Nonlinearity of Housing Price Structure:
Assessment of Three Approaches to Nonlinearity in the Previously Owned Condominium Market in the Tokyo Metropolitan Area

Chihiro SHIMIZU*
Kiyohiko G. NISHIMURA†
Koji KARATO‡

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Abstract
This paper examines three methods that explicitly take nonlinearity into account in hedonic price estimation: (1) a switching regression model (SWR) in parametric estimation, (2) a continuous dummy variable model (DmM) in non-parametric estimation, and (3) a generalized additive model (GAM) in semi-parametric estimation. These non-linear estimation methods are applied to the previously owned condominium market in the 23 wards of Tokyo, alongside a linear parametric model as a reference. We find that these nonlinear estimation methods not only increase explanatory power but also yield very similar estimates with respect to the shape of nonlinearity. However, the out-of-sample predictive power of these non-linear estimation methods is inferior to that of a reference linear parametric estimation method, suggesting that these methods are sensitive to sampling biases.

Keywords: hedonic model, nonlinearity, structure change, switching regression model, generalized additive models.

JEL Classification: C31 - Cross-Sectional Models; Spatial, R31 - Housing Supply and Markets.

* Associate Professor, Department of International Economics and Business Administration, Reitaku University
2-1-1 Hikarigaoka, Kashiwa-Shi, Chiba, 277-8686 Japan
Tel. +81-(0)4-7173-3439, Fax. +81-(0)4-7173-1100
e-mail: cshimizu@reitaku-u.ac.jp
† Deputy Governor of the Bank of Japan. Most contribution was made in the development stage of this research, before he joined its Policy Board.
‡ Associate Professor, Faculty of Economics, University of Toyama
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1. Introduction

Researches on hedonic housing price models typically assume a linear structure – either linear with respect to explanatory variables or in some transformation (typically, a logarithm) of them. In other cases, both response and explanatory variables are transformed but their relationship is assumed to be linear (Cropper, Deck and McConnel (1988), Halvorson and Pollakowski (1981), Rasmussen and Zuehlke (1990)). The premise behind this practice is that a linear structure, possibly coupled with some functional transformation of variables, is sufficiently flexible to account for the complexity of housing price formation.

In housing markets, however, there are some signs that the assumption of a linear structure (with fixed transformation of variables) may not capture the complexity. For example, markets may be segmented into several submarkets with very different coefficients for explanatory variables that cannot be captured through a linear structure with functional transformation of the variables (Bourassa, Hoesli and Peng (2003), Goodman(2003)). One possible example may be the previously owned condominium market in Tokyo, which consists of “studio”, “family-unit”, and “luxury” submarkets, each with different buyer-seller characteristics (Shimizu, Nishimura and Asami (2004)). Another example may be the effect of building age, which is often argued to have a rather complicated relationship with price in the Tokyo market because of somewhat unpredictable renovation activities. This complexity may not be captured well by pre-specified functional transformation of the variable.
In the literature, several attempts have been made to cope with this “genuine” nonlinearity, that is, nonlinearity that cannot be captured through pre-specified functional transformation (Bin (2004), Clapp (2003), Meese and Wallace (1991), Pace (1993), Pace (1998), Thorsnes and McMillen (1998), Ramazan and Yang (1996)). In this paper, we examine three among these attempts: a switching regression model in parametric estimation, a continuous dummy variable model in non-parametric estimation, and a generalized additive model in semi-parametric estimation.

Firstly, the switching regression model (SWR) is an extension of linear models, but assumes that a market is segmented into several sub-markets with different coefficients for explanatory variables (Goodman and Thibodeau (2003), McMillen (1994)). Coefficients are assumed to be different across segments but the same within each segment. The segmentation structure is estimated through segment dummy variables. Motivated by the apparent market segmentation of the abovementioned Tokyo condominium market, Shimizu and Nishimura (2007) applied this approach to the hedonic price model of this market.

Secondly, the continuous dummy variable model (DmM) overcomes restrictions on functional form imposed by linear models, or in general, models with pre-specified functional forms. For an explanatory variable to be assumed to have nonlinear effects, “continuous dummy variables” are constructed continuously with a certain bandwidth and estimated by OLS. Although this OLS estimation is parametric, the model can be considered as a non-parametric method since the explanatory variable in question is modeled and estimated as a linear combination of these continuous dummy variables.¹

¹ There is no clear definition concerning the differences between parametric regression and non-parametric regression (Yatchew, (2003)).
Thirdly, a generalized additive model (GAM) is employed to make the continuous dummy variable model’s bandwidth flexible and determined in the most fitted way to explain the data. Here, flexible bandwidth means spline fitting. There are several methods of doing this. For example, Hastie and Tibshirani (1990) use the backfitting algorithm. In this paper, we have used a more generalized version, Modified Generalized Cross Validation (MGCV) algorithm by Wood (2004).

The purpose of this paper is two-folded. Firstly, we examine how important non-linearity is in hedonic price models. In particular, we investigate how much these non-linear estimation methods improve the explanatory power of the price model compared with a reference linear model, and how consistent the results of these methods are with one another concerning the shape of the non-linearity. Secondly, we explore the predictive power of these non-linear estimation methods. We conduct an out-of-sample accuracy comparison of these models with the reference linear model.

In section 2, the data used in this study are explained and the estimation models; the SWR, DmM, and GAM models, which take into account the nonlinearity, are set up, and in section 3, assessment of Non-linearity. In section 4, we compare the predictive power of the non-linear estimation methods. Section 5 provides a conclusion.
2. Data and Three Nonlinear Estimation Methods of a Hedonic Price Equation

2.1. Data and a Reference Linear Estimation

2.1.1. Tokyo Condominium Markets and Possible Sources of Complex Nonlinearity

We examine the previously owned condominium market of the Tokyo metropolitan area, or specifically, all of its 23 wards. This area spans 621.97 square kilometers with a population of 8,489,653 in 2005. Tokyo has a long history of more than 500 years as a political and business center of Japan and has evolved gradually to its current state of complexity and heterogeneity.

In particular, Tokyo condominium markets have evolved alongside the vast expansion of the suburban railway and subway systems. These condominiums have been designed and targeted for people working in Tokyo central business districts (CBD) and/or enjoying amenities found in central areas. Thus, many researches about Tokyo condominium price structure have found that not only the physical characteristics of a condominium unit such as (1) floor space and (2) building age, but also (3) time to the nearest train station and (4) time to the nearest terminal station (a proxy for vicinity to the CBD) are the most important determinants of condominium prices. In fact, many studies found that these four variables account for almost 70% of the variation in Tokyo condominium prices (Ohnishi et al (2010)). Taking account of these facts, we have chosen these four explanatory variables as *key variables* in the following analysis.

