Is the Transport Sector Too Large?
Welfare Analysis of the Trade Model with a Transport Sector

Takaaki Takahashi*

October, 2008

Abstract
Because the demand for the transport services stems from that for the final goods, the sizes of the transport sector and the final goods sector in an economy depend on each other and are determined simultaneously. If there is a distortion in the transport sector, notably a distortion arising from the imperfect competition, however, its size realized by the market mechanism may differ from the socially desirable one. In this paper, we examine how the consequence of the market mechanism differs from the social optimum, considering the first best policy, the price regulation and the entry regulation.

Keywords: Cournot oligopoly; derived demand; entry and exit; number of transport firms; price of transport services; regulation; transport industry

JEL Classification Numbers: L13 (Oligopoly and Other Imperfect Markets); L51 (Economics of Regulation); R13 (General Equilibrium and Welfare Economic Analysis of Regional Economies); R48 (Transportation Systems: Government Pricing; Regulatory Policies)

---

*Center for Spatial Information Science, University of Tokyo, 5-1-5, Kashiwa-no-ha, Kashiwa, Chiba, 277-8568, Japan. E-mail: takaaki-t@cis.u-tokyo.ac.jp. I thank seminar participants at various institutions for valuable comments and suggestions. I gratefully acknowledges financial support by Ministry of Education, Science and Culture in Japan (Grant in Aid for Research No. 18600004).
1 Introduction

For a long period of time, economists have been discussing government interventions in transport industries. One of the main justifications for them is that the industries are usually characterized by increasing returns to scale or decreasing average cost, which allows only a small number of firms or more typically the only one firm to survive in each of the industries. In order to cope with the distortion caused by the exercise of monopoly power, governments are expected to carry out economic policies like regulations of the price of transport services and of the number of firms in an industry. Such regulation policies on decreasing returns industries have been one of the most frequented topics in the field of industrial organization, so that there is thick literature on it (see Spulber (1989), Train (1991) and the literature cited there).

Since the drastic changes in the regulation policies toward an airline industry in the US in 1978, however, many developed economies have been undertaking various forms of deregulation in the transport industries. Railroad industry is a typical example. The UK and Japan had already embarked on a reform. In the other European countries, although the speeds of deregulation differ from country to country with Sweden and Germany, for instance, being quite ahead while France much behind, a general policy of the European Union is heading toward the deregulation.\(^1\) In such circumstances, reassessing the effects of regulation/deregulation in transport industries is not a matter of minor importance.

Having said that, merely applying the framework in the industrial organization literature to transport industries can be not only insufficient but also misleading, for there is a key factor which is usually disregarded in the literature but comes to play a critical role when transport industries are concerned. It is one of the properties of the demand for transport services, the property that it is a derived demand, as such researchers as Rietveld and Nijkamp (2003) emphasize: the services themselves do not directly contribute to the consumers’ utility, but they are inevitably accompanied by the consumption of final goods.\(^2\)

To recognize this property is important because it immediately implies that the level of the other economic activities, in particular, the production of final goods depends on the size of the transport sector, and vice versa. On the one hand, the size of the transport sector affects the price of transport services, which alters the delivered prices of final goods and therefore their demand. An expansion and a shrinkage of the final good sector is, on the other hand, associated with the changes in trade flows between regions/countries and thus in the demand for transport services. Such viewpoints are indispensable when one examines the government interventions. The regulation of the price of transport services, for example, immediately affects the demand for final goods. The entry regulation in the transport sector, for another example, not only harms the demand for final goods through a rise in the price of transport services

---

\(^1\) For the movements toward deregulation in transport industries and their consequences in industrialized countries, see Winston (1993) and Quinet and Vickerman (2004).

\(^2\) Notwithstanding, in standard textbook applications, the determination of the amount of demand for transport services is, in most cases, put into a black box: the explanation usually begins with a given inverse demand curve that comes from nowhere (see Balchin et al. (2000) and O’Sullivan (2006), for instance).
due to less competitive behaviors of transport firms, but also encourages the production of final goods because the transport sector is now relieved of some of the resources previously used there for a fixed input. In this way, it is not innocuous for us to discuss, with isolating the transport sector from the others, whether the transport sector tends to become too large or too small and how government should regulate it. The linkages between sectors are all the more important because the transport sector has a considerable weight in most of the industrialized economies.3

The purpose of this paper is to evaluate the size of the transport sector realized by a market mechanism and to examine the government policies against the inefficiencies due to a monopoly power prevalent in the sector, paying special attention to the fact that the demand for transport services is a derived one. For that purpose, we construct a simple Ricardian model of trade with two regions. In order to carry goods between regions/countries, they need to use transport services, which are provided by transport firms. Thus, the demand for the transport services stems from that for the final goods. Furthermore, because of increasing returns to scale, the transport sector is characterized by the oligopolistic competition à la Cournot. We examine the levels of transport cost and the numbers of transport firms realized at a market equilibrium and at the equilibria under three types of government interventions: the regulation of both the price of the transport services (i.e., transport cost) and the number of transport firms, that is, the first best policy; the second best policy with only the price regulation; and the second best policy with only the entry regulation.

One of the most interesting findings in this paper is that the number of transport firms at the market equilibrium is too large or too small depending on the level of fixed cost in the production of transport services and the size of the economy. To understand this point, it is helpful to consider what happens if a firm enters into the transport sector. On the one hand, it intensifies the competition among transport firms, which causes a decline in the price of transport services (transport cost) and, consequently, the price of imports. As a result, consumers, who now find themselves to be able to consume more final goods, become better off. This is a competition effect of the entry. On the other hand, the economy needs to allocate a certain amount of its limited resources to the fixed input of the entrant at the sacrifice of final goods production, which aggravates the welfare of consumers. This is referred to as a fixed input effect of the entry. If the competition effect more than offsets the fixed input effect at the market equilibrium, social welfare will be improved by the entry. In this case, the number of the firms is too small. If the competition effect is dominated by the fixed input effect, to the contrary, the number is too large.

To study an optimal size of transport sector and a desirable form of its regulation with taking into account its interdependence with the other sectors has not constituted a main concern of the researchers in related fields of economics.

First of all, it has been rare that the transport sector is explicitly examined in the fields that study trade of goods and services between regions and countries. In the international economics, on the one

3Quinet and Vickerman (2004) estimate that expenditure on transport in EU countries amounted to between 9 per cent to 17 per cent of total household expenditure in 2000. Furthermore, McCarthy (2001) reports that the transportation bill occupies 16.1 % of the GDP and that the share of the employment in the transport sector accounts to 8%, both in the US in 1995.
hand, even transport costs (or more broadly trade costs) have seldom been incorporated to models, not
to mention the transport sector.\footnote{The attempts to take into account transport costs include a classical article by Sannels (1952), Herberg (1970), Falvey (1976), Cassing (1978), Casas (1983), and Casas and Kwan Choi (1990). See Steininger (2001) for the overview.} In the regional economics, on the other hand, it is true that transport
costs have been playing a key role in the explanation of locations of economic activities, in particular,
in a subfield called new economic geography (Krugman (1991), Fujita, Krugman and Venables (1999),
and Fujita and Thisse (2002), among others). For all that, most of studies in that literature treat the
level of transport cost as given, which is one of many consequences of abstracting away the transport
sector. Even when they discuss endogenous determination of transport costs, it is not, in most cases,
dealt as a result of individual behaviors of transport firms.\footnote{Mori and Nishikimi (2002) have discussed the situation in which transport cost decreases with the volume of trade. In Takahashi (2006), I have tackled the question of how economic geography affects transport cost. Finally, Kanemoto and Mera (1985), Bougeas, Demetriades and Morgenroth (1999), and Mun and Nakagawa (2005) have analyzed the impacts of infrastructure investment upon transport cost and the resulting change in the amount of goods transported.} There are two exceptions. First, Behrens et al. (2006) examine the impacts of the regulation of a transport sector upon welfare paying attention to
the incentives of transport firms, though in a very simple way. Second, Takahashi (2007) discusses the
behaviors of individual transport firms to obtain welfare implications in a general equilibrium setting.

