of price integration in local public transport, *Annals of Public and Cooperative Economics*, 76, 257-274.
- [4] Dargay, J. M., and S. Pekkarinen. (1997) Public transport pricing policy: empirical evidence of regional bus card systems in Finland, *Transportation Research Record*, 1604, 146-152.
- [5] FitzRoy, F., and I. Smith. (1998) Public transport demand in Freiburg: why did patronage double in a decade? *Transport Policy*, 5, 163-173.
- [6] FitzRoy, F., and I. Smith. (1999) Season tickets and the demand for public transport, *Kyklos*, 52, 219-238.
- [7] Gilbert, C. L., and H. Jalilian. (1991) What is a farecard worth? *Applied Economics*, 23, 1053-1058.
- [8] Giuliano, C., J. E. Moore, and J. Golob. (2000) Integrated smart-card fare system: results from field operational tests, *Transportation Research Record*, 1735, 138-146.
- [9] Hirsch, L. R., D. Jordan, R. Hickey, and V. Cravo. (2000) Effects of fare incentives on New York City transit ridership, *Transportation Research Record*, 1735, 147-157.
- [10] Lee, D. (1999) Introducing fare simplification and new convenience fares at Connecticut Transit, *Transportation Research Record*, 1669, 109-112.
- [11] Marchese, C. (2006) The economic rationale for integrated tariffs in local public transport, *Annals of Regional Science*, 40, 875-885.
- [12] Matas, A. (2004) Demand and revenue implications of an integrated public transport policy: the case of Madrid, *Transport Reviews*, 24, 195-217.
- [13] NEA. (2003) Integration and regulatory structures in public transport, *Final report of NEA Transport Research and Training to the European Commission*, NEA, Rijswijk, Netherlands.
- [14] Scottish Executive Social Research. (2004) Integrated ticketing in Scotland - needs analysis and options. *Transport Research Series*, Edinburgh, UK.
- [15] Sharaby, N. and Y. Shiftan. (2012) The impact of fare integration on travel behavior and transit ridership, *Transport Policy*, 21, 63-70.

- [16] Taylor, S., and D. Carter. (1988) Maryland mass transit administration fare simplification: effects on ridership and revenue, *Transportation Research Record*, 1618, 125-130.
- [17] TRL. (2004) The demand for public transport: a practical guide, *Transport Research Laboratory Report*, 593, TRL, Crowthorne, UK.
- [18] Ungemah, D., R. Malaika, and A. Stuart. (2006) The world can be flat: Case study of flat-rate pricing for vanpool operations, *Transportation Research Record*, 1956, 30-33.
- [19] White P. R. (1981) "Travelcard" tickets in urban public transport, *Journal of Transport Economic Policy*, 15, 17-34.