There are several possible sources suggesting a complex nonlinear relationship between key variables and the condominium price. Firstly, it has often been pointed out that Tokyo condominium markets are segmented into three distinctive sub-markets (Shimizu, Nishimura and Asami (2004)). Relatively small condominium units are often purchased for investment purposes by property investors or for residential purposes by single households, while slightly larger ones are bought by small households such as the so-called double-income no-kid (DINK) households.
In contrast, family households mostly purchase condominium units larger than a certain size. In addition, there are segments of luxury units. Although exact segmentation of the market is hard to determine, it is likely that these different segments have noticeably different pricing structures, leading to complex nonlinear price effects on key variables, particularly on floor space.\(^2\)

Secondly, the effects of “building age” (number of years since construction) might be very complex in nature, reflecting the heterogeneity of condominium owners. It is natural to expect that previously owned condominium unit prices decrease as time after construction increases because of the physical deterioration.\(^3\) Moreover, we also have to consider recent marked advances in technology in construction and facilities. They have a complicated effect on the “economic deterioration” of exiting condominium units. Also, renovation of old facilities, both in a particular unit and in the building to which it belongs greatly changes the value of the condominium unit. Renovation may involve not only individual but also collective decisions of condominium owners, which leads to added complexity.\(^4\)

Thirdly, with respect to the variable “time to the nearest station”, there are two conflicting effects of the vicinity to the station. Areas near a railway station have more shops and more convenient transport links, whereas they often suffer from a lack of parks and a poor natural environment. Moreover, the value attached to these environmental factors may differ substantially between various segments of households.\(^5\) This suggests a complex nonlinearity that

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\(^2\) In a related topic, Asami and Ohtaki, 2000 and Thorsnes and McMillen, 1998 pointed out that there is nonlinearity between land area and land value.

\(^3\) The lifespan of houses is remarkably short in Japan; the average lifespan is 30 years. (White paper by the Ministry of Land, Infrastructure, Transport and Tourism, 2009)

\(^4\) Bid prices are likely to differ between consumers who prefer new facilities and those who do not. They may be also affected by an income constraint, i.e., between high-income and low-income households. Because a depreciation curve with respect to the number of years since construction is a particularly important indicator of collateral assessment for housing loans, an earlier study focusing on this variable was reported (Clapp and Giaccotto, 1998).

\(^5\) In Japan, households, including single and DINK households, expressing a preference for convenience will probably buy property in areas near a station, while family households, particularly those with children, will tend to select areas further from the nearest station.
cannot be captured by a simple functional transformation of variables. A similar argument applies to the “travel time to the CBD” variable.

2.1.2. Brief Description of Data

This study utilizes information published in the “Shukan Jutaku Joho” (Weekly Housing Information Magazine) by Recruit Co., Ltd., one of the largest vendors of residential market information in Japan. The Recruit dataset covers the 23 special wards of Tokyo for the period 2003 to 2006. It contains 40,353 listings. This database is the most comprehensive one available to date on the previously owned condominium market in Tokyo, containing location, property characteristics, and a good proxy of transaction prices, as explained below. It should be noted that information on actual transaction prices is not available in Japan.

Shukan Jutaku Joho provides time-series of housing prices from the week they are first posted until the week they are removed due to successful transaction. We use only the price in the final week. This final-week price is shown in our follow-up study to be a good proxy of the actual contract price in the transaction.

Transportation convenience at each point was first represented by “time to the nearest station” (TS) and “travel time to the CBD” (TT).

As information regarding the characteristics of previously-owned condominiums themselves, information on “Floor space” (FS), “Age of building” (Age), “Balcony space” (BS), and

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6 There are two reasons for the listing of a unit being removed from the magazine: a successful deal or a withdrawal (i.e. the seller gives up looking for a buyer and thus withdraws the listing). We were allowed access information regarding which of the two reasons applied for individual cases and discarded those where the seller withdrew the listing.

7 Recruit Co., Ltd. provided us with information on contract prices for about 24 percent of all listings. Using this information, we were able to confirm that prices in the final week were almost always identical with the contract prices (i.e., they differed at a probability of less than 0.1 percent).

8 Only data related to condominiums within walking distance were extracted, and the walking time to the nearest station (in minutes) was adopted.

9 Travel time to the CBD is measured as follows. The metropolitan area of Tokyo is composed of 23 wards centering on the Tokyo Station area and containing a dense railway network. Within this area, we choose seven railway/subway stations as the central stations, which include Tokyo, Shinagawa, Shibuya, Shinjuku, Ikebukuro, Ueno, and Otemachi. Then, we define travel time to the CBD by the minutes needed to commute to the nearest of the seven stations in the daytime.
“Number of units” (NU) was used. In addition, regarding items considered to affect condominium value, including whether condominiums are on the highest floor, whether they are on the ground floor, and the direction in which windows face,\(^{10}\) data for analysis were obtained using relevant data published in the housing magazine, and dummy variables were produced from the information.

Table 1 shows the list of explanatory variables and their descriptions and Table 2 presents their summary statistics.

Comparing condominium “resale prices” (RP) first of all, we can see that there was a gradual upward trend in the average value from 2003 through 2006, with the standard deviation being larger in 2003-2004 and 2006 than in 2005. There were no significant differences in floor space (FS), time to the nearest station (TS), balcony space (BS), and total number of units (NU) over the three points in time. However, with regard to travel time to the CBD (TT), we can see that in 2005 only, the transacted properties were relatively far from the center.

### 2.1.3. A Reference Linear Estimation Model

Let’s define the models using hedonic equations. The simplest model is set up below as the base model.

\[
\log RP / FS = a_0 + \sum_{h} a_{ih} \log X_h + \sum_{j} a_{ij} \log Z_j + \sum_{l} a_{il} \log BC_l + \sum_{k} a_{ik} \log LD_k + \sum_{m} a_{im} \cdot TD_m \cdot \epsilon
\]

(1)

\(^{10}\) Assuming that the prices of condominiums with windows facing south are higher in Japan, a “south-facing dummy” was set.
$RP$: Resale price of condominium (yen)

$X_i$: Key variables

$FS$: Floor space (square meters)
$Age$: Age of building (months)
$TS$: Time to the nearest station (minutes)
$TT$: Travel time to the CBD (minutes)

$Z_i$: Other variables

$BS$: Balcony space (square meters)
$NU$: Number of units (units)
$TM$: Time on the market (weeks)

$BC$: Building characteristics dummy (first floor dummy, highest floor dummy, south-facing dummy, and ferroconcrete dummy)

$LD_j$: Location (ward) dummy ($j = 0 \ldots J$)
$RD_k$: Railway line dummy ($k = 0 \ldots K$)
$TD_l$: Time dummy ($l = 0 \ldots L$)

This model for explaining previously owned condominium prices incorporates the floor space ($FS$), the number of years since construction ($Age$), the time to the nearest station ($TS$), and the travel time to the CBD ($TT$), which are variables considered to have important effects on previously owned condominium prices. In addition to these variables, information such as the amount of balcony space ($BS$) and the number of units ($NU$), available from magazines containing information on properties, was also incorporated in the models. Such information also relates to the condominium location or the building characteristics. The railway line dummy ($RD_k$) takes regional characteristics into account, and the time dummy ($TD_l$) takes temporal changes in the market into account.
The objective of this study is to clarify the price structure of previously owned condominiums in the 23 wards of Tokyo, with the analysis focusing on the four main variables, $FS$, $Age$, $TS$, and $TT$.