Second, in a field of urban economics, a number of researchers have accumulated studies on the
government interventions to a transport sector aimed at the congestion externalities (see Small (1992),
and Small and Gómez-Ibáñez (1999) for the review). Notwithstanding, it seems that there is a big gap
between their concerns and ours. First, they consider the commuting cost from residences to workplaces
rather than the transport cost to ship goods. This is because their main interest lies in a spatial structure
of cities but not in regional trade. Second, as its consequence, they do not attempt to fully embody the
idea of the derived demand property for transport services: it is usually assumed that each worker makes a
given amount of trip, often normalized to one trip, between their homes and offices. True the commuting
distance for each worker and its aggregate are variables to be determined, but with the number of trips
given, the assertion that the demand for transport services is the derived one holds only partially.

Lastly, it is worth adding another literature in relation to the conclusion that the number of transport
firms can be too small for some values of parameters, as has been mentioned. It makes a sharp contrast
to the result that oligopoly involves excessive entry because of the existence of a business stealing effect
(Mankiew and Whinston (1986), and Suzumura and Kiyono (1987), among others). The reason for this
disparity is that the literature focuses on a partial equilibrium whereas this paper on a general equilibrium.
That is, we consider the situation in which the market of transport services is interrelated to that of final
goods: an entry of a firm into the transport sector affects not only the demand for transport services but
also that for final goods. More specifically, an entry stimulates the consumption of final goods through
a decline in the prices of transport services (transport cost) and final goods, as we have explained as the
competition effect. This mitigates the business stealing effect and consequently, the result is not always
the excess entry.

The paper is organized as follows. In the subsequent section, basic framework is presented. In section
3, we formulate the behaviors of the oligopolistic transport firms that make a decision on the quantity of transport services to produce and on the entry and exit. In section 4, government policies are examined. Three types of policies, namely, price and entry regulation (the first best policy), price (transport cost) regulation and entry regulation are examined. We show that the price regulation is better than the entry regulation. Section 5 deals with a market equilibrium. We ask whether the market equilibrium is associated with too many or too few transport firms. Section 6 concludes.

2 Basic framework

We consider a simple Ricardian economy with two regions, East and West (which may be denoted as region $E$ and region $W$ when it is more convenient), and two goods, $E$ and $W$. The total population of the economy is fixed at $L$. The share of the population in East, which is endogenous in the long-run, is denoted by $\theta$.

Both goods are produced through a constant returns to scale technology. Let $a_{ij}$ be the unit labor requirement to produce good $i$ in region $j$. East has an absolute advantage in the production of good $E$ while West in the production of good $W$: $a_{WE} = \beta_E a_{EE}$ for some $\beta_E > 1$ and $a_{EW} = \beta_W a_{WW}$ for some $\beta_W > 1$. Obviously, East (West, resp.) has not only the absolute advantage but also a comparative advantage in the production of good $E$ (good $W$, resp.). For simplicity, furthermore, we focus on the symmetric case where the degrees of the absolute advantage are equal: $\beta_E = \beta_W \equiv \beta$. Finally, one can choose units so that $a_{EE} = a_{WW} = 1$. Then, $a_{WE} = a_{EW} = \beta$.

Furthermore, workers have Cobb-Douglas preference with the share of spending on each good being equal to $1/2$. This rather restrictive assumption of symmetry is made for the sake of simplicity, because without it the model would become too complicated for us to use as a basis for the subsequent general equilibrium analysis with the explicit consideration of the behaviors of transport firms. Besides, the symmetric case is a useful benchmark and yields the results that are qualitatively similar to those obtained in asymmetric cases. Under this preference, the demand functions for the two goods become

$$X_{ij} = \frac{Y_j}{2p_{ij}}, \quad (i, j = E, W)$$

where $p_{ij}$ and $X_{ij}$ are price and aggregate consumption of good $i$ in region $j$, respectively, and $Y_j$ is aggregate income of region $j$.

In addition, because good $E$ is produced in East while good $W$ is in West no matter whether the economy engages in trade or not, we always have

$$p_{ii} = a_{ii}w_i = w_i, \quad (i = E, W)$$

where $w_i$ is wage rate in region $i$. Let us consider good $E$ in East a numéraire, that is, $w_E = 1$ and $w_W = p$, where $p$ is a relative price of good $W$ in West. On the other hand, the price of good $E$ in West and that of good $W$ in East hinge on whether trade occurs or not. If it does not occur, they are given by $p_{ij} = a_{ij}w_j = \beta w_j$ for $i, j = E, W$ with $i \neq j$. If the trade occurs, instead, $p_{ij}$’s depend on transport costs.
Thus, let us introduce a transport sector, which consists of \( n \) transport firms. They provide transport services necessary to ship goods between regions, and charge an ad valorem price. Let \( t_E \) be the price of the transport services that ship 1 dollar of good \( E \) from East to West: the transport cost to have 1 unit of that good shipped in that direction is equal to \( p_E t_E \). \( t_W \) is similarly defined. Then, when trade occurs, the price of the good \( i \) produced in region \( i \) and consumed in region \( j \) becomes equal to

\[
p_{ij} = (1 + t_i)p_{ii} = (1 + t_i)w_i, \quad (i, j = E, W; i \neq j)
\]

where (2) is used. In what follows, we focus on the case where trade occurs and each region specializes in the production of one good, namely, East produces only good \( E \) while West produces only good \( W \). This occurs if the size of the comparative advantage is sufficiently large, that is, if

\[
1 + t_E < \beta \quad \text{and} \quad 1 + t_W < \beta,
\]

which is assumed.

Transport firms are owned by workers in the economy. For the sake of simplicity, we assume that each of them has an equal share. Then, the profit raised in the transport sector is distributed to the two regions in proportion to their populations. That is, out of total profit, \( \Pi \), portion \( \theta \) goes to East and the rest, portion \( 1 - \theta \), goes to West. Since there is only one factor of production, labor, regional income consists of labor income and profit:

\[
Y_E = \theta(Lw_E + \Pi), \quad \text{and} \quad Y_W = (1 - \theta)(Lw_W + \Pi).
\]

When the profit of the transport sector is negative, the loss is borne by workers, probably in the form of tax. In this case, (4) implies that the tax burden to finance the loss is equally shared by workers.