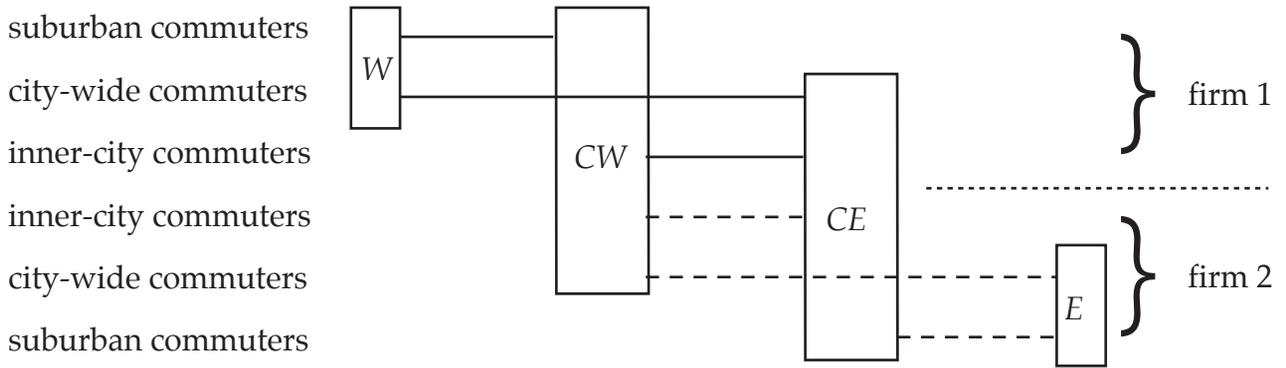


Figure 1: Transport routes

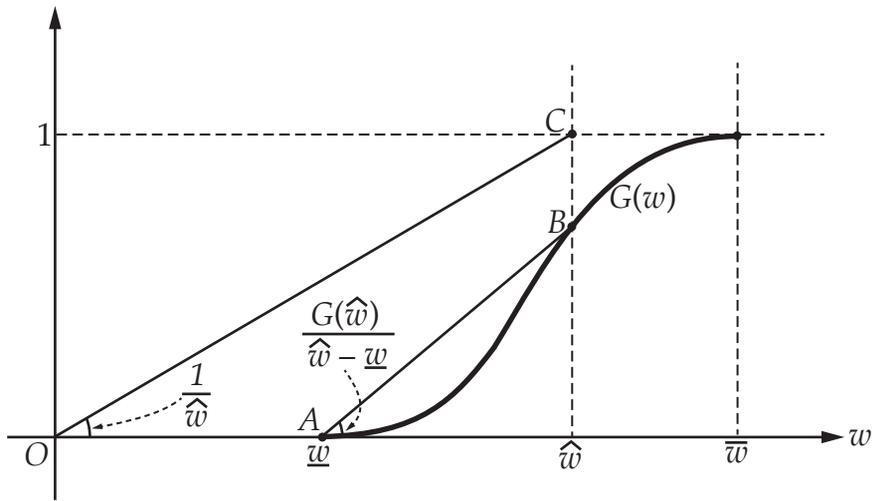


Figure 2: The weak Pareto-improvement condition

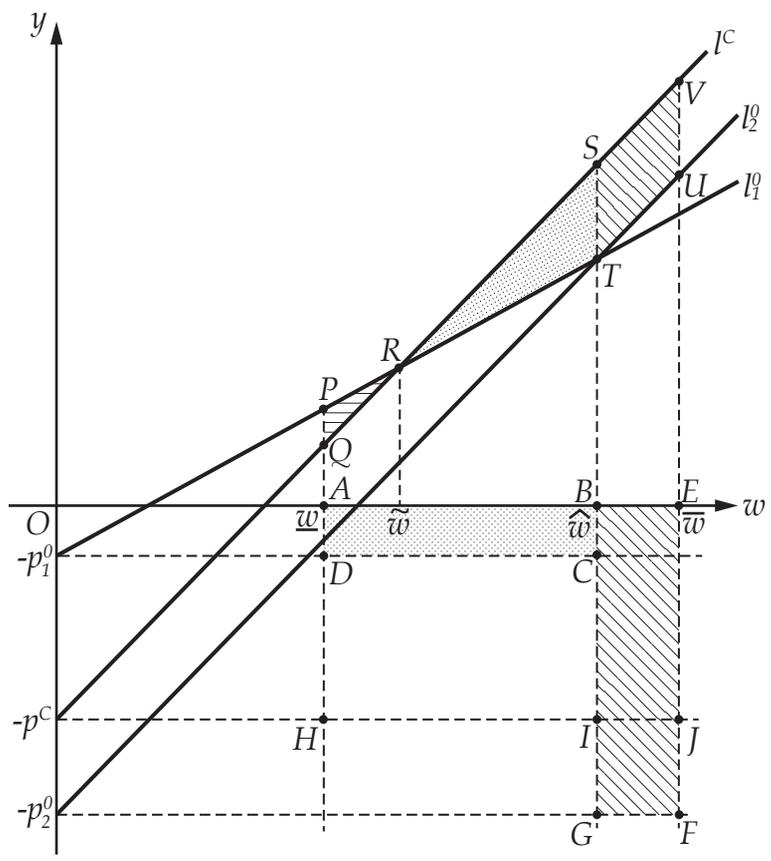


Figure 3: The case of uniform distribution