2.2. A Switching Regression Model in Parametric Estimation

Next, we modified the base model considering structural change. It is assumed that there are two points of structural change and three price-bidding curves. For example, if attention is paid to the location-centered attribute, the following three types of purchases are expected: i) studio type condominiums in which a single person lives, ii) family type condominiums in which a small household (for instance, a couple) lives, and iii) luxury type condominiums in which a family with children lives. Depending on such differences in the entities, the single person and the small household will probably select a more convenient region, while the family will tend to attach more importance to the living environment. Hence, $TS$ is also considered to be divided into the following: i) an area in which households attaching more importance to convenience are located, ii) an area that is within walking distance of the station but in which a good living environment is maintained, and iii) an area that requires the use of buses or a car to reach the nearest station.

Given the above perspective, it may be thought that on the whole, three groups having three different preferences exist in Japan.

As described above, if the market structure is divided into three markets, two points of structural change should exist for the variable group thought to have nonlinearity. It is unknown, however, at what point the market structure changes (Jushan and Perron, 1998). Under such circumstances, the basic model is modified, and an estimate is made through exploratory analysis of variables thought to have nonlinearity, i.e., $FS$, $Age$, $TS$, and $TT$. Specifically, the following
two dummy variables are introduced on the assumption that the market is divided at points \( l \) and \( m \) for each main variable \( X_h \).

\[
D_{m(h \leq X_h < m)}: \text{if } l_h \leq X_h < m_h, \text{ then } 1, \text{ otherwise } 0
\]

\[
D_{m(h \leq X_h < m)}: \text{if } m_h \leq X_h, \text{ then } 1, \text{ otherwise } 0
\]

\( l < m \)

A model such as the one shown below is estimated by introducing the above dummy variables.

\[
\log \text{RP} / \text{FS} = a_0 + \sum_h a_{ih} \log X_h + \sum_i a_{1i} \log Z_i + \sum_j a_{3j} \log BC_j + \sum_k a_{4k} \cdot LD_k + \sum_l a_{5l} \cdot RD_l + \sum_m a_{6m} \cdot TD_m +
\]

\[
+ a_i D_{m(h \leq X_h < m)} + a_i D_{m(h \leq X_h < m)} + a_i (\log X_h) \left( D_{m(h \leq X_h < m)} \right)
\]

\[
+ a_i (\log X_h) \left( D_{m(h \leq X_h < m)} \right) + \varepsilon
\]

\( (2) \)

This model is the switching regression model (SWR), and we assume that the regression model is switched at points \( l \) and \( m \). For further information on SWR, see McMillen (1994), and Shimizu and Nishimura (2007).

2.3. A Continuous Dummy Variable Model in Non-Parametric Estimation

In SWR, we hypothesized that the market structure is divided into three markets and two points of structural change exist. If there were more than two structural change points, SWR could not explain the nonlinearity appropriately. We conduct a hedonic model with the four main variables used as parametric variables in the base model made into dummy variables to estimate the
nonlinearity. In forming the dummy variables, an arbitrary bandwidth (\( \beta \)) was set up for each variable unit. The model obtained by forming the dummy variables based on the four main variables is referred to as the *continuous dummy model* (DmM), and can be expressed as:

\[
\log \frac{RP}{FS} = a_0 + \sum_i a_i \log Z_i + \sum_j a_j \log BC_j + \sum_k a_k \cdot LD_k + \sum_l a_l \cdot RD_l + \sum_m a_m \cdot TD_m + \\
+ \sum_{\rho} a_{\rho} \cdot Dm(FS_{\rho}) + \sum_{\sigma} a_{\sigma} \cdot Dm(Age_{\sigma}) + \sum_{\tau} a_{\tau} \cdot Dm(TS_{\tau}) + \sum_{\iota} a_{\iota} \cdot Dm(TT_{\iota}) + \varepsilon
\]

(3)

\( Dm(X_h) \): Continuous changing dummy (Dm) variables with a bandwidth (\( \beta \)) determined by main variables \( (X_h) \)

2.4. A Generalized Additive Model in Semi-Parametric Estimation

The DmM model assumes that the previously owned condominium price structure changes continuously by the bandwidth (\( \beta \)) unit set for each variable. In the actual market, however, it is not likely that the same bandwidth for all regions of the variable.

In this paper, we have applied calculations based on a more generalized version of DmM, the *generalized additive model* (Hastie and Tibshirani(1986),(1990)), hereafter referred to as GAM.

In general, GAM has a structure with the smooth function as follows:

\[
g(\mu) = \sum_m \gamma_m W_m + \sum_h s_h(X_h),
\]

(4)

where \( s_h \) are smooth functions of the covariates \( X_h \). The model allows for rather flexible specification of the dependence of response on the covariates. Smooth functions are represented
using penalized splines with smoothing parameters selected by generalized cross-validation (GCV).\textsuperscript{11}

The spline curve calculated based on the $S_h$ functions is a smooth curve passing through the multiple provided points. Increasing the number of these multiple points increases the fit. However, a problem similar to the arbitrariness problem in determining bandwidth for DmM occurs.\textsuperscript{12}

To prevent too much wiggliness in the estimated curve, a special term that penalizes rapid changes in smooth term is added to the fitting criteria. A common penalty is a smoothing parameter and an integrated squared second derivative of the function $s_h$.

The estimation model of the hedonic function with GAMs is as follows:

$$
\log \frac{RP}{FS} = a_0 + \sum_h s_h(X_h) + \sum_i a_i \log Z_i + \sum_j a_{2j} \log BC_j + \sum_k a_{3k} \cdot LD_k + \sum_m a_{5m} \cdot TD_m + \varepsilon
$$

(5)

where $\log \frac{RP}{FS}$ is the identity link, and $s_h(X_h)$ is an unspecified smooth function for the main variable ($FS$, $Age$, $TS$, $TT$). The other terms are predictors ($\log Z, LD, RD, TD$) via a linear combination term with the parameters $a_2, a_3, a_4, a_5$.

As well, we employed the Modified Generalized Cross Validation (MGCV) algorithm of Wood (2006) for calculating the $s_h$ functions.\textsuperscript{13} The MGCV algorithm apply to the equation (5).

\begin{itemize}
  \item There are multiple characteristics that can be used as smoothing parameter selection criteria. Generalized cross-validation is one of these criteria (Wood, 2006, pp. 175-177).
  \item For example, when performing an approximation based on a cubic curve, the curve’s slope has three distinct sections. In this case, one must determine four knots representing the upper and lower limits of the section. If there are two knots, the splines will be linear; conversely, if there are many knots, the splines will resemble an interpolated line chart showing the data in detail. The form that the splines will take is dependent on the inclination of the slope between the knots, and by increasing their number, it is possible to increase the apparent fit.
  \item Hastie and Tibshirani (1990) proposed the backfitting algorithm to fit the smooth function $sh$. However, the stability of the backfitting algorithm had problems, particularly in datasets with high collinearity among the explanatory variables (Schimek 2009). Another limitation of the GAM estimator is the requirement to select a smoothing parameter (namely, the number of degrees of freedom). A more preferable approach is to determine the degree of smoothing of $sh$ in an endogenous way that depends on the examined data. The automatic selection of the smoothness criteria in the GAM model is possible with the Modified Generalized Cross Validation (MGCV) algorithm of Wood (2004). We applied the MGCV algorithm by using R software (R Development Core Team, 2009) with the MGCV library.
\end{itemize}
3. Assessment of Nonlinearity

3.1. Brief Summary of Estimation Results

**Base Model: Linear Model**

In this section, we estimate four models, the base model, *switching regression model* (SWR), *continuous dummy variable model* (DmM), and *generalized additive model* (GAM) using 2005 data.