Transport technology exhibits increasing returns to scale. For one thing, transport firms use labor for a variable input. In particular, they use the labor in the region of the origin of shipment. In order to ship 1 unit of good from East to West (from West to East, resp.), each firm needs to employ \( c \) units of labor in East (in West, resp.).

In addition, they use labor for a fixed input as well. Each of them uses \( F \) units. In what follows, however, we rather use a per capita variable, \( f \equiv F/L \), instead of \( F \) for the sake of convenience. Here, two assumptions are imposed for the fixed input. First, notice that the total amount of labor used as a fixed input in the economy is equal to \( Fn = Lf n \). In order to focus on the non-trivial cases where at least some final goods is produced, we assume that the fixed cost is so small that \( Lf n < L \), or equivalently,

\[
f < \frac{1}{n},
\]

for any admissible value of \( n \). The second assumption is that the regional distribution of the labor used for a fixed input coincides with that of workers. In other words, \( \theta Lf n \) units of labor are employed in East while \( (1 - \theta)f n \) units are employed in West.\(^6\) There are three reasons for the assumption. First,
it is often observed that the fixed input is used mainly for the construction of the facilities necessary to handle shipment in each region, such as terminal buildings in airports and railway stations. In this case, it is natural to suppose that the amount of fixed input used in each region depends on the size of that region. At its extreme, it may be permissible to consider the amount to be proportional to the regional size. Second, it may be the case that the firms owned by the workers in a particular region are somehow destined to employ for a fixed input the labor in that region. One possibility is that the firms are located in the region where their shareholders are located, with no access to the labor in the other region, probably due to the existence of commuting costs or by other technological, institutional and/or social reasons. In such a case, the regional distribution of the labor used for a fixed input coincides with that of shareholders and, therefore, that of population. Third and finally, the assumption greatly simplifies the analysis.

Profit of the sector is, therefore, given by

\[
\Pi = (p_{EE}t_E - w_{EC})X_{EW} + (p_{WW}t_W - w_{WC})X_{WE} - Lfn[\theta w_E + (1 - \theta)w_W].
\]

Finally, because we are focusing on the case of complete specialization, the markets clear if

\[
\begin{align*}
\theta L &= X_{EE} + (1 + c)X_{EW} + \theta Lfn, \\
(1 - \theta)L &= X_{WW} + (1 + c)X_{WE} + (1 - \theta)Lfn.
\end{align*}
\]

This completes the description of a short run, in which the regional distribution of workers, \(\theta\), is given. In this paper, we concentrate on the equilibrium at which the prices of transport services are equal with respect to the two directions, that is, \(t_E = t_W = t\). Although there remains a possibility that asymmetric prices are supported by an equilibrium, the analysis becomes much simpler by focusing on the cases with symmetric prices. Later, we will come back to this point to show that this symmetry is indeed consistent with the behaviors of transport firms. Solving the system of (2), (3), (4), (6) and (7) along with \(t_E = t_W = t\), we obtain

\[
p = \frac{A_E\theta}{A_W(1 - \theta)},
\]

where

\[
A_E \equiv fn(1 + t)[1 + t - \theta(t - c)] - \frac{(1 - \theta)(t - c)^2}{2} - (1 + c)(1 + t) \quad \text{and} \\
A_W \equiv fn(1 + t)[1 + t - (1 - \theta)(t - c)] - \frac{\theta(t - c)^2}{2} - (1 + c)(1 + t).
\]

In a long run, regional distribution of workers is determined. Here, we follow the convention in the literature that workers’ decisions on their regions of residence are based on the relative levels of indirect utility achieved in the two regions. It is straightforward to compute the indirect utility of a worker in each region:

\[
\begin{align*}
v_E &= \frac{1}{2\sqrt{p(1 + t)}} \left(1 + \frac{\Pi}{L}\right), \\
v_W &= \frac{1}{2\sqrt{p(1 + t)}} \left(p + \frac{\Pi}{L}\right).
\end{align*}
\]
The long-run equilibrium must satisfy \( v_E = v_W \), which implies that \( p = 1 \): At the long-run equilibrium, the prices of the two goods become equal to each other. Then, (8) is reduced to
\[
A_E \theta = A_W (1 - \theta).
\]
(9)

It is readily verified that \( A_E \theta \gtrless A_W (1 - \theta) \) if \( \theta \lesssim 1/2 \), provided that (5) holds. Therefore, (9) has a unique solution, \( \theta = 1/2 \). This establishes the following lemma:

**Lemma 1.** When the prices of transport services are symmetric with respect to the directions of shipment, workers are distributed equally in the two regions at the long-run equilibrium.

It immediately follows that, at the long-run equilibrium, the wage rates in the two regions become equal. Furthermore, the aggregate consumptions are reduced to:
\[
X_{EE} = X_{WW} = \frac{L(1 - fn)(1 + t)}{2(2 + c + t)}, \quad \text{and}
\]
\[
X_{WE} = X_{EW} = \frac{L(1 - fn)}{2(2 + c + t)}.
\]

The condition (5) guarantees that all of these amounts are positive. The equilibrium level of the profit per transport firm, \( \Pi/n \), is given by
\[
\pi(t, n) = \frac{L[t - c - 2fn(1 + t)]}{n(2 + c + t)},
\]
which decreases with \( n \) for given \( t \).

Moreover, the total amount of labor used in the transport sector is equal to \( c(X_{EW} + X_{WE}) + Lfn \). Therefore, the share of labor in that sector is given by
\[
\lambda(t, n) = \frac{c + fn(2 + t)}{2 + c + t}.
\]

Furthermore, the indirect utility of each consumer in each region becomes equal to
\[
v_E = v_W = v(t, n) \equiv \frac{1 - fn}{2 + c + t} \sqrt{1 + t}.
\]
(11)

In order to understand the structure of the model, it would be helpful to examine the effects of changes in cost parameters, \( c \) and \( f \), upon the level of indirect utility. We can decompose the total effects into three parts:
\[
\frac{dv(t, n)}{dx} = \frac{\partial v(t, n)}{\partial x} + \frac{\partial t}{\partial x} \frac{\partial v(t, n)}{\partial t} + \frac{\partial n}{\partial x} \frac{\partial v(t, n)}{\partial n} + \frac{\partial v(t, n)}{\partial x} \frac{\partial t}{\partial n} \frac{\partial v(t, n)}{\partial t} + \frac{\partial v(t, n)}{\partial n} \frac{\partial t}{\partial n} \frac{\partial v(t, n)}{\partial t} \quad x \in \{c, f\}.
\]
(12)

The first term, which is negative for both \( x = c \) and \( x = f \), represents an adverse effect due to the shift of labor from the final goods sector to the transport sector (direct cost effect): if the price of transport services and the number of transport firms remain unchanged, the rise in the production cost in the transport sector reduces the amount of labor available in the final goods sector. Next, the second term
measures an effect through the change in the price of transport services, provided that the number of firms remains unchanged (transport price effect). Because a higher price is obviously associated with a lower level of the indirect utility \((\partial v(t, n)/\partial t < 0)\), the effect has an opposite sign of \(\partial t/\partial x\). Finally, the last term represents an effect through the change in the number of transport firms (number-of-firms effect). It is further decomposed into two parts. First, the number of firms affects the indirect utility through a change in the price of transport services. This is represented by the first term in the parentheses in the second line. When transport firms are free to choose their quantity of production, a shrinkage of the transport sector will mitigate the competition among them, which renders them a stronger monopoly power and results in a higher price. To the contrary, an expansion of the sector will provoke fiercer competition, which brings about a lower price. Because \(\partial v(t, n)/\partial t < 0\), this competition effect has an opposite sign of \(\partial n/\partial x \cdot \partial t/\partial n\). Second, the change in the size of the transport sector directly affects the indirect utility. This effect is represented by the last term in the parentheses in the second line. As the transport sector shrinks, a certain amount of labor previously used for a fixed input is released from the sector and becomes available for the final good production, which raises the level of the indirect utility: \(\partial v(t, n)/\partial n < 0\). This fixed input effect has an opposite sign of \(\partial n/\partial x\).