First, we begin by estimating a base model, as shown in column (i) of Table 3. The construction cost per square meter is expected to diminish with an increase in condominium size, but if condominium size and grade are positively correlated, the unit resale price increases with the size of FS and the other related variables of “Balcony space” (BS) and “Number of units” (NU). This holds true for BS, because it is expected that BS tends to become larger with increasing condominium grade. Because of this, BS is considered to be positively estimated for the condominium scale. NU is a representative index showing the resale price of a condominium as a whole rather than the price of each unit. For example, because shared space tends to be ampler as NU increases, this space is considered to affect the condominium unit price.

In addition, convenience in commuting to offices or schools decreases as “time to the nearest station” (TS) increases. Because there are fewer shops and services and daily life is less convenient far from stations, condominium resale prices are expected to decrease. Furthermore, because, in general, more people commute to the CBD, not only commuting expenses but also commuters’ opportunity costs increase, which is thought to contribute to the decreased condominium prices.
Because there are not only the abovementioned factors that are specific to real estate but also broad disparities in the housing environment among administrative municipalities or areas along railway lines, which cannot be considered in the functions estimated in this paper, such disparities are estimated using the dummy variables.

With the base model as the starting point, it is modified to give the other estimation models below. In concrete terms, regarding the variables, except for those to be improved as instrumental variables, all those adopted for the base model are forcibly incorporated into the other models.

**Switching Regression Model**

In the function that was estimated as the base model, it was assumed that there was a simple linear relationship between the unit resale price and each variable. However, in actuality, it was difficult to assume that each variable had a simple linear relationship with the unit resale price.

We conducted a *switching regression model (SWR)* which is explained in Section 2.2 and found switching (break) points in four key variables, “Floor space” (FS), “Age of building” (Age), “Time to the nearest station” (TS), and “Travel time to the CBD” (TT).

In the individual index models (FS, Age, TS, and TT), two structurally different sections were estimated through an exploratory approach for FS, Age, TS, and TT. By extracting optimum switching (structural change) points for measurement using AIC as an assessment index, a structural change test was conducted using the F-test. Two structural change points were detected for FS, Age, and TS. It was found, however, that there was only one point of structural change for TT. (Refer to the Appendix.)

The model was formulated on the basis of equation (2) by coupling with the structurally changed sections extracted using the individual models. The estimation results using the model
are shown below. Column (iii) of Table 3 shows estimated coefficients as constant-term dummies and estimated statistics obtained as cross-terms.

**Continuous Dummy Variable Model**

An estimation was conducted using a *continuous dummy variable model* (DmM) in accordance with the model shown in equation (3). Here, in forming a dummy variable corresponding to each main variable, the problem was how to set its bandwidth ($\beta$). For example, it is unlikely that consumers change their preference based on 1 m$^2$ of "Floor space" ($FS$), and because of this, the bandwidth was set at $\beta = 5$. It was considered unproblematic to set $\beta = 1$ for the "Age of Building" ($Age$), "time to the nearest station" ($TS$), and "travel time to the CBD" ($TT$).

\[
Dm(FS_\rho) : \rho = 15,20,25,30,\ldots,135
\]
\[
Dm(Age_\sigma) : \sigma = 1,2,3,4,5,\ldots,35
\]
\[
Dm(TS_\varsigma) : \varsigma = 1,2,3,4,5,\ldots,30
\]
\[
Dm(TT_\tau) : \tau = 1,2,3,4,5,\ldots,30
\]

An Estimation result of DmM is shown in column (ii) of Table 3. The impacts of $FS$, $Age$, $TS$, and $TT$ change at each specified break point. The result is omitted because there are many dummy variables (87).

**A Generalized Additive Model**

Next, an estimation for a *generalized additive model* (GAM) was conducted. Equation (5) was set up as a semi-parametric regression model containing both parametric terms and non-parametric
terms for smoothing. In column (iv) of Table 3, the determination coefficient adjusted for the degrees of freedom was 0.814, a value that indicates that the estimated model has an explanatory power similar to those of DmM and SWR. Note that the explanatory powers of DmM, SWR, and GAM were improved equivalently compared with that of the base model.

Table 4 shows the estimated performance of the smoothing function by GAM. In column (i), no great difference was observed in the coefficients other than the smoothed parameters in comparison with the base model. The degrees of freedom of the smoothing term obtained in terms of the GCV were not integral. The F-value is a statistic that shows whether there was any difference between the effect of smoothed cases and that of non-smoothed cases, and it was indicated that smoothing produced a significant difference between the models.

3.2. Improvement in Explanatory Power

A comparison of the results obtained using the linear model with those obtained using the continuous dummy model (DmM), switching regression model (SWR), and generalized additive model (GAM) showed that the predictive power is improved by considering nonlinearity. In DmM, SWR, and GAM, the determination coefficients adjusted for the degrees of freedom are 0.816, 0.812, and 0.810, an improvement over the 0.775 for the base model. The estimation parameters related to structural disparities are, in general, accurately estimated.

We examined the predictive power using in-samples with different statistical value. For comparison of the hedonic models (the base model, DmM, SWR, and GAM) with in-samples, we use the “Residual Sum of Squares” (RSS) as an evaluation statistic of hedonic models.

The RSS of nonlinear regressions using the SWR (190.708), DmM (185.377), and GAM (188.708) are smaller than that of the base model (228.471) estimated with the linear model.
As a result of comparing the estimated hedonic models with the determination coefficient adjusted for the degrees of freedom, RSS, DmM, or GAM were found to be the most powerful hedonic models for estimating the Tokyo metropolitan condominium market.

3.3. Shape of Nonlinearity: Consistency among the Three Methods

A comparison of the estimated shapes of curves for the variables based on the results described above for a series of estimations revealed the following.

Firstly, we can see the nonlinear relationship between “Floor space” ($FS$) and unit prices. In small condominiums with $FS$ of approximately 20 m$^2$, the marginal effect on the unit resale price per m$^2$ area is high but gradually decreases. It was found, however, that when $FS$ is larger than 80 m$^2$, the marginal effect increases rapidly (see Figures 1 and 2). This tendency is shown by DmM, SWR, and GAM, but not by the base model. DmM, SWR, and GAM show the same tendency, so it can be said that this structure is stable. It is seen that in the base model, the relationship between $FS$ and unit resale prices can be estimated as a monotonically increasing function, and inaccurate prices can be obtained if the nonlinearity is not taken into account. The theoretical values for smoothing terms in GAM were normalized so that their total was 0.

This tendency of the relationship is considered to be attributable to differences in the characteristics and thickness of the market. First, condominiums of approximately 20 m$^2$ for single-person households are often purchased as investments, whereas purchasers are increasingly likely to be the persons living in the condominiums as condominium size increases. Also, regarding construction costs, equipment such as kitchen and restroom equipment (the construction cost of which is significant) affects the area per unit price. Hence, construction cost
per unit tends to increase as the area decreases. For these reasons, it is thought that the unit resale prices of condominiums with small areas tend to become higher.

The average $FS$ is 61.82 m$^2$. Condominiums of approximately 55 to 70 m$^2$ for families are supplied in large numbers in Japan, and the market becomes thinner as the condominium area increases. Because of this, as the area of a condominium increases, a premium is expected to be placed on units having $FS$ above a certain value.

Next, attention was paid to the relationship between $Age$ and unit resale prices. It was indicated that in the relationships between price reduction based on the values estimated using the base model formulated as a simple linear structure and those estimated using the three models, DmM, SWR, and GAM, greater discrepancies were observed in the price as the age departed from the average value (16.51 years). Regarding DmM, SWR, and GAM, we found that prices increased from around 12 years from construction and then decreased after 23 years from construction (see Figures 3 and 4).

Reasons for the above trend may be that large-scale repairs are first required ten years after construction. In addition, similar large-scale repairs are required around 20 years after construction, that is, ten years after the first repairs. Because the rate of depreciation is particularly rapid from ten to 20 years after construction and because the building price value subsequently diminishes, the ratio of the land value increases in condominiums (condominium value is equal to land value and building value), so the depreciation proportion is expected to decrease.

A comparison of the results obtained using the linear model with those obtained using DmM, SWR, and GAM showed that the price structure yielded by the linear model differed greatly from those given by DmM, SWR, and GAM, and that, particularly in the case of the base model, as the price structure departed from the average price, greater discrepancy was observed.
Analysis of the relationship between “time to the nearest station” (TS) and unit resale prices showed that the price gradient increased slightly when TS was more than 12 minutes, and that it declined rapidly when TS was more than 17 minutes.

The above tendency was also observed in the cases of DmM, SWR, and GAM (see Figures 5 and 6). Hence, when estimation was conducted using a linear model, price differences increased when previously owned condominiums were further from the nearest station. First, prospective buyers selecting a condominium site showed a high preference for transportation convenience, and when TS was more than ten minutes, the price declined. Because the analysis was made only for the walking time in this study, it is seen that there is a limiting point when TS is more than ten minutes. A greater price decline is observed when TS exceeds 17 minutes. This is the limit for the perceived accessibility to the nearest station on foot, and if TS exceeds this limit, access to the nearest station is thought to be by an alternative means such as bicycle, bus, or car.

Analysis of the relationship between TT and unit resale prices showed that there was only one point of structural change (when TT was more than 15 minutes) at which the prices of previously owned condominiums declined rapidly. A similar tendency was also detected using DmM (β = 1) and SWR. In other words, previously owned condominium resale price levels did not differ from one another within an area of about ten minutes from any one of the seven stations set as CBDs, but upon reaching a travel time of about 15 minutes, the prices declined. When using DmM, SWR, and GAM, but not the linear model, previously owned condominium resale price levels are estimated to increase slightly as the travel time increases up to about ten minutes (see Figures 7 and 8). It seems more appropriate to consider this as no price change, rather than as an increased price, because of the great impact of estimation errors resulting from small changes in price
levels. However, previously owned condominium prices declined when $TT$ increased by more than 15 minutes.

The above analysis indicates that if estimation were to be conducted using the linear model, previously owned condominium resale price levels would appear to decline more slowly away from CBDs, and for condominiums farther from CBDs, greater errors would arise. In GAM, which was affected by the large number of ten-minute samples, it was implicitly estimated that prices of previously owned condominiums first increased, then declined rapidly, then increased when $TT$ was ten to 15 minutes, and ultimately declined again. This suggests that the use of GAM may result in errors if the data used for GAM distribution are discontinuous.

4. Predictive Power of the Nonlinear Estimation Methods

4.1. Out-of-Sample Tests: Accuracy of Predictions

In the in-samples test with RSS, the nonlinear regressions are better than the base model and the continuous dummy variable model (DmM) and general additive model (GAM) were found to be the most powerful hedonic models for estimating the Tokyo metropolitan condominium market.

Next, we conducted an examination using out-of-samples data. The Residual Sum of Squares (RSS) value of DmM was the smallest of the four models. To measure the quality of the fit, we conducted an out-of-sample prediction analysis. The estimation results in Table 3 are housing data (number of observations: 9,682) concerning transactions conducted in 2005.

Table 2 presents the summary statistics for the 2003-2004 data (number of observations: 17,913) and 2006 data (number of observations: 11,877) used in the out-of-sample test, along with the 2005 data.
With regard to the out-of-sample test, we verified predictive accuracy by repeatedly and randomly selecting the same amount of data as the in-sample data used to calculate the four models (9,682) from the above data and observing the differences between the actual prices and the forecast prices calculated from the four models.

Specifically, in the linear model estimation, a forecast price \( \hat{y}_n = x_n^\prime \hat{a} \), where \( x_n \) is the explanatory variables vector in the \( n \)th house of data in 2003/2004 and 2006 (out-of-sample in multi-period) and \( \hat{a} \) is the estimated coefficients vector (from 2005 data; shown in Table 2).

In this way, based on the “estimated forecast prices” and “actual prices”, we calculated two evaluation indexes: the mean absolute percent error (MAPE) and the symmetric mean absolute percent error (SMAPE) (Bin (2004); Ramazan and Yang (1996)). These were defined as follows:

\[
MAPE_n = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{|y_n - \hat{y}_n|}{y_n} \right) \cdot 100\
\]

\[
SMAPE_n = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{|y_n - \hat{y}_n|}{(y_n + \hat{y}_n)/2} \right) \cdot 100\
\]

where \( y_n \) is actual price of data in 2003-2004 and 2006 (out-of-sample). For each index, if the forecast price matches the actual price, the value is 0; the greater the discrepancy between the two, the greater the index value becomes. These calculations were repeated 500 times for each index, and Table 5 shows their distribution.

First, if we compare the MAPE and SMAPE for 2003 and 2004 with 2006, the values are relatively high for 2003 and 2004. This difference occurs because we pooled data over a two-year period covering 2003 and 2004, which meant that temporally distant data were included. As a result, it is not possible to make a simple comparison between the two periods, so we focused on the relative difference between the base model (linear model) of the respective tests.
In analysis focusing on 2003 and 2004, the linear model had the worst predictive power, and the more nonlinearity was taken into account, the smaller predictive errors tended to become (in the following order: SWR, GAM, DmM).