3 Oligopolistic transport sector

In this section, we give more concrete structure to the transport sector to obtain some key results on the prices of transport services and the number of transport firms when the sector is characterized by the oligopolistic competition à la Cournot.

Suppose that \(n\) transport firms produce transport services, which are distinguished only by the direction of shipments, that is to say, they are not differentiated as long as the direction is the same. Let \(z_E(k)\) be the quantity of the service produced by the \(k\)-th firm to carry good \(E\) from East to West. \(z_W(k)\) is similarly defined. For simplicity, we choose the unit so that 1 unit of transport services can ship exactly 1 unit of final goods. Then, the market for the transport services clear when

\[
\sum_k z_E(k) = X_{EW} \\
\sum_k z_W(k) = X_{WE}.
\]

Given the amounts of services produced by each firm, their aggregates determine the prices of the services, \(t_E\) and \(t_W\), through inverse demand functions: \(X_{ij} = \alpha_i Y_j/p_{ii}(1 + t_i)\), where \(Y_j\) and \(p_{ii}\) depend on \(t_E\) and \(t_W\). (As has been mentioned, \(t_E\) and \(t_W\) become equal to each other \textit{ex post}.)

In the rest of this section, we first analyze the decision making of each transport firm on the scale of production given the number of its competitors, and then examine the number of firms determined as a result of free entry and exit.
3.1 Scale of production and prices of transport services

First, let us discuss the decision making of each transport firm on the scale of production when the number of its competitors is given. Firm $k$ chooses $z_E(k)$ and $z_W(k)$ that maximize its profit, given by:

$$\pi(k) = \sum_{i=E,W} [t_i z_i(k)p_{ii} - cz_i(k)w_1] - L f[\theta w_E + (1-\theta)w_W].$$

Two assumptions are made. First, we assume that each firm makes Cournot conjecture, or, to put it another way, that a coefficient of conjectural variation for each firm is equal to 1. It means that each firm sticks to its belief that the other firms will not change their behaviors (see Seade (1980)). Then, we have $\partial t_i/\partial z_i(k) = \partial t_i/\partial X_{ij}$. Second, we assume that transport firms do not take into account all the effects of their decisions but only a part of them. Theoretically, a choice of a firm affects its profit through many channels: it affects the prices of transport services, which, in turn, alters all the other variables including the price of final goods, wage rates and regional incomes. However, it seems farfetched and unnecessary to suppose that transport firms well consider such far-flung effects. In this paper, therefore, we assume that they consider only two types of effects, namely, a direct effect, $\partial \pi(k)/\partial z_i(k)$, and the effect transmitted through the change in the prices of transport services, $\partial \pi(k)/\partial t_i = \partial t_i/\partial z_i$. These two assumptions imply that $\partial t_i/\partial z_i(k) = \partial t_i/\partial X_{ij} = -(1+t_i)/X_{ij}$, because the inverse demand is given by $t_i = Y_j/(2X_{ij}p_{ii}) - 1$ (see (1) and (3)). Assuming the symmetry with respect to transport firms, $z_i(k) = z_i = X_{ij}/n$, we obtain the condition for the profit maximization: $t_i = (p_{ii} + cn)w_1/[p_{ii}(n-1)]$. Then, (2) and $w_1 = w_2 = p = 1$ give the following prices of transport services:

$$t_E = t_W = t^*(n) \equiv \frac{1 + cn}{n-1}. \quad (13)$$

Two observations follow. First, the prices of transport services indeed become equal to each other, as has been assumed in the previous section. Therefore, the symmetry of the prices of final goods, $p = 1$, is consistent with that of the prices of transport services, $t_E = t_W$: $p = 1$ and $t_E = t_W$ actually constitute an equilibrium. Second, the price of transport services decreases with the number of firms: $dt^*(n)/dn < 0$.

This is because competition among firms becomes severer when there are a greater number of firms.

For the price given by (13), the indirect utility of workers, given by (11), is reduced to

$$v^*(n) \equiv v(t^*(n),n) = \frac{1 - fn}{2n - 1} \sqrt{\frac{n(n-1)}{1+c}}. $$

Furthermore, the share of the labor employed in the transport sector becomes equal to

$$\lambda^*(n) \equiv \lambda(t^*(n),n) = \frac{c(n-1) + fn(2n - 1 + cn)}{(1+c)(2n - 1)}.$$ 

In the earlier discussion on the number-of-firms effect, we have paid an attention to the impact of a change in the number of transport firms upon the indirect utility. It is worth studying the impact

---

\textsuperscript{7}Suzumura and Kiyono (1987) discuss the entry and exit behaviors of firms for a general level of the coefficient of conjectural variations. As a matter of fact, however, we could not treat the coefficient as given, because it usually depends on the number of firms in the transport sector. Yet explicitly incorporating this effect into the model brings about too much complication.
again for the specific case with the price being given by (13), because it plays a key role in the following sections. It is straightforward to see that

\[
\frac{dv(t^*(n),n)}{dn} = \frac{dv(t^*(n),n)}{dn} + \frac{\partial v(t^*(n),n)}{\partial t} + \frac{\partial v(t^*(n),n)}{\partial n}
\]

\[
= K_v(n) \left[ (1 - fn) - 2fn(n-1)(2n-1) \right]
\]

\[
\equiv K_v(n)\psi(n),
\]

(14)

where

\[
K_v(n) \equiv \frac{1}{2(2n-1)^2 \sqrt{(1+c)n(n-1)}} > 0,
\]

\[
\psi(n) \equiv 1 - fn - 2fn(n-1)(2n-1).
\]

The term \( dt^*(n)/dn \cdot \partial v(t^*(n),n)/\partial t = K_v(n)(1 - fn) \) in (14) measures the competition effect, which is positive because of (5). As the number of firms increases, the competition in the transport sector becomes fiercer, which induces them to produce more and results in a lower price. This has a favorable effect on the welfare of workers. The term \( \partial v(t^*(n),n)/\partial n = -K(n)2fn(n-1)(2n-1) \), on the other hand, measures the fixed input effect, which is negative as has been explained. Since the directions of the two effects are opposite, sign of the total effect is ambiguous. When the competition effect dominates the fixed input effect, the total effect is positive. When the opposite holds, it is negative. As \( n \) increases, moreover, the competition effect shrinks whereas the fixed input effect grows, and, therefore, \( d\psi(n)/dn < 0 \). Consequently, when the number of firms is greater, it is more likely that the total effect becomes negative, and that a rise in the number of firms gives an adverse impact on the welfare of workers.

Similarly, we can investigate the impact of a change in the number of transport firms upon the share of labor in the transport sector, \( \lambda \), when the price is given by (13). It can be shown that the impact is unambiguously positive: as the number increases, the share of labor always rises. To see this, note that

\[
\frac{d\lambda^*(n)}{dn} = \frac{d\lambda(t^*(n),n)}{dn} = \frac{dt^*(n)}{dn} \frac{\partial \lambda(t^*(n),n)}{\partial t} + \frac{\partial \lambda(t^*(n),n)}{\partial n}
\]

\[
= K_\lambda(n) \left[ c(1 - fn) + f(2n-1 + cn)(2n-1) \right] > 0,
\]

where

\[
K_\lambda(n) \equiv \frac{1}{(1+c)(2n-1)^2} > 0.
\]

The total effect is decomposed into two parts. First, as the number of transport firms increases, the price of transport services declines. This stimulates the trade between regions and therefore increases the demand for the labor used for a variable input in the transport sector. However, it follows that the amount of final goods production declines because only a smaller amount of labor becomes available for it, on the other hand. This reduces the demand for transport services and thus the labor used for a variable input in the transport sector. Although part of the increase in the labor demand in the sector is, in this way, offset by the secondary effect, the primary effect dominates it. Consequently, the labor used for a variable input in the transport sector increases. This effect is captured by the first term. Second, as the
number of transport firms increases, the transport sector needs to hire more labor for a fixed input. To the extent that the amount of the labor available for the final good production decreases, the final good production shrinks. We have again a counteractive effect. However, this counteractive effect is not too large, and consequently the share of labor in the transport sector rises. This direct effect is represented by the second term.

### 3.2 Number of transport firms

Next, we discuss the number of transport firms, $n^0$, determined as a result of free entry and exit. What is important is that it is an integer. Thus, it can differ from a 0-profit number of firms, $\bar{n}$, which is a solution to $\pi(t, \bar{n}) = 0$. In this profit function, the price of transport services is either a constant or a function of the number of firms, depending on the situation we analyze.

Two observations follow with respect to $n^0$. First, free entry/exit implies that the profit of each firm becomes no lower than 0: $\pi(t, n^0) \geq 0$. In what follows, it will be always the case that the profit decreases with the number of firms, that is, $d\pi(t, n)/dn < 0$. Then, $\pi(t, n^0) \geq 0$ is equivalent to $n^0 \leq \bar{n}$. Second, the profit would be negative when one additional firm entered, that is, $\pi(t, n^0 + 1) < 0$. This is equivalent to $n^0 > \bar{n} - 1$ as long as $d\pi(t, n)/dn < 0$. Putting these two observations together, we have $\bar{n} - 1 < n^0 \leq \bar{n}$. Now, let us introduce function $I(x)$ which gives the maximum integer that does not exceed $x$, that is,

$$x - 1 < I(x) \leq x \quad \text{with} \quad I(x) \in \mathbb{Z},$$

(15)

where $\mathbb{Z}$ is a set of integers. Then, the above finding implies that

$$n^0 = I(\bar{n}),$$

(16)

because $n^0$ is an integer. In addition, it immediately follows that a necessary and sufficient condition for $n^0$ to be no smaller than 1 is that $\bar{n} \geq 1$.

In the following analysis, we will use some properties of the function $I(\cdot)$, which are summarized as follows:

**Lemma 2.**

i) $dI(x)/dx \geq 0$ for any $x$;

ii) $I(x + z) = I(x) + z$ for any $x$ and any $z \in \mathbb{Z}$; and

iii) $x < z$ if and only if $I(x) < z$ for any $x$ and any $z \in \mathbb{Z}$.

The proof is relegated to the appendix.

### 4 Social optima

In order to judge whether the transport sector is too large or not, we need a yardstick. In this section, three types of social optima are introduced as such yardsticks. They are the first best optimum when
a social planner can choose both the price of transport services and the number of transport firms, the second best optimum when the planner sets the price while the number of firms is determined through individual decision makings by transport firms, and finally the second best optimum when the planner sets the number of firms and firms determine the price.

First best optimum. When a social planner can set both the price of transport services and the number of transport firms, it chooses marginal cost pricing having one transport firm operate. This is obvious since distortion arises only from the transport sector, and, furthermore, the welfare increases by the relocation of the labor used for a fixed input in that sector to the final good sector as long as the marginal cost pricing prevails. Indeed, we can solve the problem to maximize the indirect utility given by (11) subject to constraint $n \geq 1$ by choosing $t$ and $n$, to obtain $t = c$ and $n = 1$. It is worth adding that at the first best optimum, the profit of the (unique) transport firm is negative and the loss is equal to the fixed cost. Therefore, to achieve the first best, the tax whose total amount is equal to the fixed cost must be collected.

Second best optimum when the social planner can set only the price of transport services. The second case to consider is the second best optimum where a social planner can set only the price of transport services. This is described by a two stage game in which, at the first stage, the social planner sets the price, and, at the second stage, transport firms make entry and exit until incumbent firms earn nonnegative profit but a potential entrant cannot do so.

Notice that the number of transport firms determined at the second stage is prescribed by (16). Here, the 0-profit number of firms, $\pi$, can be regarded as a function of $t$ and written as $\pi^P(t) \equiv (t - c)/[2f(1 + t)]$ where superscript $P$ stands for the second best with the price regulation. Therefore, the equilibrium number of firms, $I(\pi^P(t))$, also becomes a function of the regulated price. Thus, we denote it as $n^P(t)$. The problem of the planner is, then, to choose the price that maximizes the indirect utility, $v(t, n)$, given by (11), subject to the constraint on the number of transport firms, $n^P(t) = I(\pi^P(t))$. It turns out that the solution is the price that entices exactly one firm to operate in the transport sector. We can prove the following result whose proof is relegated to the appendix:

**Lemma 3.** When the social planner can control only the price of transport services, it chooses the price

$$t^P \equiv \frac{c + 2f}{1 - 2f},$$

which induces one firm to operate.

It is not difficult to explain this result intuitively. Suppose that the planner lowers the price of transport services. It affects the indirect utility through two channels. First, it enables workers to spend

---

In this paper, we do not explicitly deal with the outside option that no transport firm operates in the economy. In other words, we assume that social welfare is always higher when at least one firm operates than when no firm operates. The latter situation might be explicitly formulated by introducing a ‘traditional transport technology’ that enables each consumer to carry goods by themselves, which, however, makes the analysis much more complicated without yielding any additional insight.
more money upon the final goods. This corresponds to what we have seen as the transport price effect. Second, it reduces the number of firms in the transport sector; and therefore, some of the labor used for a fixed input in that sector is released and becomes available for the final goods production, namely, the fixed input effect. Both effects are favorable and, consequently, the planner attempts to lower the price as much as possible. Provided that at least one firm operates in the transport sector, this implies that it chooses such a low price that only one firm enters into the sector.