On the other hand, the results for 2006 show that the linear model and DmM had almost the same predictive accuracy, while predictive accuracy was poor with GAM and particularly SWR.

4.2. Discussion

When we verify predictive accuracy using out-of-sample data, why do the results vary based on the time period? Why do significant predictive errors occur in 2006 with GAM and SWR in particular, even though in 2003-2004 the results are the same as for the in-samples test?

We believe it is due to the problems of temporal sample selection bias and structural change, which we had anticipated.

A problem with the linear model is that is not possible to estimate characteristics toward the end of the distribution. On the other hand, with calculations that take nonlinearity into account, while it is possible to properly estimate characteristics at the end of the distribution, once a change in the data distribution occurs, errors end up being magnified because they are overly responsive to such characteristics.

This research covers the period from the lengthy recession brought about by the collapse of the 1980s Bubble to the time when prices started to rise again. Specifically, prices were declining in 2003-2004, and they hit bottom in 2004. In 2005, prices began to rise, and 2006 saw them
increase at a high rate. In other words, there were significant structural changes between 2003-2004, 2005, and 2006.\textsuperscript{14}

In this kind of situation, it is possible that the distribution of transactions that took place changed significantly.\textsuperscript{15}

In particular, when there are significant structural changes following macro-level shocks, nonlinearity conditions change due to changes in the distribution of transactions. As a result, since the transaction distribution for 2003-2004 was the same as the distribution for 2005, the same structure was observed for the predictive accuracy of prices with in-sample data as with out-of-sample data. In 2006, however, when prices began to rise, it is highly probable that predictive errors were magnified by taking nonlinearity into account.

5. Conclusions

It has been pointed out that, in many countries, the housing market is not homogeneous but rather is a heterogeneous market. As a result, in recent years it has been noted that nonlinearity should be taken into account in hedonic model estimates.

All previous studies comparing linear models and nonlinear models have reported that taking nonlinearity into account increases a model's explanatory power.

However, these previous studies performed in-sample and out-of-sample tests using separate data from the same point in time. Moreover, in these studies, the calculations for estimating models that took nonlinearity into account were difficult, and there were also not many data available: for in-sample data, models were estimated using 1,000 samples or less, while

\textsuperscript{14} We performed an F-test-based structural change test, which showed structural change at a 1% level of significance for each.
\textsuperscript{15} Shimizu, Nishimura, and Watanabe (2010) point out that in periods when prices are rising, repeat sales samples (properties that are resold on the market multiple times) tend to increase.
predictive accuracy was verified using only a few hundred out-of-sample data items. For our study, we constructed a unique dataset of around 40,000 items focusing on the area of Tokyo's 23 wards. In addition to estimating models using around 10,000 in-sample data items, we verified their predictive accuracy using approximately 30,000 out-of-sample items at various points in time before and after the period for which the model was estimated.

First, with regard to verification of in-sample predictive accuracy, in comparison to a linear regression model, both the coefficient of determination adjusted for degrees of freedom and the RSS improved with each of SWR, DmM, and GAM. Furthermore, when we verified the estimation results by plotting graphs for the four key parameters taking nonlinearity into account, we obtained almost identical results (i.e., similar slopes) for each of the key parameters of the three models that accounted for nonlinearity. What’s more, the slopes obtained from the three models taking nonlinearity into account diverged considerably from the slope of the linear regression. On this basis, with regard to in-sample data, the findings strongly support the importance of taking nonlinearity into account.

Next, we verified predictive accuracy using out-of-sample data. In Japan, the hedonic model is a built-in element of many policies. For example, cost-benefit analysis is obligatory when implementing urban (re)development projects, and it is the hedonic method that is used to evaluate their benefit. Financial institutions and the Financial Services Agency use an auto appraisal system to evaluate mortgage risk, with the hedonic method also being used for the estimates in this model. In addition, the government is promoting the development of a housing price index, and here as well the hedonic method is expected to be used as the estimation method.

With regard to the hedonic model used in these economic policies, there is a need for an accurate, time-oriented model, in order to estimate models using data obtained in the past and use them in policies employing the estimated parameters.
In this research, we estimated a linear model and three models taking nonlinearity into account using 2005 data, and tested their predictive accuracy with respect to a 2003-2004 data group and a 2006 data group. The results we obtained show that for 2003-2004, as with the in-samples test, predictive accuracy was greater for the models taking into account nonlinearity in comparison to the linear model. However, in 2006, the opposite result was obtained: taking into account nonlinearity significantly lowered predictive accuracy. It was determined that the reason for this is that in periods when structural changes occur, taking nonlinearity into account tends to amplify errors.

To date, no other research on this issue has performed time-oriented out-of-sample tests of predictive accuracy with such large-scale data. We thus believe this study has provided new insights into the practical application of hedonic-related research.
References


Appendix: Estimation of Switching Regression

A-1. Estimation of Switching (Structural Change) Points by AIC

Assuming that points of price structure change exist in the relationship between unit resale price and condominium characteristics, structural estimation was carried out using the *switched regression model* (SWR). Here, on the assumption that there were two such points in each relationship, these change points were explored. Under ordinary circumstances, \( l \) and \( m \) were set for every main variable \( X_h \). Because it was difficult to optimize them simultaneously, optimization was carried out for each variable using the base model as a starting point. A model assessment was performed on the basis of Akaike’s information criterion (AIC).

To confirm whether a structural change occurred in the detected \( l \) and \( m \) following the above model estimation, a structural change test was conducted using the F-test.

*Estimation Results for “Floor space” (FS)*

\( FS \) was changed by units of 5 m\(^2\) for consistency with DmM. The ranges of combinations of \( l \) and \( m \) in \( Dm_{(l>5, l<h)} \) and \( Dm_{(m<5, l<h)} \) were \( l > 15, m < 135, \) and \( l < m \), and there were 253 combinations. By estimating all the 253 combinations of 253 functions, their AIC values were compared. Estimation results showed that AIC was minimized at \( l = 40 \) and \( m = 90 \), and the determination coefficient adjusted for the degrees of freedom was 0.779, showing an improvement in the explanatory power. Figure 1 shows the combinations of \( l \) and \( m \) and changes in AIC.
Estimation Results for “Age of Building” (Age)

On the basis of the distribution of data on Age, the range of analysis was from more than one year to 35 years. The range of combinations of \( l \) and \( m \) in \( Dm_{(l(0<,Xh<mh)<)} \) and \( Dm_{(mh<,Xh)} \) were \( l > 2, \ m < 35, \) and \( l < m \), and there were 561 combinations. By estimating all the 561 combinations of 561 functions, their AIC values were compared. Estimation results showed that AIC was minimized at \( l = 12 \) and \( m = 23 \), and the determination coefficient adjusted for the degrees of freedom was 0.801, showing an improvement in the explanatory power compared with that of the base model. Figure 1 shows the combinations of \( l \) and \( m \) and changes in AIC.

Estimation Results for “Time to the Nearest Station” (TS)

On the basis of the distribution of data on TS, the range of analysis was from more than one minute to 30 minutes. The ranges of combinations of \( l \) and \( m \) in \( Dm_{(l(h<,Xh<mh)} \) and \( Dm_{(mh<,Xh)} \) were \( l > 2, \ m < 30, \) and \( l < m \), and there were 300 combinations. By estimating all the 300 combinations of 300 functions, their AIC values were compared. Estimation results showed that AIC was minimized at \( l = 12 \) and \( m = 17 \), and the determination coefficient adjusted for the degrees of freedom was 0.777, showing an improvement in the explanatory power compared with that of the base model. Figure 1 shows the combinations of \( l \) and \( m \) and changes in AIC.

Estimation Results for “Travel Time to the CBD” (TT)

On the basis of the distribution of data on TT, the range of analysis was from more than 0 minutes to 30 minutes. The ranges of combinations of \( l \) and \( m \) in \( Dm_{(l(h<,Xh<mh)} \) and \( Dm_{(mh<,Xh)} \) were \( l \geq 1, \ m < 30, \) and \( l < m \), and there were 406 combinations. By considering all the 406 combinations of 406 functions, their AIC values were compared. Estimation results showed that AIC was
minimized at \( l = 11 \) and \( m = 15 \), and the determination coefficient adjusted for the degrees of freedom was 0.777, showing an improvement in the explanatory power compared with that of the base model. Figure 1 shows combinations of \( l \) and \( m \) and changes in AIC.

A-2. Confirmation by Structural Change Test

Optimum models were selected from the possible combinations in the above estimation. However, there was no evidence that structural change occurred in the sections extracted here. To demonstrate the presence of this change, a structural change test (an F-test) was conducted (Table 3).

Specifically, the three groups divided by \( l \) and \( m \) in \( Dm_{(l \leq h, h < m)} \) and \( Dm_{(m \leq h, h < m)} \) were subjected to an F-test. Group I was set as \( X_{(h<l)} \), group II was set as \( X_{(c \leq h < m)} \), and group III was set as \( X_{(m<h)} \). Three tests were conducted, between group I and group II, group II and group III, and group I and group III, for each variable (\( FS, Age, TS \), and \( TT \)). It is particularly important to verify whether or not there was a structural change between group I and group II, and group II and group III. If a structural change can be verified, then there is a nonlinear relationship between the unit resale price and each variable. If the F-test detects structural change between group I and group II and between group II and group III, but not between group I and group III, then the structure is different only within \( l < h < m \).

The results of the structural change test showed that a structural change occurred at the two previously determined values of \( l \) and \( m \) for \( FS, Age \), and \( TS \) with a significant difference of 10%. For the \( TT \), no structural change was observed between group I and group II, but a structural change was found to exist between group II and group III. A structural change was also observed.
between group I and group III. These findings indicated that the structure changed only for $m = 15$ minutes or more.
Table 1. List of analyzed data

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Variables</th>
<th>Contents</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>Resale price of condominium</td>
<td>Resale price in last week listed in a housing information magazine</td>
<td>10,000 yen</td>
</tr>
<tr>
<td>FS</td>
<td>Floor space</td>
<td>Floor space</td>
<td>m²</td>
</tr>
<tr>
<td>Age</td>
<td>Age of building: Number of years since construction</td>
<td>Period between the date when the data is deleted from the magazine and the date of construction of the building</td>
<td>year</td>
</tr>
<tr>
<td>TS</td>
<td>Time to the nearest station</td>
<td>Distance in time (walking time) to the nearest station</td>
<td>minute</td>
</tr>
<tr>
<td>TT</td>
<td>Travel time to the CBD</td>
<td>Minimum railway riding time in the daytime to seven terminal stations in 2005*.</td>
<td>minute</td>
</tr>
<tr>
<td>BS</td>
<td>Balcony space</td>
<td>Balcony space.</td>
<td>m²</td>
</tr>
<tr>
<td>NU</td>
<td>Number of units</td>
<td>Total units of the condominium.</td>
<td>unit</td>
</tr>
<tr>
<td>TM</td>
<td>Time on the market</td>
<td>Period between the date when the data appear in the magazine for the first time and the date when they are deleted.</td>
<td>week</td>
</tr>
<tr>
<td>FF</td>
<td>First floor dummy</td>
<td>The property is on the ground floor 1, on other floors 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>HF</td>
<td>Highest floor dummy</td>
<td>The property is on the top floor 1, on the other floors 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>SD</td>
<td>South-facing dummy</td>
<td>Window facing south 1, other directions 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>FD</td>
<td>Ferroconcrete dummy</td>
<td>Steel reinforced concrete frame structure 1, other structure 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>LDj (j=0,…,J)</td>
<td>Location (Ward) dummy</td>
<td>j th administrative district 1, other district 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>RDk (k=0,…,K)</td>
<td>Railway line dummy</td>
<td>k th railway line 1, other railway line 0.</td>
<td>(0,1)</td>
</tr>
<tr>
<td>TDl (l=0,…,L)</td>
<td>Time dummy (quarterly)</td>
<td>l th quarter 1, other quarter 0.</td>
<td>(0,1)</td>
</tr>
</tbody>
</table>

*Terminal stations: Tokyo, Shinagawa, Shibuya, Shinjuku, Ikebukuro, Ueno and Otemachi
Table 2. Summary statistics of the variables

<table>
<thead>
<tr>
<th></th>
<th>2003/2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RP</strong>: Resale price of condominium</td>
<td>3264.44</td>
<td>3253.89</td>
<td>3378.73</td>
</tr>
<tr>
<td>(10,000 yen)</td>
<td>(-2348.29)</td>
<td>(-1858.83)</td>
<td>(-2314.76)</td>
</tr>
<tr>
<td><strong>FS</strong>: Floor space (m²)</td>
<td>63.38</td>
<td>61.82</td>
<td>62.29</td>
</tr>
<tr>
<td></td>
<td>(-26.74)</td>
<td>(-19.83)</td>
<td>(-22.97)</td>
</tr>
<tr>
<td><strong>Age</strong>: Age of building (year)</td>
<td>16.69</td>
<td>16.51</td>
<td>18.19</td>
</tr>
<tr>
<td></td>
<td>(-10.23)</td>
<td>(-9.92)</td>
<td>(-11.09)</td>
</tr>
<tr>
<td><strong>TS</strong>: Time to the nearest station (minute)</td>
<td>7.68</td>
<td>7.45</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>(-4.29)</td>
<td>(-4.19)</td>
<td>(-4.29)</td>
</tr>
<tr>
<td><strong>TT</strong>: Travel time to the CBD (minute)</td>
<td>10.35</td>
<td>14.83</td>
<td>10.52</td>
</tr>
<tr>
<td></td>
<td>(-6.71)</td>
<td>(-5.23)</td>
<td>(-6.82)</td>
</tr>
<tr>
<td><strong>BS</strong>: Balcony space (m²)</td>
<td>7.88</td>
<td>8.14</td>
<td>8.13</td>
</tr>
<tr>
<td></td>
<td>(-6.39)</td>
<td>(-5.96)</td>
<td>(-6.19)</td>
</tr>
<tr>
<td><strong>NU</strong>: Number of units (unit)</td>
<td>89.7</td>
<td>88.03</td>
<td>87.92</td>
</tr>
<tr>
<td></td>
<td>(-125.38)</td>
<td>(-122.48)</td>
<td>(-124.85)</td>
</tr>
<tr>
<td><strong>TM</strong>: Time on the market (minute)</td>
<td>10.68</td>
<td>9.33</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>(-9.66)</td>
<td>(-8.37)</td>
<td>(-8.48)</td>
</tr>
</tbody>
</table>