Three comments are in order. First, it is important to note that with the price in Lemma 3, the (unique) transport firm earns 0 profit. Therefore, it is actually the average cost pricing or Ramsey-Boiteux pricing (Ramsey (1927), Boiteux (1956)).9 To put it differently, $t^P$ solves the problem to maximize the welfare subject to the constraint that the profit of transport firms does not become negative. Second, $t^P$ increases with $c$ and $f$: a rise in the marginal and/or fixed cost makes the price higher. Finally, the indirect utility under the price regulation is given by

$$v^P \equiv v(t^P, 1) = \frac{1}{2} \sqrt{\frac{1 - 2f}{1 + c}}.$$ 

**Second best optimum when the social planner can set only the number of transport firms.**

Next, I examine the situation in which the social planner regulates the number of transport firms. The following game depicts this situation. At the first stage, the planner issues permits for firms to operate in the transport sector. For the sake of simplicity, I assume that they are allocated to firms randomly and free of charge. At the second stage, the firms that have acquired them produce transport services. Here, the planner controls the number of permits to issue.

Let us solve the planner’s optimization problem. Because firms’ decision on the amount of transport services to produce is made competitively à la Cournot, their price is given by $t^*(n)$ for any $n$ set by the planner. The planner’s problem is thus to choose an integer $n \geq 1$ that maximizes $v^*(n)$. Now, let us suppose for a while that the constraint that the number of firms be no smaller than 1 ($n \geq 1$) is satisfied. The first order condition without considering the integer constraint is that $dv^*(n)/dn = K_v(n)\psi(n) = 0$ (see (14)). Recall that $\psi(\cdot)$ is a decreasing function. Since $\psi(1) = 1 - f > 0$ (see (5)) and $\psi(1/f) = -2(1-f)(2-f)/f^2 < 0$, there is unique $n \in (1, 1/f)$ that solves $\psi(n) = 0$ or $dv^*(n)/dn = 0$. We denote such $n$ by $\hat{n}$. Because $dv^*(n)/dn > 0$ for $n < \hat{n}$ and $dv^*(n)/dn < 0$ for $n > \hat{n}$, $\hat{n}$ gives a unique maximum of $v^*(n)$. By construction, the optimal number of firms, denoted by $n^E$, is $\hat{n}$ when it happens to be an integer, or either of the two integers that surround $\hat{n}$ otherwise. In other words,

$$
\begin{align*}
&n^E = I(\hat{n}) & \text{if } v^*(I(\hat{n})) > v^*(I(\hat{n}) + 1) \\
&n^E \in \{I(\hat{n}), I(\hat{n}) + 1\} & \text{if } v^*(I(\hat{n})) = v^*(I(\hat{n}) + 1) \\
&n^E = I(\hat{n}) + 1 & \text{if } v^*(I(\hat{n})) < v^*(I(\hat{n}) + 1).
\end{align*}
$$

In addition, we need to take into account the constraint that the number of firms be no smaller than 1. It is straightforward to see that $n^E$ indeed satisfies it: because $\hat{n} > 1$ implies that $I(\hat{n}) \geq 1$, it must

---

9Indeed, one can confirm that the total cost, $c(z_E w_E + z_W w_W) + Lf(\theta w_E + (1 - \theta) w_W)$, divided by the amount of production, $z_E + z_W$, becomes equal to $t^P$, provided that $n = 1$, $w_E = w_W = 1$, $\theta = 1/2$ and $z_E = X_{EW} = X_{WE} = z_W$. 

---
be the case that \( n^E \geq 1 \). Now, we denote the indirect utility realized under the entry regulation with \( n = n^E \) by \( v^E \). By construction, we have \( v^E = v^*(n^E) \).

The mechanism working behind this result is quite simple. Suppose that the social planner reduces the regulated number of firms. This works against workers through the competition effect: the competition among firms in the transport sector becomes less severe, which results in a higher price of transport services. On the other hand, it works in favor of workers through the fixed input effect: the transport sector needs a smaller amount of labor for a fixed input, and therefore, more labor becomes available in the final goods sector. In this way, there are two forces working in the opposite directions. The solution \( n^E \) is a point where these two are balanced with each other (as long as we ignore the integer constraint).

**Lemma 4.** When the social planner can control only the number of transport firms, it sets the number at \( n^E \).

**Comparison of the two second best optima.** One of the important policy implications of this paper is related to a comparison of the two second best optima, that with the price regulation and that with the entry regulation. We sometimes encounter the situations in which the first best cannot be implemented due to some political, institutional and/or historical reasons. In such situations, however, a government might still have an option to attain one of the second best optima. In this part, we answer the question of which is more desirable.

One may notice that there is an asymmetry between the two second best optima in the following sense. The ideal for the social planner is that the price of transport services is as low as possible while the number of firms is as small as possible so that the economy can save the fixed cost. In the case of the price regulation, it can suppress the number of firms by imposing a lower price. Thus, there is no tradeoff between lowering the price and reducing the number of firms. In the case of the entry regulation, however, this is no longer true. As the planner diminishes the number of transport firms, the price of transport services determined in the market rises. Here, it faces a tradeoff between a higher price and fewer firms. This asymmetry brings us a conjecture that the price regulation can achieve higher welfare than the entry regulation.

To verify this conjecture, let us define function \( \nu(n) \) as \( \nu(n) \equiv v^P - v^*(n) \), where \( v^P \) is independent of \( n \). By construction, \( \nu(n^E) \) gives the difference between the level of indirect utilities at the price regulation and that at the entry regulation: \( \nu(n^E) = v^P - v^E \). From the preceding discussion about the entry regulation, we know that \( v^*(n) \) has a unique maximum at \( \hat{n} \geq 1 \). It immediately follows that \( \nu(n) \) reaches a unique minimum at that point. Formally,

\[
\frac{d\nu(n)}{dn} \leq 0 \quad \text{if} \quad n \leq \hat{n} \quad \text{for any} \quad n \geq 1,
\]

which leads to the following result.

**Proposition 1.** The price regulation always yields a higher level of indirect utility than the entry regulation, that is, \( v^P > v^E \).
The proof is tedious and relegated to the appendix.

5 Market equilibrium

In this section, we derive the market equilibrium realized when transport firms freely make decisions on the amounts to produce and on the entry and exit, and then evaluate the associated outcome, in particular, the number of firms in the transport sector, in terms of the social optima discussed in the preceding section.

Consider the two stage game in which transport firms make entry and exit at the first stage and decide the price of transport services they produce at the second stage. Earlier discussion suggests that the solution to the second stage subgame is given by $t^*(n)$. This implies that the 0-profit number of firms is equal to $n = n^M \equiv (2f)^{-1/2}$, where superscript $M$ stands for the market equilibrium (see (10)). Since the solution to the entire game is given by (16), we have

$$n^M = I(\pi^M) = I\left(\frac{1}{\sqrt{2f}}\right), \quad \text{and} \quad t^M = t^*(n^M),$$

where $t^M$ and $n^M$ are the price and the number of firms at the market equilibrium, respectively.

Now, let us conduct some comparative statics analyses. In the first place, we examine a change in $c$. Suppose that it rises. First, the equilibrium number of transport firms is independent of $c$. Second, because the equilibrium number remains unchanged and $\partial t^M / \partial c > 0$, the equilibrium price of the transport services rises. Lastly, let us examine the effect upon the indirect utility, which consists of three components as has been explained (see (12)). First, we have a negative direct cost effect. Second, $\partial t^M / \partial c > 0$ implies that the transport price effect is negative. Third and finally, the number-of-firms effect disappears since $n^M$ does not depend on $c$. Therefore, the total effect, which becomes equal to the sum of the negative direct cost effect and the negative transport price effect, is negative: the rise in the marginal cost necessarily aggravates the welfare of workers.

Next, we turn to a change in the per capita fixed cost, $f$. Suppose that it rises. First, the rise in the fixed cost deters entry of transport firms and/or prompts their exit and results in a decline in the number of firms: $\partial n^M / \partial f \leq 0$ (see Lemma 2 i) and (17)). Second, the shrinkage of the transport sector brings about a higher price because the competition among transport firms becomes less fierce. (Notice that $t^M$ does not directly depend on $f$.) Third and finally, the direction of the effect upon the indirect utility is ambiguous. For one thing, the direct effect is negative because more labor needs to be allocated to the transport sector as a fixed input. Moreover, the transport price effect disappears since $t^M$ does not directly depend on $f$. In addition, the direction of the number-of-firms effect is ambiguous. As has been explained earlier, the shrinkage of transport sector gives an adverse effect on the indirect utility by relaxing the competition among transport firms (negative competition effect), on the one hand, and a favorable effect by saving the fixed cost (positive fixed cost effect), on the other hand. When the former dominates the latter, the indirect utility goes down as a result of the decline in the number of firms, that is, the number-of-firms effect is negative. When the opposite holds, it is positive. Because the direct
cost effect is negative, consequently, the entire effect is negative when the fixed input effect is too weak, whereas it is positive when the latter effect is sufficiently strong.

In addition, since $\partial n^M / \partial f \leq 0$, the necessary and sufficient condition for $n^M \geq 1$ is that $f \leq 1/2$. In order to focus on non-trivial cases, we assume that it is satisfied.

Now, we are ready to ask the question of whether the size of the transport sector realized at the market equilibrium is too large or too small.

For one thing, we can say that in light of the first best optimum, too many firms enter into the transport sector at the market equilibrium, because the first best solution involves only one firm. The same statement holds true when we evaluate the market equilibrium in light of the second best optimum when the social planner can control only the price of transport services.

However, if we use the second best optimum when the planner can control only the number of transport firms as a benchmark, the result changes: depending on the values of parameters, the number of firms in the transport sector at the market equilibrium can be too small. By the comparison of $n^M$ and $n^E$, we can obtain the following results: When the fixed cost in the transport sector is too high and/or the economy is too small, too few firms enter the transport sector at the market equilibrium. Instead, when the fixed cost is too low and/or the economy is too big, there is an excess entry.

**Proposition 2.**

i) A sufficient condition for $n^M \leq n^E$, is that $f \geq 2/9$. A necessary condition for $n^M < n^E$ is, on the other hand, that $f > g$, where $g$ is a real number.\(^{10}\)

ii) A sufficient condition for $n^M \geq n^E$ is that $f \leq g$. A necessary condition for $n^M > n^E$ is, on the other hand, that $f < 2/9$.

The proof is relegated to the appendix.

To understand these results intuitively, note that as $f$ rises, both $\pi^M$ and $\hat{n}$ decline:

$$\frac{\partial \pi^M}{\partial f} = -\frac{1}{2f \sqrt{2}} < 0 \quad \text{and} \quad \frac{\partial \hat{n}}{\partial f} = -\frac{\hat{n} \left[1 + 2(\hat{n} - 1)(2\hat{n} - 1)\right]}{3f(2\hat{n} - 1)^2} < 0.$$

Since $|\partial \pi^M / \partial f| > |\partial \hat{n} / \partial f|$, however, $\pi^M$ declines faster than $\hat{n}$.\(^{11}\) Because $n^M$ is a maximum integer that is no greater than $\pi^M$ and $n^E$ is one of the two integers surrounding $\hat{n}$, $n^M$ tends to decline faster than $n^E$. Therefore, as $f$ rises, it becomes more likely that $n^M$ becomes lower than $n^E$, and consequently that there are too few firms at the market equilibrium.

Two comments are worth adding.

First, when too few firms enter the transport sector at the market equilibrium, we have $d\psi^*(n)/dn > 0$ at $n = \pi^M$. As has been discussed in Section 3.1, this implies that the competition effect dominates the fixed input effect. That is, an additional firm would intensify the competition so much that the benefit from the decline in price more than offsets the loss caused by the additional expense for the fixed cost.

\(^{10}\)Numerically, we can solve for $g$ to obtain $g \approx 0.0484$.

\(^{11}\)Solving $\psi(\hat{n}) = 0$ for $f$ yields $f = \hat{n} \left[1 + 2(\hat{n} - 1)(2\hat{n} - 1)\right]^{-1}$. Substituting this value of $f$ into $\partial \pi^M / \partial f$ and $\partial \hat{n} / \partial f$, and rearranging, we can conclude that $|\partial \pi^M / \partial f| > |\partial \hat{n} / \partial f|$ if $9(2\hat{n} - 1)^4 > 8\hat{n} \left[1 + 2(\hat{n} - 1)(2\hat{n} - 1)\right] > 0$, which is indeed satisfied for any $\hat{n} \geq 1$. 

17
On the other hand, when there are too many firms, the fixed input effect dominates the competition effect.

Second, we have shown that the number of firms at the market equilibrium can be too small. This makes a sharp contrast to the ‘excess entry’ result by Mankiw and Whinston (1986), and Suzumura and Kiyono (1987), among others. They insist that the oligopolistic competition involves an excessive number of firms if evaluated in light of the second best outcome with the entry regulation. The reason is a so-called business stealing effect: when an additional firm enters, the marginal revenue of the incumbents decreases. This externality effect, according to them, causes a gap between the firm’s benefit and the benefit for the economy as a whole from the entry of an additional firm. Their conclusion is, however, based on a partial equilibrium setting. In the model presented here, the market of the transport services is not independent of the market of the final goods. Indeed, entry of a firm into the transport sector stimulates the consumption of the final goods through the competition effect, that is, through the decline in the price of transport services. This mitigates the business stealing effect and consequently, there is not always the excess entry.

Now, recall that the share of the labor employed in the transport sector, $\lambda$, increases with the number of firms when the price is given by $t^*(n)$: $d\lambda^*(n)/dn > 0$. Therefore, one can conclude that, when the number of transport firms at the market equilibrium is smaller (larger, resp.) than that at the entry regulation, the share of labor in the transport sector is lower (higher, resp.) than the counterpart at the entry regulation. Hence, the next corollary immediately follows from Proposition 2.

**Corollary 1.** When $f \geq S/p$, the share of labor in the transport sector at the market equilibrium is no higher than that at the entry regulation. When $f \leq g$, instead, it is no lower than the counterpart at the entry regulation.