Number of Observations: 18794 9682 11877

( ): standard deviation
Table 3. Estimation results of base model, DmM, SWR, and GAM

<table>
<thead>
<tr>
<th>(i) Base</th>
<th>(ii) SWR</th>
<th>(iii) DmM</th>
<th>(iv) GAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef.</td>
<td>t-value</td>
<td>Coef.</td>
<td>t-value</td>
</tr>
<tr>
<td>log(FS)</td>
<td>0.047</td>
<td>8.984</td>
<td>-0.094</td>
</tr>
<tr>
<td>log(Age)</td>
<td>-0.188</td>
<td>-96.379</td>
<td>-0.086</td>
</tr>
<tr>
<td>log(TS)</td>
<td>-0.054</td>
<td>-21.510</td>
<td>-0.046</td>
</tr>
<tr>
<td>log(TT)</td>
<td>-0.017</td>
<td>-5.237</td>
<td>-0.009</td>
</tr>
</tbody>
</table>

Dm (Xh)    Yes
DM_FS (40 ≤ FS < 90) 0.387 5.149
DM_Age (12 ≤ Age < 23) 0.579 11.733
DM_TS (12 ≤ TS < 17) 0.216 2.130
DM_FS (90 ≤ FS) -1.374 -6.058
DM_Age (23 ≤ Age) -0.109 1.521
DM_TS (17 ≤ TT) 0.458 10.901
log(FS)* -0.110 5.188
log(Age)* 0.579 11.733
log(TS)* 0.216 2.130
log(FS)* -1.374 -6.058
log(Age)* -0.109 1.521
log(TS)* 0.458 10.901
log(FS)* 0.110 5.188
log(Age)* -0.241 -14.059
log(TS)* -0.099 -2.522
log(FS)* 0.339 6.711
log(Age)* -0.106 -4.831
log(TS)* -0.296 -2.993
log(TT)* -0.163 -11.236

s(Xh)    –    –    Yes
BS 0.012 4.471
NU 0.020 10.190
TM -0.006 -3.331
FF -0.034 -6.198
HF 0.054 5.365
FD -0.012 -3.226
SD 0.003 0.965
LD Yes
RD Yes
TD Yes
Const. 3.931 155.275

Residual Sum of Squares 228.471 190.708 185.337 188.166
Adj. R² 0.775 0.812 0.816 0.810
Number of Observations 9,682 9,682 9,682 9,682
Table 4. Estimated smoothing parameters

<table>
<thead>
<tr>
<th></th>
<th>Estimated d.f</th>
<th>F-statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{FS}$</td>
<td>7.573</td>
<td>33.41</td>
<td>0.000</td>
</tr>
<tr>
<td>$s_{Age}$</td>
<td>8.518</td>
<td>1366.12</td>
<td>0.000</td>
</tr>
<tr>
<td>$s_{TS}$</td>
<td>7.779</td>
<td>96.77</td>
<td>0.000</td>
</tr>
<tr>
<td>$s_{TT}$</td>
<td>8.983</td>
<td>26.72</td>
<td>0.000</td>
</tr>
</tbody>
</table>

GCV score: 0.019

Deviance explained: 81.60%

Number of Observations: 9,682

(Note: GCV score is an indicator of the error of the generalized cross-validation method, and the ‘deviance explained’ is an index of the applicability of the theoretical figure to the actual performance.)

Table 5. The mean absolute percent error (MAPE) and the symmetric mean absolute percent error (SMAPE)

<table>
<thead>
<tr>
<th></th>
<th>MAPE</th>
<th>SMAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>s.d.</td>
</tr>
<tr>
<td>Base</td>
<td>25.457</td>
<td>0.081</td>
</tr>
<tr>
<td>SWR</td>
<td>21.566</td>
<td>0.084</td>
</tr>
<tr>
<td>DmM</td>
<td>15.519</td>
<td>0.073</td>
</tr>
<tr>
<td>GAM</td>
<td>19.079</td>
<td>0.079</td>
</tr>
</tbody>
</table>

* The experiment is repeated 500 times per $t-1$[year2003&2004], $t+1$[year2006], $t$=year2005

Table A-1. Test results for structural change test (Prob>0)

<table>
<thead>
<tr>
<th></th>
<th>I vs. II</th>
<th>II vs. III</th>
<th>I vs. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{(h&lt;l)}$ vs. $X_{(i&lt;h&lt;=m)}$</td>
<td>0.00003</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>$X_{(i&lt;h&lt;=m)}$ vs. $X_{(m&lt;h)}$</td>
<td>0.00179</td>
<td>0.08101</td>
<td>0.05582</td>
</tr>
<tr>
<td>$X_{(h&lt;l)}$ vs. $X_{(m&lt;h)}$</td>
<td>0.00000</td>
<td>0.00001</td>
<td>0.01115</td>
</tr>
</tbody>
</table>

FS: Floor space (m$^2$)

<table>
<thead>
<tr>
<th></th>
<th>I vs. II</th>
<th>II vs. III</th>
<th>I vs. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age: Age of building (months)</td>
<td>0.00179</td>
<td>0.08101</td>
<td>0.05582</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I vs. II</th>
<th>II vs. III</th>
<th>I vs. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>TS: Time to the nearest station (minutes)</td>
<td>0.00000</td>
<td>0.00001</td>
<td>0.01115</td>
</tr>
</tbody>
</table>

TT: Travel time to the CBD (minutes)

<table>
<thead>
<tr>
<th></th>
<th>I vs. II</th>
<th>II vs. III</th>
<th>I vs. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT: Travel time to the CBD (minutes)</td>
<td>0.22236</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>
Figure 1. Relationship between “Floor space” ($FS$) and unit resale prices 1: OLS, DmM, and SWR

Figure 2. Relationship between “Floor space” ($FS$) and unit resale prices 2: GAM
Figure 3. Relationship between “Age of building” (Age) and unit resale prices 1: OLS, DmM, and SWR

Figure 4. Relationship between “Age of building” (Age) and unit resale prices 2: GAM
Figure 5. Relationship between “Time to the nearest station” (TS) and unit resale prices 1:

OLS, DmM, and SWR

Figure 6. Relationship between “Time to the nearest station” (TS) and unit resale prices 2:

GAM
Figure 7. Relationship between “Travel time to the CBD” (TT) and unit resale prices 1 OLS, DmM, and SWR

Figure 8. Relationship between “Travel time to the CBD” (TT) and unit resale prices 2: GAM
Figure A-1. AIC/three segments in SWR