### 6 Concluding remarks

In this paper, we have evaluated the size of the transport sector realized by the market mechanism and have studied the government policies against the inefficiencies caused by the monopoly power prevalent in the sector. In doing so, special attention is paid to the fact that the demand for the transport services is derived from that for the final goods. Constructing a simple Ricardian model of trade with two regions, we have examined the price of transport services and the number of firms in the transport sector realized at the market equilibrium and at the equilibria under three types of government interventions, namely, the first best optimum, the price (transport cost) regulation and the entry regulation. It is shown that the number of firms in the transport sector at the market equilibrium is too large or too small depending on the amount of fixed cost in the production of transport services and the size of the economy. It is also revealed that the price regulation always yields higher welfare than the entry regulation.
References


Appendix

Proof of Lemma 2.
i) Suppose that $dI(x)/dx < 0$ for some $x$. Then, there exist $x'$ and $x''$ such that $x' < x''$ and $I(x') > I(x'')$. The last inequality implies that $I(x') > I(x'') + 1$ because, by definition, $I(x')$ and $I(x'')$ are integers. Furthermore, by definition, $I(x') ≤ x'$ and that $I(x'') > x'' - 1$ (see (15)). Combining the last three inequalities yields $x' ≥ I(x') ≥ I(x'') + 1 > x''$. Consequently, $x' > x''$, which is a contradiction. Hence, $dI(x)/dx ≥ 0$ for any $x$.

ii) From the definition of $I(·)$, it immediately follows that $x + z - 1 < I(x + z) ≤ x + z$ and that $x + z - 1 < I(x + z) ≤ x + z$ (see (15)). However, both $I(x + z)$ and $I(x) + z$ are integers. Therefore, the two inequalities imply that $I(x + z)$ and $I(x) + z$ coincide with each other.

iii) Look at the ‘if’ part, first. Since $z$ is an integer, $I(x - z)$ implies that $I(·) ≤ z - 1$. Using the fact that $x - 1 < I(·) ≤ z - 1$ (see (15)), we have $x < z$. The ‘only if’ part, on the other hand, immediately follows from (15).

Proof of Lemma 3.
First, note that $d\pi(t)/dt ≥ 0$ for any $t$, because $\pi(t)$ is an increasing function of $t$ and $I(·)$ is a non-decreasing function (see Lemma 2 ii)). Second, it is readily verified that

$$\frac{dv(t, n^P(t))}{dt} = -\frac{\sqrt{1 + t}}{2(2 + c + t)} \left[ 2f(2 + c + t) \frac{dn^P(t)}{dt} + (t - c)(1 - fn^P(t)) \right].$$

It must be the case that $n^P(t) ≤ \pi(t)$ by (16). In order that $n^P(t) = (t - c)/[2f(1 + t)] ≥ 1$, therefore, we need $\pi(t) ≥ 1 > 0$. Consequently, $t > c$. Hence, $dn^P(t)/dt ≥ 0$ implies $dn/\pi/tdt < 0$. The social planner chooses $t$ that is as low as possible. However, since we must have $1 ≤ n^E(t) ≤ \pi(t)$, $\pi(t)$ cannot be lower than 1 and, therefore, $t$ cannot be lower than $t^P$. Hence, the optimal $t$ is given by $t^P$ and the corresponding number of firms is equal to 1.

Proof of Proposition 1.
It is convenient to distinguish three cases. First, suppose that $n^E = 1$. Then, it is straightforward to see that $\nu(n^E) = \sqrt{1 - 2f}/(2\sqrt{1 + c}) > 0$. Second, suppose that $n^E = 2$. In this case, we have

$$\nu(n^E) = \frac{1}{6\sqrt{1 + c}} \left[ 3\sqrt{1 - 2f} - 2\sqrt{2}(1 - 2f) \right] > 0.$$  

Here, the inequality follows from the fact that $(3\sqrt{1 - 2f})^2 - [2\sqrt{2}(1 - 2f)]^2 = (1 - 2f)(1 + 16f) > 0$, because $1 - 2f > 0$ due to (5). Finally, suppose that $n^E ≥ 3$. Recall that $n^E$ is equal to either of the two integers that surround $\hat{n}$, that is, either $I(\hat{n})$ or $I(\hat{n}) + 1$. If $n^E = I(\hat{n})$, on the one hand, $n^E ≥ 3$ implies that $\hat{n} ≥ 3$, because $\hat{n} ≥ I(\hat{n})$ (see (15)). If $n^E = I(\hat{n}) + 1$, on the other hand, $n^E ≥ 3$ (or equivalently $I(\hat{n}) ≥ 2$) implies that $\hat{n} ≥ 2$. Therefore, in both cases, $\hat{n} ≥ 2$. Now, we know that $\hat{n}$ is a solution to $\psi(n) = 0$. Solving $\psi(\hat{n}) = 0$ for $f$ yields $f = \left[ \hat{n}(1 + 2(\hat{n} - 1)(2\hat{n} - 1)) \right]^{-1}$. Substituting this value of $f$ into $\nu(\hat{n})$, we can verify that $\nu(\hat{n}) > 0$ if $\xi(\hat{n}) > 0$ where $\xi(n) = 12n^2 - 28n^2 + 21n - 6$. Note that $d\xi(n)/dn > 0$ for $n ≥ (14 + \sqrt{7})/18 = 0.63$. Therefore, $\xi(n)$ is increasing in $n$ for any $n ≥ 1$. This, along with the fact that $\xi(2) = 20 > 0$, implies that $\xi(\hat{n}) > 0$ for any $\hat{n} ≥ 2$. Consequently, $\nu(\hat{n}) > 0$ for any $\hat{n} ≥ 2$. Because $\hat{n}$ gives a minimum of $\nu(\cdot)$, however, $\nu(n^E) ≥ \nu(\hat{n}) > 0$ for any $\hat{n} ≥ 2$.

Proof of Proposition 2.
First, we prove the sufficient condition for $n^M ≤ n^E$. Suppose that $f ≥ 2/9$. It follows that $\psi(\pi^M) = [4\sqrt{f} - 3f - 2]/\sqrt{f} ≥ 0$. This implies that $\pi^M ≤ \hat{n}$ because $\psi(\hat{n}) = 0$ and $\psi(\cdot)$ is a decreasing function. Since $I(\cdot)$ is a non-decreasing function, however, we have $I(\pi^M) ≤ I(\hat{n})$ and $n^M ≤ n^E$ by
Hence, \( f \geq 2/9 \) is a sufficient condition for \( n^M \leq n^E \). Next, let us obtain the necessary condition for \( n^M < n^E \). Suppose that \( n^M < n^E \). Since \( n^E \leq I(\widehat{n}) + 1 \) by definition, \( n^M < n^E \) implies that \( I(\pi^M) < I(\widehat{n}) + 1 \). However, \( I(\widehat{n}) + 1 = I(\widehat{n} + 1) \) as has been discussed in Lemma 2 ii). Because \( I(\cdot) \) is a non-decreasing function, we establish \( \pi^M < \widehat{n} + 1 \). The definition of \( \widehat{n} \) implies that \( \pi^M < \widehat{n} + 1 \) or, equivalently, \( \widehat{n}^M - 1 < \widehat{n} \), if and only if \( \psi(\pi^M - 1) > 0 \). The last inequality is reduced to \( 338f^3 - 209f^2 + 92f - 4 > 0 \), which turns out to be equivalent to \( f > g \) for some real number \( g \). Hence, \( f > g \) is a necessary condition for \( n^M < n^E \). One can similarly prove that the sufficient condition for \( n^M \geq n^E \) and the necessary condition for \( n^M > n^E \) are \( \psi(\pi^M - 1) \leq 0 \) and \( \psi(\pi^M) < 0 \), respectively, which establishes the proposition.